Materials Physics and Mechanics. 2023;51(5): 1-8
Research article
http://dx.doi.org/10.18149/MPM.5152023_1

Submitted: May 3, 2023
Approved: September 19, 2023
Accepted: October 9, 2023

On the exact solution of mixed problems for multicomponent multilayer materials

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Abstract. For the first time, an accurate analytical solution of mixed or contact problems for multicomponent multilayer materials has been constructed. It is assumed that the contact problem is formulated at the boundary of a multilayer multicomponent material in a semi-infinite region. These can be contact problems for a multilayer medium that simultaneously includes thermoelectroelastic, magnetoelastic, piezoelastic, water-saturated, nanomaterials and other layers described by linear partial differential equations. In the contact area, there can be any conditions of mechanical, physical or chemical properties that lead the boundary problem to a system of arbitrary finite number of Wiener-Hopf integral equations with a meromorphic matrix in the core. The article uses a new universal modeling method that allowed factorizing the operator of an infinite system of linear algebraic equations.

Keywords: multicomponent materials, mixed, contact problems, system of Wiener-Hopf integral equations, Galerkin transformation, factorization.

Acknowledgements. *The work was carried out with the financial support of the Russian Science Foundation, project 22-21-00129.*

Citation: Babeshko VA, Evdokimova OV, Babeshko OM. On the exact solution of mixed problems for multicomponent multilayer materials. *Materials Physics and Mechanics.* 2023;51(5): 1-8. DOI: 10.18149/MPM.5152023_1.

Introduction

Mixed, including contact tasks for multicomponent layered media play an important role in a wide variety of practice areas. They arise in the problem of assessing the strength of structures [1–3], the dynamic behavior of deformable bodies [4,5], wave scattering by defects [6], ultrasonic testing [7], the study of semiconductor properties [8,9], the wave phenomena in liquid media [10], seismology [11], evaluation of useful signals [12–15], in banking [16], in probability theory [17–19], in the theory of resonances and localization of processes [20] in the development of block element theory in differential and integral equations [21] and in other fields. As a rule, they are reduced to solving systems of Wiener-Hopf integral equations. In the case of a multicomponent layered medium of finite thickness, these systems of integral equations they have a matrix kernel, which has the following property. Fourier transforms of elements of the matrix kernel of integral equations are meromorphic functions. As described in article [22], methods for the exact solution of such a system of integral equations have not been developed. In this article, thanks to the development of the factorization method in the form of the sum of the operator of an infinite system, it is possible to accurately solve infinite systems of integral equations, and with them, the systems of integral equations themselves. This work should be considered as a development of the previously described approach [22] to the study of the properties of multicomponent materials. A special case of the problem under consideration for a two-component material is considered in [23].

Publisher: Peter the Great St. Petersburg Polytechnic University

Problem statement

The contact problem for a semi-infinite stamp on a multicomponent multilayer medium is investigated. It is assumed that the layers consist of various materials, the thickness of the multilayer medium has a finite value. In the area of contact of the stamp with the base, all types of permissible mechanical, physical, chemical, biological boundary conditions can be set with the requirement that they lead to a system of Wiener-Hopf integral equations of the order N. It is required that the Fourier transform of the kernel matrix of this system of integral equations has meromorphic functions as its elements. Here is the parameter N is an arbitrary bounded integer. Similarly, a semi-infinite Griffiths crack in the described layered medium can be considered. Mixed problems by known methods [2,4,5,22] are reduced to a system of integral equations of the form:

$$\int_0^\infty \mathbf{k}(x-\xi)\boldsymbol{\varphi}(\xi)d\xi=\mathbf{f}(x), \quad 0\le x<\infty \quad ,$$

$$\mathbf{k}(x)=\frac{1}{2\pi}\int_\Gamma \mathbf{K}(\alpha)e^{-i\alpha x}d\alpha, \quad \mathbf{K}(\alpha)=\left\|K_{mr}(\alpha)\right\|, \quad m,r=1,2,...,N,, \tag{1}$$

$$\boldsymbol{\varphi}=\left\{\varphi_1,\varphi_2,...,\varphi_N\right\}, \quad \mathbf{f}=\left\{f_1,f_2,...,f_N\right\}, \quad f_s(x)=A_s(\eta)e^{-i\eta x},$$

where the vector $\boldsymbol{\varphi}$ is the desired one, and the vector $\mathbf{f}(x)$ is the specified one.

We assume that the elements $K_{mp}(\alpha), \quad m,p=1,2,...,N$ of the matrix-function $\mathbf{K}(\alpha)$ in (1) are generally meromorphic functions of the variable α. In mixed problems of mechanics and mathematical physics, meromorphic functions $K_{mp}(\alpha)$ and the determinant $det\,\mathbf{K}(\alpha)$ have the following representation and asymptotic behavior [2,5]:

$$K_{mp}(\alpha)=D^{-1}(\alpha)L_{mp}(\alpha)\ ,\ det\,\mathbf{K}(\alpha)=D^{-N}(\alpha)\Delta(\alpha), \quad \Delta(\alpha)=det\left\|L_{mp}(\alpha)\right\|.$$

$$K_{mp}(\alpha)=T_{mp}\left|\alpha\right|^{-1}(1+o(\alpha)), \quad m=p, \quad K_{mp}(\alpha)=T_{mp}\alpha^{-1}(1+o(\alpha)),$$

$$m\ne p, \quad \left|\alpha\right|>>1, \quad p=1,2,...,N.$$

Here , the functions $L_{mp}(\alpha),\ D(\alpha),\ \Delta(\alpha)$ are integer functions of first order and finite type, that is, exponential type, in particular, polynomials. It is assumed that integer functions $D(\alpha),\ \Delta(\alpha)$ vanish on sets of values $\pm\zeta_n$ and $\pm z_n$, accordingly, having condensation points at infinity in some wedge-shaped regions of the upper, plus, and lower, minus, parts of the complex plane, as a rule, in the vicinity of the imaginary axis. For the sake of simplicity, we will not complicate the properties of matrix functions that have zero general and partial indices, and the system of integral equations is uniquely solvable in some $L_p, \quad p>1$. The properties of the matrix-function elements are described in more detail in [2,5] and here we repeat only the following. The determinant $\Delta(\alpha)$ of the matrix-function is an even whole function of type $N\sigma$ [24] and has a countable set of single zeros z_m going to infinity in the upper half-plane. Let 's denote this set W. The denominator $D(\alpha)$ is an even whole function of type σ and has a countable set of zeros ξ_m. Whole functions $L_{mp}(\alpha)$ have a type σ. Let 's construct N aggregates W_p containing zeros z_{mp}, which are selected from the number of zeros z_m of the determinant $\Delta(\alpha)$. Each aggregate W_p will have a countable number of them. When forming them, it is required that each zero z_{mp} number m, from the aggregate W_p, corresponds to the zero number ξ_m of the denominator, such that when $m\to\infty$, the behavior took place $\left|\xi_m z_{mp}^{-1}\right|\to v=const$. This requirement is met in most practically important tasks if the following rule applies. When constructing W_p, as the first zeros from the upper half-plane $Rez_{mp} \geq 0$ of the specified aggregates W_p, a sequence of the first zeros z_m of the determinant $\Delta(\alpha)$ is taken in ascending order of modules, or when counting counterclockwise, that is $z_{1p} = z_p, \quad p = 1, 2, ..., N$. In the future, each subsequent zero z_{np} of the aggregate W_p will be taken from among all the zeros z_m of the determinant $\Delta(\alpha)$ according to the rule $m = p + (n-1)N, \quad n \to \infty$ for each. Obviously, the collections $W_p, \quad p = 1, 2, ..., N$ of infinite sets of zeros z_{mp} constructed in this way, are independent, non-intersecting and their union contains the entire set of zeros z_m of the determinant $\Delta(\alpha)$, i.e., $W = \cup W_p$. We construct whole functions $M_p\left(\alpha, z_p\right)$ in the form of infinite products [2,5,24] with the help of the zeros included in W_p the number N [2,5,24], taking $\pm z_{sp} = z_{sp}^{\pm}$.

$$M_p\left(\alpha, z_p\right) = M_{pm}\left(\alpha, z_p^{\pm}\right) M_{p\pm}\left(\alpha, z_p^{m}\right),$$

$$M_{pm}\left(\alpha, z_p^{\pm}\right) = T_{pm} e^{mi\alpha} \prod_{s=1}^{\infty} \left(1 - \frac{\alpha}{z_{sp}^{\pm}}\right) e^{\frac{\alpha}{z_{sp}^{\pm}}}, \tag{2}$$

$$T_{p\mp} = const, \quad p = 1, 2, ..., N,$$

which, after dividing by $D(\alpha)$, will give meromorphic functions denoted by $M_p(\alpha)$. Their zeros are $\pm z_{mp}$. We take the components of the vector of the right part $\mathbf{f}(x)$ of the system of integral equations (1) in the form $A_p(\eta) e^{-i\eta x}, \quad p = 1, 2, ..., N, \quad Im\eta = 0$. Such component values make it possible to obtain arbitrary right-hand sides of a system of integral equations using Fourier transforms, in the form

$$f_p(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A_p(\eta) e^{-i\eta x} d\eta, \quad p = 1, 2, ... N. \tag{3}$$

Solution method

To attract to the study a new universal modeling method [21] based on the ideas of fractals, we present a system of Wiener-Hopf integral equations, taking into account the properties of matrix-function elements, in coordinate form:

$$\begin{gathered} \frac{1}{2\pi} \sum_{p=1}^{N} \int_{-\infty}^{\infty} K_{mp}(\alpha) \Phi_p(\alpha) e^{-i\alpha x} d\alpha = A_m(\eta) e^{-i\eta x}, \; 0 \leq x < \infty, \\ \Phi_p(\alpha) = \int_0^{\infty} \varphi_p(x) e^{i\alpha x} dx, \; p = 1, 2, ..., N. \end{gathered} \tag{4}$$

Then, taking into account the properties of whole functions having countable numbers of zeros, we have [24]:

$$k_{mp}(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} K_{mp}(\alpha)e^{-i\alpha x}d\alpha, \quad K_{mp}(\alpha) = \frac{L_{mp}(\alpha)}{D(\alpha)}, \quad f = \{f_1, f_2, \dots f_N\},$$

$$L_{mp}(\alpha) = \prod_{n=1}^{\infty} L_{mpn}(\alpha), \qquad K_{mp}(\alpha) = \frac{L_{mp}(\alpha)}{D(\alpha)} = \prod_{n=1}^{\infty}\frac{L_{mpn}(\alpha)}{D_n(\alpha)}, \tag{5}$$

$$L_{mpn}(\alpha) = (\alpha^2 - \tau^2_{mpn}), \quad D_n(\alpha) = (\alpha^2 - \xi_n^2).$$

Here τ_{mpn} are the zeros of whole functions representing the numerators of the matrix elements, which may not coincide with the zeros of the determinant. The system of Wiener-Hopf integral equations (1) can, taking into account (5), be represented using a system of differential equations in the form:

$$\sum_{p=1}^{N}\prod_{n=1}^{\infty} L_{mpn}(i\frac{\partial}{\partial x})\varphi_p(x) = \prod_{s=1}^{N} D_s(i\frac{\partial}{\partial x}) f_m(x), \quad m = 1, 2, \dots, N,$$

$$L_{mpn}(i\frac{\partial}{\partial x}) = \frac{\partial^2}{\partial x^2} + \tau^2_{mpn}, \quad D_p(i\frac{\partial}{\partial x}) = \frac{\partial^2}{\partial x^2} + \xi_r^2. \tag{6}$$

Taking into account (3), we limit ourselves to the representations of the right part of the functions $B_p e^{-i\eta x}$:

$$\prod_{s=1}^{N} D_s(i\frac{\partial}{\partial x}) f_s(x) = Be^{-i\eta x}. \tag{7}$$

Thus, assuming that the parameter η does not coincide with any of the poles ξ_r, we conclude that the differential operators leave the exponential function unchanged on the right, only changing the coefficient in front of it.

We apply the transformation of Galerkin [21]. to a system of the differential equations (6), (7). To do this, we will construct determinants containing new unknown functions χ_p and operators.

$$\mathbf{\Phi}_p(x) = \mathbf{E}_p(\mathbf{X}), \quad \mathbf{X} = \{\chi_1, \chi_2, \dots, \chi_N\}, \quad \mathbf{\Phi}_p = \{\varphi_1, \varphi_2, \dots, \varphi_N\},$$

$$\mathbf{E}_1(\mathbf{X}) = \begin{vmatrix} \chi_1 & L_{12} & \cdots & L_{1N} \\ \chi_2 & L_{22} & \cdots & L_{2N} \\ \vdots & \vdots & \ldots\vdots & \vdots \\ \chi_N & L_{N2} & \cdots & L_{NN} \end{vmatrix}, \quad \mathbf{E}_2(\mathbf{X}) = \begin{vmatrix} L_{11} & \chi_1 & \cdots & L_{1N} \\ L_{21} & \chi_2 & \cdots & L_{2N} \\ \vdots & \vdots & \ldots\vdots & \vdots \\ L_{N1} & \chi_N & \cdots & L_{NN} \end{vmatrix}, \dots, \tag{8}$$

$$\mathbf{E}_N(\mathbf{X}) = \begin{vmatrix} L_{11} & L_{12} & \cdots & \chi_1 \\ L_{21} & L_{22} & \cdots & \chi_2 \\ \vdots & \vdots & \ldots\vdots & \vdots \\ L_{N1} & L_{N2} & \cdots & \chi_N \end{vmatrix}.$$

As a result of calculations and simplifications in (8), the following system of N independent differential equations is obtained to determine the functions χ_p:

$$L\chi_p = B_p e^{-i\eta x}, \quad L = det\|L_{mr}\| \quad m, r = 1, 2, \dots, N. \tag{9}$$

Having opened the determinant (9), and having carried out possible transformations, we obtain differential equations. They do not depend on the order of calculation of the determinant, since all elements are differential operators with constant coefficients and are commuting. We obtain N infinite systems of differential equations with constant coefficients described by a single differential operator arising from the determinant (9) of the entire system of Wiener-Hopf integral equations. Thus, we obtain an entire function, the arguments of which will be, in accordance with the conditions of the problem, the products of second-order differential operators of the form:

$$\Delta(i\frac{\partial}{\partial x}) = \prod_{n=1}^{\infty} G_n, \quad G_n(i\frac{\partial}{\partial x}) = \frac{\partial^2}{\partial x^2} + z_n^2 \quad . \tag{10}$$

It follows that the homogeneous equation (11) for each function χ_p has the form:

$$\prod_{n=1}^{\infty} G_n(i\frac{\partial}{\partial x})\chi_p = 0, \quad p = 1,2,...,N .$$

Since the equation $L = \prod_{n=1}^{\infty} G_n$ has constant coefficients, the characteristic equation for homogeneous solutions of a differential equation is an integer function $\Delta(\alpha)$. As a result, a homogeneous solution for each χ_p is represent able in the form: $\chi_p = \sum_{m=1}^{\infty} \chi_m e^{iz_m x}, \quad p = 1,2,...,N .$

The solutions of each equation $G_m(i\frac{\partial}{\partial x})\chi_p = 0$ are taken here. The presence of right-hand sides in a system of integral equations leads systems of differential equations to inhomogeneous ones. To search the general solution of each inhomogeneous differential equation we will look for them in the following form.

The presence of right-hand sides in a system of integral equations leads systems of differential equations to inhomogeneous ones. Taking into account the fact that the general solutions of differential equations must also satisfy the inhomogeneous conditions of the system of integral equations, we will define the parameter B_p from this condition later. To search the general solution of each inhomogeneous differential equation (7) we will look for them in the following form:

$$\chi_{p\eta}(x) = B_p e^{-i\eta x} + \sum_{m=1}^{\infty} y_m e^{iz_m x}, \quad p = 1,2,...,N . \tag{11}$$

Here B_p, y_m are independent of x and are unknown. Thus, all functions $\chi_{p\eta}(x)$ in (11) have the same decomposition, but with different coefficients B_p.

For mathematically rigorous finding of its expansion terms $y_{pm}e^{iz_{mp}x}$ for each function $\chi_{p\eta}(x)$, a new, previously unknown, factorization method has been developed in the form of the sum of the operator of an infinite system of linear algebraic equations, which is a discrete analog of continuous factorization used in the Wiener-Hopf method.

Solution of the Galerkin equation

A new universal modeling method is applicable for the solution [21]. Unlike traditional finite systems of Galerkin differential equations, in the case of differential equation (9) generated by a system of integral equations (1), a differential equation of infinite order is obtained. For it, it is necessary to construct linearly independent, complete solutions of this equation. Linear independence refers to the absence of identical exponential harmonics in these solutions. Completeness refers to the use of solutions N of all exponential harmonics contained in the characteristic equation in the aggregate $\Delta(\alpha) = 0$. The coefficients of the solutions constructed in this way must be uniquely determined from the requirement of satisfying the system of integral equations (1).

The developed method for solving such infinite systems of linear algebraic equations, technically quite complex, allowed us to obtain the result set out below.

Let the system of integral equations (1) be uniquely solvable in some Banach space [2,5] and the rank of the matrix $\|K_{nr}(z_m)\|$ is equal to $N-1$. Let's introduce notation using previously accepted parameters:

$$\mathbf{A}=\|(\xi_k\text{-}z_m)^{-1}\|,\qquad \mathbf{A}^{-1}=\|\tau_{gr}\|,\quad \tau_{gr}=\frac{1}{\Delta_+'(-z_g)(\xi_r-z_g)[\ D_+^{-1}(-\xi_r)\ ]'},$$

$$\mathbf{A}_p=\|(\xi_k\text{-}z_{mp})^{-1}\|,\mathbf{A}_p^{-1}=\|\tau_{pgr}\|,\quad \tau_{pgr}=\frac{1}{M_{p+}'(-z_{gp})(\xi_r-z_{gp})[\ D_+^{-1}(-\xi_r)\ ]'},$$

$$,\quad \mathbf{C}_p=\|K_{nr}(z_{mp})\|\quad n,r=2,3,....,$$

$$N_{1p}(\alpha)=\prod_{s=1}^{N}{}'M_s(\alpha,z_{ms})+M_p(\alpha,z_{mp}),$$

$$N_{2p}(\alpha)=N_{1p}(\alpha)-2M_p(\alpha,z_{mp}),\quad N_p(\alpha)=N_{2p}^{-1}(\alpha)N_{1p}(\alpha),$$

$$\mathbf{N}_p(\alpha)=\|N_p(\alpha)\|.$$

The factorization of the following functions is used here $\Delta(\alpha)=\Delta_+(\alpha)\Delta_-(\alpha),\quad D(\alpha)=D_+(\alpha)D_-(\alpha),\quad M_p(\alpha)=M_{p+}(\alpha)M_{p-}(\alpha)$. In the course of the study, the lemma is proved.

Lemma. Let $\det\mathbf{C}\neq 0$. Then the complete set of linearly independent solutions of the inhomogeneous Galerkin equation for the right - hand sides $B_p(\eta)e^{-i\eta x}$ is given by the relations:

$$\mathbf{X}_p=\{\chi_{p\eta}\},\quad \mathbf{Y}_p=\{y_{pm}\},$$

$$\chi_{p\eta}(x)=G^{-1}(\eta)B_p(\eta)e^{-i\eta x}+\sum_{m=1}^{\infty}y_{pm}e^{iz_{mp}x}.$$

The designations are accepted here:

$$\mathbf{Y}_1=\mathbf{A}_1^{-1}\left\langle\ \mathbf{F}_1-\frac{1}{2}\sum_{p=2}^{N}[\mathbf{G}_p+\mathbf{G}_1]\ \right\rangle,\quad p=1$$

$$\mathbf{Y}_p=\mathbf{A}_p^{-1}\left\langle\ \frac{1}{2}[\mathbf{G}_p+\mathbf{G}_1]\ \right\rangle,\quad p=2,3,...,N,\quad \mathbf{F}_p=\left\{-\frac{G^{-1}(\eta)B_p}{\eta+\xi_r}\right\}$$

$$\mathbf{G}_1=\mathbf{F}_1,\quad \mathbf{G}_p=\mathbf{A}\mathbf{N}_p(z_{mp})\mathbf{C}_p^{-1}\mathbf{A}^{-1}\mathbf{F}_p,\quad p=2,3,...,N,$$

To obtain a vector $\boldsymbol{\varphi}(x)=\{\varphi_1,\varphi_2,...,\varphi_N\}$ representing the solution of the Wiener-Hopf system of integral equations (1), we use the Galerkin transformation formulas (8). The constructed system of solutions $\mathbf{X}(x)$ to the Galerkin differential equation (9) is complete, exhaustive of all harmonics of the zeros of the characteristic equation $\Delta(\alpha)=0$, which is required when applying the Galerkin transformation. Therefore, to determine the vector $\boldsymbol{\varphi}=\{\varphi_1,\varphi_2,...,\varphi_N\}$ of the solution of the integral equation (1), it is necessary to use the formulas (8). The complete system of solutions of the Galerkin equation (9), functions $\chi_{p\eta}(x)$, are introduced into the determinants (8) and, after their disclosure and performing the necessary differential operations, give an exact solution to the system of integral equations (1).

The theorem. Under the conditions of the lemma, the exact solution of the system of integral equations (1) for the right parts $A_p(\eta)e^{-i\eta x}$ has the form:

$$\mathbf{\Phi}_p(x) = \mathbf{E}_p(\mathbf{X}), \quad \mathbf{X}(x) = \left\{ \chi_{1\eta}, \chi_{2\eta}, \ldots, \chi_{p\eta} \right\},$$

$$\mathbf{X}(x) = \mathbf{K}^{-1}(\eta)\mathbf{f} + \mathbf{Z}(x), \quad \mathbf{Z}(x) = \left\{ \sum_{m=1}^{\infty} y_{pm} e^{iz_{mp}x} \right\},$$

$$\mathbf{f} = \left\{ f_1, f_2, \ldots, f_N \right\}, \quad \mathbf{F} = -\frac{\mathbf{K}^{-1}(\eta)\mathbf{f}}{\eta + \xi_r}, \quad \mathbf{F} = \left\{ \mathbf{F}_p \right\}.$$

The validity of both the lemma and the theorem can be verified by directly substituting the corresponding solutions into systems of differential and integral equations.

In the event that the matrix-function ***K(α)*** continuously transitions into a functional-commutative one, the constructed solution passes into the solution obtained by simple handling of a system with functionally commutative matrix functions for a particular case. From this formula, as a special case, the result of a two-dimensional problem of the work [24] is obtained.

Conclusions

For the first time, an accurate analytical solution of mixed or contact problems for multicomponent multilayer materials has been constructed.

The article uses a new universal modeling method, [21] which made it possible to overcome the problem of factorization of the operator of an infinite system of linear algebraic equations. The exact solution is given in the form of formulas and accompanying parameters that allow the result to be applied in research. The considered contact problem is formulated on the boundary of a multilayer multicomponent material in a semi-infinite region. These can be contact problems for a multilayer medium, which simultaneously includes thermoelectroelastic, magnetoelastic, piezoelastic, water-saturated, nanomaterials and other layers described by linear partial differential equations. In the contact area, there can be any conditions of a mechanical, physical or chemical nature that lead the boundary problem to a system of arbitrary finite number of Wiener-Hopf integral equations with a meromorphic matrix in the core. This method is applicable to solving a mixed problem about the behavior of a semi-infinite crack in a multicomponent multilayer foundation. Unlike other approaches, the proposed method allows you to obtain the desired solution in an analytical form without requiring additional research.

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