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EFFECT OF THE SPECIMEN SIZE ON NECKING DEVELOPMENT IN METALS AND ALLOYS DURING SUPERPLASTIC DEFORMATION

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Abstract. A model is proposed that describes the development of individual and multiple necks in superplastically deformed materials. Within the model, the examined samples have the form of round bars and are subjected to tensile superplastic deformation without strain hardening. It is demonstrated that neck development and necking-induced failure occur faster with a decrease in strain rate sensitivity and/or an increase in the fraction of the sample length occupied by necks. This means that high values of strain to failure observed in small specimens of superplastically deformed ultrafine-grained metals and alloys, where diffuse necking happens in the whole specimen, can be significantly reduced in larger specimens where the necking regions occupy only a small part of the sample.
Keywords: superplastic deformation, necking, ductility, failure, ultrafine-grained materials

1. Introduction

It is known that the ductility of metals and alloys under superplastic deformation is often limited by cavitation or diffuse necking (e.g., [1]). In particular, diffuse necking is often observed in ultrafine-grained (ufg) alloys demonstrating superplastic deformation or superplastic behavior (e.g., [2,3]). Due to the difficulty in making nanostructured materials large enough for standard mechanical testing, to measure the mechanical properties of such materials, many researchers have been using small samples [4]. In such samples the neck, if it appears, propagates over the entire sample length and represents a gradual decrease in the sample thickness from the sample edges to its center. This is in contrast to the case of large samples, where a neck during superplastic deformation can occupy only a part of the sample length, although multiple necking can occur [5-7].

At the same time, recent investigations [4,8,9] demonstrated that the sample length can have a significant effect on the ductility of metals. In particular, it was experimentally demonstrated that the gauge length can affect the onset of necking and postnecking behaviour of ufg and coarse-grained Cu [8,9]. The gauge length effect on the ductility of Cu has been attributed [4,9] to the difference in the fractions of the specimen length occupied by the neck in small and large samples. However, no relation has been established that would allow one to estimate the speed of the neck development and strain to failure in large samples based on the corresponding parameters in small samples and the strain rate sensitivity of specimens. To fill this gap, here we suggest a model that describes the development of individual and multiple necks in superplastically deformed materials and calculate the effect of the specimen length on the ductility of such materials.

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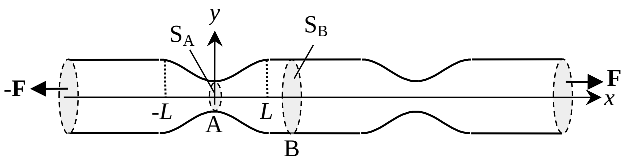

Fig. 1. Deformed bar with multiple necks

2. Model

Consider a plastically deformed round bar under a uniaxial tension (Fig. 1). Assume that plastic deformation of the bar is accompanied by the formation of identical multiple necks that develop during plastic deformation. We denote the fraction of the specimen length occupied by necks as f_{neck}. In considering plastic deformation of the bar, we will use the long wavelength approximation [10-13]. This approximation assumes that the stress state and strain rate are the same at every cross section of the bar, and the stress state represents uniaxial tension σ. This approximation neglects the Bridgman effect [14] associated with the formation of hydrostatic tension, in addition to uniaxial tension, in the neck regions. However, the comparison of the long wavelength approximation with more refined models [10] demonstrated that in the case of high strain rate sensitivity (which is realized in the examined situation of superplastic deformation), the relatively simple long-wavelength approximation becomes very accurate. Therefore, we will use this approximation in further analysis.

3. Calculation of necking development during superplastic deformation

Within this approximation, we will expand the approach adopted by [10] for the analysis of a bar with a single neck to the examined case of multiple necks. To do so, we consider two cross sections, A and B, of the bar. The true stresses, σ_A and σ_B, in these cross sectional areas, S_A and S_B, are related as

$$\sigma_A S_A = \sigma_B S_B = F, \tag{1}$$

where F is the magnitude of the tensile force acting on the bar (Fig. 1). We then consider one of the neck regions in the bar and suppose that there is an initial nonuniformity. Let us introduce the coordinate system (x, y) with the origin in the center of the neck region (Fig. 1) and denote the length of the neck as $2L$ (so that the neck occupies the region $-L \le x \le L$). Then the variation of the initial cross sectional area $S^i(x)$ in the neck region is assumed to be [10]

$$S^i(x) = S_B^i f(\tilde{x}), \tag{2}$$

where S_B^i is the area of the cross section in the regions without necking, and $\tilde{x} = x / L$. Following [10], we take the initial imperfection of the cross sectional area in the form

$$f(\tilde{x}) = 1 - \eta \frac{1 + \cos(\pi \tilde{x})}{2}, \tag{3}$$

where $\eta > 0$ is the imperfection amplitude. For simplicity, in considering superplastic deformation or superplastic behavior, we focus on the common situation where strain hardening is absent. In this case, the true strain σ does not depend on true strain ε but depends on strain rate $\dot{\varepsilon}$. We also employ the model strain-rate-dependent constitutive law (e.g., [10])

$$\sigma = \sigma_R (\dot{\varepsilon} / \dot{\varepsilon}_R)^m, \tag{4}$$

where σ_R and $\dot{\varepsilon}_R$ are the reference stress and strain rate, respectively. The true strain ε is related to the engineering strain ε_e as $\varepsilon_e = e^{\varepsilon} - 1$, which yields: $\dot{\varepsilon}_e = e^{\varepsilon}\dot{\varepsilon}$. From the condition of the volume conservation during plastic deformation, we have: $S(1+\varepsilon_e) = S^i$, which can be rewritten as $S = S^i e^{-\varepsilon}$. Substitution of the latter relation to formulae (1)–(4) gives:

$$e^{-\varepsilon/m}\dot{\varepsilon} = f(\tilde{x})^{-1/m} e^{-\varepsilon_B/m}\dot{\varepsilon}_B . \tag{5}$$

From (5), we have:

$$\int_0^{\varepsilon} e^{-\varepsilon'/m}\, d\varepsilon' = f(\tilde{x})^{-1/m} \int_0^{\varepsilon_B} e^{-\varepsilon_B'/m}\, d\varepsilon_B' , \tag{6}$$

which yields:

$$\varepsilon = -m \log[1 - f(\tilde{x})^{-1/m}(1 - e^{-\varepsilon_B/m})] . \tag{7}$$

From (7) the stretch λ (defined as $\lambda = \varepsilon_e + 1 = e^{\varepsilon}$) can be related to the stretch λ_B at cross section B as

$$\lambda = [1 - f(\tilde{x})^{-1/m}(1 - \lambda_B^{-1/m})]^{-m} . \tag{8}$$

The average stretch of the bar λ_{av} (defined as the ratio of the lengths of the strained and unstrained bar) can be obtained from (8) as

$$\lambda_{av} = \lambda_B(1 - f_{neck}) + f_{neck}\int_0^1 [1 - f(\tilde{x})^{-1/m}(1 - \lambda_B^{-1/m})]^{-m}\, d\tilde{x} . \tag{9}$$

To analyze the development of necking with plastic deformation, we calculate the ratio Δ of the cross sectional area, S_A, at the center of the neck to the cross sectional area, S_B, of the regions without necking. The ratio Δ follows as

$$\Delta = \frac{S_A}{S_B} = \frac{\sigma_B}{\sigma_A} = \frac{\dot{\varepsilon}_B^m}{\dot{\varepsilon}_A^m} . \tag{10}$$

To relate the strain rates appearing on the right hand side of formula (10) to the stretch λ_B, using (7), we express the strain rate $\dot{\varepsilon}$ as

$$\dot{\varepsilon} = \frac{f(\tilde{x})^{-1/m} e^{-\varepsilon_B/m}}{1 - f(\tilde{x})^{-1/m}(1 - e^{-\varepsilon_B/m})}\dot{\varepsilon}_B . \tag{11}$$

Substituting (11) to (10) and using the relations $\lambda_B = e^{\varepsilon_B}$ and $\varepsilon_A = \varepsilon(\tilde{x} = 0)$, one obtains:

$$\Delta = [1 - (1-\eta)^{-1/m}(1 - \lambda_B^{-1/m})]^m (1-\eta)\lambda_B . \tag{12}$$

4. Results and discussion

Using formulae (9) and (12), we plot the dependencies of Δ on the average engineering strain $\varepsilon_e^{av} = \lambda_{av} - 1$. These dependencies are shown in Fig. 2, for $\eta = 0.01$ and various values of the parameters m and f_{neck}. Figure 2 demonstrates the known and intuitively evident fact that the development of necking (characterized by a decrease of Δ from 1 (when necking is absent) down to zero (when necking leads to failure) occurs faster for the materials with smaller values of strain rate sensitivity m.

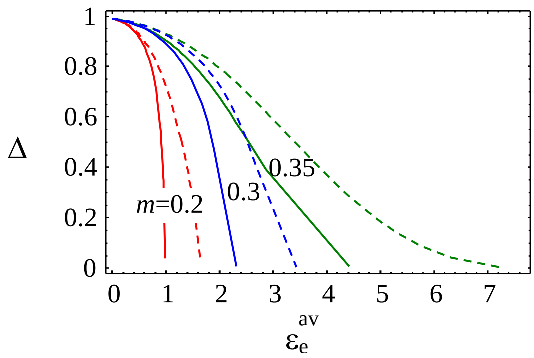

Fig. 2. Dependencies of the ratio Δ of the cross sectional area at the neck center to that in the regions without necks on the engineering strain ε_e^{av} for a plastically deformed round bar, at various values of strain rate sensitivity m and two different values of the fraction f_{neck} of neck regions. The solid lines correspond to the case $f_{neck} = 0.2$ while the dashed lines depict the situation where $f_{neck} = 1$

Also, in the initial stage of necking development (characterized by $\Delta > 0.9$), for a specified engineering strain ε_e^{av}, the parameter Δ is nearly independent of f_{neck}. At the same time, when necking becomes pronounced (that is, Δ becomes sufficiently small), the development of necking occurs at smaller strains for a smaller value of the fraction f_{neck} of necking regions. This effect becomes especially pronounced for high values of strain rate sensitivity m. If we assume that in small specimens there is only one neck, which occupies the whole length of the bar (that is, $f_{neck} = 1$), while in larger ones necks occupy only a small part of the bar, we can conclude from the above than in large specimens necking develops faster (that is, at smaller average strain) than in small ones.

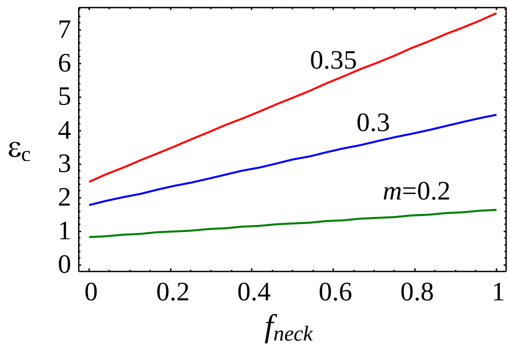

Fig. 3. Dependencies of the engineering strain to failure ε_c for a plastically deformed round bar on the fraction of neck regions f_{neck}, for various values of strain rate sensitivity m

Now calculate the engineering strain to failure for the plastically deformed bar that fails by necking. The critical value λ_c of the average stretch λ_{av} at which necking leads to failure is obtained from the relation $\Delta = 0$ and formula (12) as [10]

$$\lambda_c = [1-(1-\eta)^{1/m}]^{-m}, \qquad (13)$$

and the corresponding critical value ε_c of the average engineering strain ε_e^{av} (that is, strain to failure) is given by $\varepsilon_c = \lambda_c - 1$. The dependencies of the engineering strain to failure ε_c on the fraction of neck regions f_{neck} are plotted in Fig. 3, for $\eta = 0.01$ and various values of strain rate sensitivity m. Figure 3 clearly demonstrates that the strain to failure ε_c strongly increases with an increase in the fraction f_{neck} of necking regions, and this effect becomes especially strong for large values of strain rate sensitivity typical of superplastic deformation. This implies that very high strain to failure observed (e.g., [2,3,15-17]) in short ufg specimens with high strain rate sensitivity (characterized by $f_{neck} = 1$) may not be observed in similar long specimens where the value of f_{neck} can be small.

5. Conclusions

Thus, in this paper, we have suggested a model describing the development of individual and multiple necks in superplastically deformed materials. In the framework of the model, we have considered a round bar under tension, which is superplastically deformed without strain hardening. Within the long wavelength approximation, we have calculated the normalized cross sectional area in the center of neck regions and strain to failure as functions of strain rate sensitivity and volume fraction occupied by necks. We have demonstrated that neck development and necking-induced failure speed up with a decrease in strain rate sensitivity and/or an increase in the volume fraction occupied by necks. The latter effect (faster neck development and a decrease in the strain to failure with an increase of the fraction of sample length occupied by necks) is especially pronounced in the case of a high strain rate sensitivity. This implies that record values of strain to failure observed (e.g., [2,3,15-17]) in ufg metals and alloys in the form of bars with a small length (where the neck occupies the entire specimen) can be significantly reduced in large enough specimens where the necking regions occupy only a small part of the sample.

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