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DAMPED VIBRATION ANALYSIS OF LUMPED MASS ON BIDIRECTIONALLY GRADED BEAM RESTED ON TORSION SPRING HINGES

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Abstract. This article is concerned with the vibration control of a bidirectionally graded beam supported on torsion spring hinges using a lumped mass damper. The conventional mixing rule is used to model the material gradation in the thickness direction. The material gradation in the axial direction is modeled by an exponential function. The spectral Ritz method is employed to minimize the total potential energy and calculate the fundamental angular frequency of free vibration and the corresponding mode shape. The characteristic vibration equation of the system is obtained by calculating the determinant of the Hessian of the total potential energy. For the first time, the modified Taylor basis is introduced, which eliminates the drawbacks and the necessity of using auxiliary functions in the spectral Ritz method. For this purpose, the modified Taylor basis is calculated by satisfying the boundary conditions and the natural conditions at the ends of the beam. The effects of the dimensionless rotational stiffness at the ends of the beam, the material gradations in the axial and transverse directions, the amount and position of the lumped mass on the fundamental angular frequency of the free vibration are investigated.
Keywords: damped vibration, mass damper, bidirectional gradation, semi-rigid support, spectral Ritz method, modified Taylor basis

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1. Introduction

Unwanted vibration can be caused by environmental factors such as wind or earthquakes acting on a structure, or by a seemingly innocuous vibration source causing resonances that can be destructive, unpleasant, or just plain annoying. Masses of people going up and down stairs at the same time, or large numbers of people stomping in unison, can cause serious problems in large structures such as stadiums if those structures do not have damping measures. Distribute the wave energy inside the structure with properly designed dampers that spread the wave energy over a wider frequency range and absorb the resonant portions of the entire wave frequency band with so-called mass dampers.

The accelerating mass moving on the beam was modelled as a moving finite element to account for inertial effects in addition to the gravitational force of the mass. The effect of

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longitudinal force due to acceleration of the moving mass is also considered [1]. The combined differential equations of the FGM beam are obtained using the first-order shear deformation theory (FSDT). In these equations, the interaction terms of the inertia are derived from exact second order differentials of the displacement functions with respect to the mass contact point. The FGM beam consists of two different materials that vary with a power law [2]. A modified finite element method (FEM) is presented that can be used to analyse the transverse vibrations of a Timoshenko beam made of functionally graded materials (FGM) on a two-parameter foundation subjected to a mass moving with variable velocity [3]. The dynamic response of FG Timoshenko beam in a thermal environment subjected to an accelerating load is investigated [3]. The dynamic responses of a symmetric and sigmoidal FG Timoshenko beam resting on an elastic foundation and subjected to a moving mass are studied [4]. The free vibration and buckling stability of FG nanobeams subjected to magnetic and thermal fields are studied [5] The dynamic behavior of FG Timoshenko beam on a linear-elastic four-parameter foundation under the influence of a high velocity moving mass with variable velocities is investigated [6]. A theory of vibrations of beams with continuous cracks is used to predict the changes in transverse vibrations of a simply supported beam with a breathing crack. The equation of motion and boundary conditions of the cracked beam, which is considered as a one-dimensional continuum, have been used. It is shown that the changes in natural frequency due to a breathing edge crack depend on the bi-linear character of the system. The associated linear problems are solved over their respective domains of definition and the solutions are fitted by the initial conditions. The changes in vibration frequencies for a breathing fatigue crack are smaller than those caused by open cracks. The method was tested for the evaluation of the lowest natural frequency of lateral vibration for beams with a unidirectional breathing crack. Experimental results of aluminum beams with fatigue cracks are used for comparison with the analytical results [7]. In a proposed element model, the stress equilibrium equation is used to derive the rational distribution of the transverse shear stress. Numerical examples have been investigated to illustrate the effectiveness and applicability of the proposed mixed beam element model in vibration analysis. The numerical results show that the vibration frequencies obtained with the mixed beam element model are much more satisfactory compared to the classical beam element model and beam element models based on conventional first-order or higher-order shear deformation theory [8]. A novel higher-order cubic-quintic nonlinear model for the nonlinear free vibration of damped and undamped bidirectional functionally graded beams (2D FGBs) is proposed. As far as researchers are aware, no previous study has focused on the damping properties of 2D FG beams. Therefore, the present work extends the previous studies in this field. It is assumed that the material properties of the beam change simultaneously in the axial and lateral directions according to exponential and power law functions, respectively. A new neutral surface is defined to eliminate the effect of strain and bending coupling. The variational iteration method (VIM) and the Hamiltonian approach (HA) are applied to obtain closed-form analytical solutions for the nonlinear vibrations of damped and undamped 2D FG beams. The results show that increasing the material classification indices can similarly decrease the damping coefficient for an undamped 2D FG beam and more time is required for the vibration to decay. Moreover, a significant difference is found between the cubic and cubic-quintic models compared to the homogeneous beams, indicating the importance of applying higher order nonlinear models for the nonlinear analysis of 2D FG beams [9]. Using a well-established model consisting of an equivalent single-layer Timoshenko beam coupled with mass-spring-pot subsystems representing the resonators, novel and exact analytical expressions for the frequency response and modal behaviour under arbitrary loads are presented. In particular, the frequency response is obtained by a direct integration method, while the modal impulse and frequency response functions are derived by a complex modal analysis approach where appropriate orthogonality conditions are introduced for the complex modes. The expressions for the frequency response and the modal response are valid for an arbitrary number of resonators and degrees of freedom within the resonators, an arbitrary number of loads and positions of the loads relative to the resonators. The proposed complex modal analysis approach solves the difficult problem of computing all complex eigenvalues without overlooking a single one. For this purpose, a recently introduced contour integral algorithm is applied to an exact dynamic stiffness matrix whose size here depends only on the number of degrees of freedom at the beam ends, independent of the number of resonators and degrees of freedom within resonators. Numerical applications prove the exactness and robustness of the proposed system [10] beam finite element model is proposed for static and vibration analysis of FGM sandwich beams with viscoelastic nonlinear material behavior. Zigzag theory for displacement fields is used for the analysis. Timoshenko's 1st order shear models and Reddy's higher order shear models are used for static and vibrational behaviors, respectively. Various viscoelastic frequency dependent laws are considered. The resulting stiffness matrix is nonlinear and frequency dependent. Solutions are possible using a powerful asymptotic method in combination with a new method for the power series terms. It is found that the behavior of the beam is very sensitive to the loss factor. In the case of vibrations, the damping characteristics depend nonlinearly on the power law index. The boundary conditions have an influence on the vibration modes. The case of the cantilever is particularly interesting and requires an optimization process [11]. Many other relevant works can be found in the literature [12-21].

In this paper, the effect of the position of the lumped mass damper on the reduction of the fundamental frequency of the bidirectional functionally graded material (BDFGM) beam with rectangular cross section is presented. The volume fraction distribution of the functionally graded material is used to model the mass density and elastic modulus in the thickness direction. The lumped mass is connected to the beam at an arbitrary location without sliding. The BDFGM beam has a small thickness to length ratio, so the total potential energy is derived based on classical beam theory. The total potential energy is minimized by applying the spectral Ritz method to calculate the fundamental frequency and the corresponding mode shape. For the first time, the modified Taylor basis is introduced, which eliminates the disadvantages and the necessity of using adhesive functions in the spectral Ritz method. For this purpose, the modified Taylor basis is computed by satisfying the boundary and natural conditions at the ends of the beam. The BDFGM rests on elastic torsion springs and displacement restraints at the ends. In the presence of a lumped mass damper, a reduction in frequency is observed. However, the dimensionless frequency reduction is a function of the amount and position of the lumped mass, with the position of the lumped mass having a more important effect on the vibration control of the BDFGM beam. The convergence of the numerical results is observed by using small terms in the basis.

2. Vibration Analysis

Figure 1 shows a beam made of bidirectional functionally graded material (BDFGM) and a lumped mass damper attached to it. The length, width, and height of the beam are L, b and h, respectively. The position of the lumped mass damper is measured from the left end of the FGM beam. The mass of the ring is considered in relation to the total mass of the FGM beam.

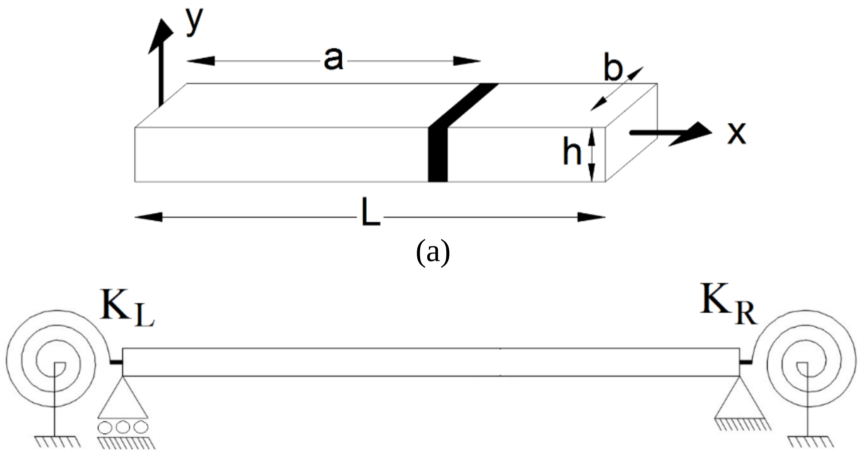

Fig. 1. The BDFGM beam with (a): lumped mass damper (b): rested on rotational spring hinges

The total potential energy of BDFGM beam and attached lumped mass, m, at position a from left end of the BDFGM beam is written as follows:

$$\Pi = \frac{1}{2}\int_0^L \psi\left(EI_{eq}(Y'')^2 - \bar{m}_{eq}Y^2\omega^2\right)dx - \frac{1}{2}m(Y|_{x=a}\omega)^2, \tag{1}$$

where EI_{eq} and $\bar{m}_{eq}$ are equivalent flexural rigidity and equivalent mass per unit length of the BDFGM beam, respectively. The dimensionless shape function, ψ shows the variation of the mass density and elasticity modulus in longitudinal direction of the BDFGM beam. The deflection of the BDFGM beam is shown by the parameter Y. The parameter ω denotes natural angular frequency of the free vibration. The distance between neutral axis location and mid-axis location is e. As a result, one can write

$$EI_{eq} = b\int_{-\frac{h}{2}-e}^{\frac{h}{2}-e} Ey^2 dy. \tag{2}$$

The parameter e can be obtained as follows:

$$e = \frac{\int_{-\frac{h}{2}}^{\frac{h}{2}} Ey dy}{\int_{-\frac{h}{2}}^{\frac{h}{2}} E dy}. \tag{3}$$

The elastic modulus and mass density distributions in thickness direction are assumed as follows:

$$\rho(y) = \rho_m V_m + \rho_c V_c, \tag{4}$$

$$E(y) = E_m V_m + E_c V_c, \tag{5}$$

in which, the subscripts m and c denotes metallic and ceramic constituents, respectively. The volume fraction of the phase materials are

$$V_c = \left(\frac{y}{h} + \frac{1}{2}\right)^n, \tag{6}$$

$$V_m = 1 - \left(\frac{y}{h} + \frac{1}{2}\right)^n, \tag{7}$$

where n is gradient index or material exponent parameter which takes non-negative real amounts. The linear mass density of the BDFGM beam is calculated as follows:

$$m_{eq} = \frac{(\rho_c + n\rho_m)bh}{n+1}. \tag{8}$$

The eccentricity of the neutral axis is calculated and presented in Eq. (9).

$$e=\frac{nh(E_c-E_m)}{(n+2)(E_c+nE_m)}. \quad (9)$$

The equivalent flexural rigidity of the neutral axis is calculated and presented in Eq. (10).

$$EI_{eq}=(\lambda_1 E_c^2+\lambda_2 E_c E_m+12E_m^2)/(\lambda_3 E_c+\lambda_4 E_m). \quad (10)$$

The constants λ_1 to λ_4 are defined as follows:

$$\lambda_1=n^4+4n^3+7n^2, \quad (11)$$
$$\lambda_2=4n^3+16n^2+28n, \quad (12)$$
$$\lambda_3=(n^2+5n+6)(n^2+2n), \quad (13)$$
$$\lambda_4=(n^2+5n+6)(n+2). \quad (14)$$

The spectral Ritz method is applied to calculate first frequencies of the system. The truncated Taylor series expansion of the deflection is selected as basis function and unknown coefficients.

$$Y\approx\{a_0\,a_1\,\cdots\,a_m\}\{1\,x\,\vdots\,x^m\}\qquad m\in N. \quad (15)$$

After satisfying boundary conditions at pinned supports and natural conditions at rotational spring hinges, the coefficients a_0 to a_3 are calculated in terms of the remained coefficients, a_4 to a_m.

$$a_1=\frac{2EI_{eq}\psi_0\left(\alpha+\beta-\gamma\left(3k_{\theta r}L+6EI_{eq}\psi_L\right)\right)}{\left(L^2k_{\theta l}k_{\theta r}+4EI_{eq}\psi_L k_{\theta l}L+4EI_{eq}k_{\theta r}\psi_0 L+12EI_{eq}^2\psi_0\psi_L\right)L}, \quad (16)$$

$$a_2=\frac{k_{\theta l}\left(\alpha+\beta-\gamma\left(3k_{\theta r}L+6EI_{eq}\psi_L\right)\right)}{\left(L^2k_{\theta l}k_{\theta r}+4EI_{eq}\psi_L k_{\theta l}L+4EI_{eq}k_{\theta r}\psi_0 L+12EI_{eq}^2\psi_0\psi_L\right)L}, \quad (17)$$

$$a_3=2\gamma\xi_1-\beta\xi_2-\alpha\xi_3. \quad (18)$$

The unknown parameters are introduced in Eq. (19) to Eq. (24).

$$\xi_1=k_{\theta l}k_{\theta r}L+k_{\theta r}EI_{eq}\psi_0+k_{\theta l}EI_{eq}\psi_L, \quad (19)$$
$$\xi_2=k_{\theta l}+2\frac{EI_{eq}\psi_0}{L}, \quad (20)$$
$$\xi_3=k_{\theta l}-\frac{2EI_{eq}\psi_0}{L}, \quad (21)$$
$$\alpha=k_{\theta r}L\sum_{n=4}^{m}a_nL^n n, \quad (22)$$
$$\beta=EI_{eq}\psi_L\sum_{n=4}^{m}a_nL^n n(n-1), \quad (23)$$
$$\gamma=\sum_{n=4}^{m}a_nL^n, \quad (24)$$

where ψ_c is amount of the function ψ at $x=c$. Substituting Eq. (15) into Eq. (1), one has

$$\Pi=f(a_4,a_5,\dots,a_m,\omega). \quad (25)$$

Minimizing total potential energy, yields

$$\frac{\partial\Pi}{\partial a_i}=0\qquad 4\le i\le m. \quad (26)$$

The characteristic equation will be obtained by considering nontrivial solution. To this purpose, determinant of the matrix of the coefficients must be vanished. The matrix of the coefficients is Hessian of the total potential energy.

$$|H_{ij}|=\left|\frac{\partial^2\Pi}{\partial a_i\partial a_j}\right|=0\qquad 4\le i\le m, 4\le j\le m. \quad (27)$$

After substituting angular frequency of damped vibration into Hessian of the total potential energy, $[H]$, the unknown coefficients can be calculated in terms of an arbitrary coefficient, say a_m, as follows:

$$\{a_1 \ \vdots \ a_{m-1}\} = -a_m [H^*]^{-1}_{(m-1)\times(m-1)} \{\widetilde{H}\}_{1\times(m-1)}. \quad (28)$$

The square matrix $[H^*]$ is calculated by deleting last row and last column of the matrix $[H]$. The vector $\{\widetilde{H}\}$, is the last column of the matrix $[H]$, except the last row.

3. Results and Discussion

The dimensionless parameters for numerical examples are introduced in Eq. (29) to Eq. (36).

$$\bar{x} = \frac{a}{L}, \quad (29)$$

$$\bar{X} = \frac{x}{L}, \quad (30)$$

$$\bar{Y} = \frac{Y}{Y_{max}}, \quad (31)$$

$$\bar{M} = \frac{M}{m_{eq}L}, \quad (32)$$

$$\bar{k}_{\theta l} = \frac{k_{\theta l}L}{EI_{eq}\psi_0}, \quad (33)$$

$$\bar{k}_{\theta r} = \frac{k_{\theta r}L}{EI_{eq}\psi_0}, \quad (34)$$

$$\psi = e^{\frac{\zeta x}{L}}, \quad (35)$$

$$\bar{\omega} = \frac{\omega}{\omega_0}. \quad (36)$$

The parameter ω_0 is fundamental frequency for $\bar{M} = 0$. The numerical values of parameters for numerical solutions are presented in Table 1.

Table 1. The numerical values for numerical examples

$L(m)$	$b(m)$	$h(m)$	n	ζ	$\bar{x}$	$\bar{M}$	$\rho_m\left(\frac{kg}{m^3}\right)$	$\rho_c\left(\frac{kg}{m^3}\right)$	$E_m\left(\frac{N}{m^2}\right)$	$E_c\left(\frac{N}{m^2}\right)$	$\bar{k}_{\theta l}$	$\bar{k}_{\theta r}$
1	0.12	0.15	2	0.1	0.6	0.1	2700	3750	$7e10$	$3e11$	300	600

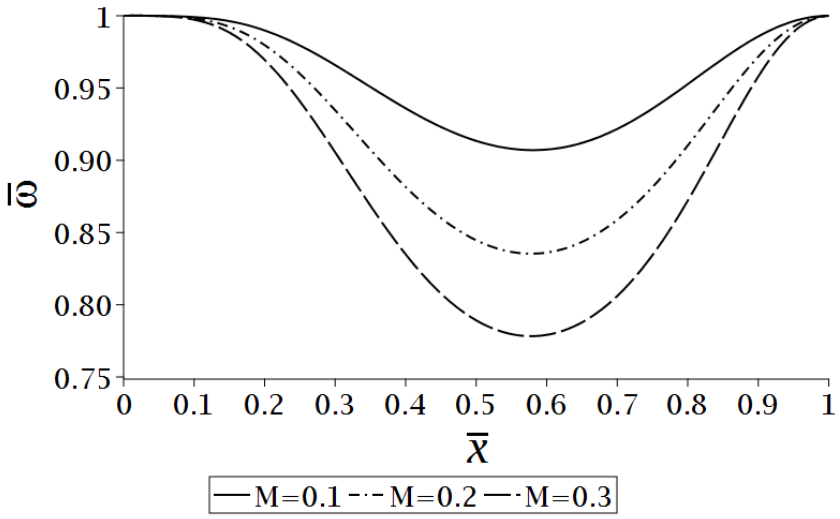

Fig. 2. The frequency reduction of the clamped-pinned BDFGM beam caused by mass damper

The dimensionless fundamental angular frequency of the free vibration for clamped-pinned BDFGM beam ($\bar{k}_{\theta l} = \infty, \bar{k}_{\theta r} = 0$) with various amounts of lumped mass against the position of the lumped mass damper is presented in Fig. 2. Figure 3 depicts the effect of axial gradation on damped frequency of BDFGM rested on torsion spring hinges. The effect of mass damper on reduction of frequency is increased by decreasing axial gradation parameter.

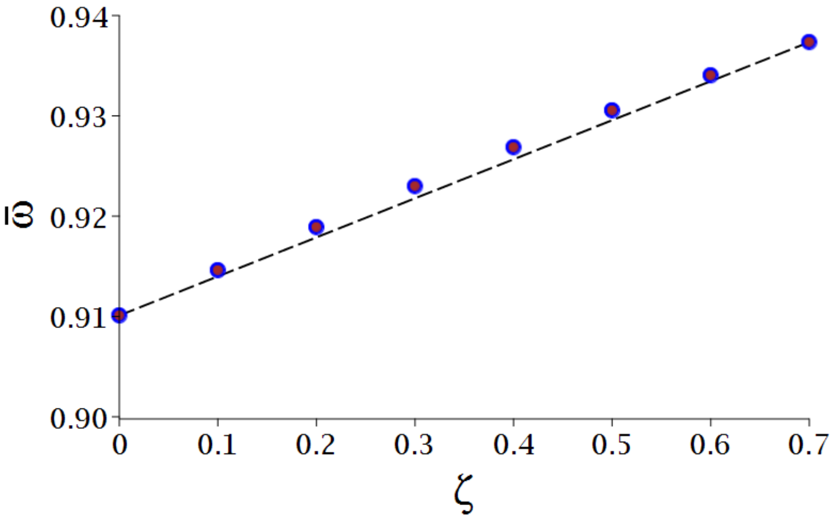

Fig. 3. The effect of axial gradation on fundamental frequency of BDFGM rested on semi-rigid supports in the presence of the lumped mass damper

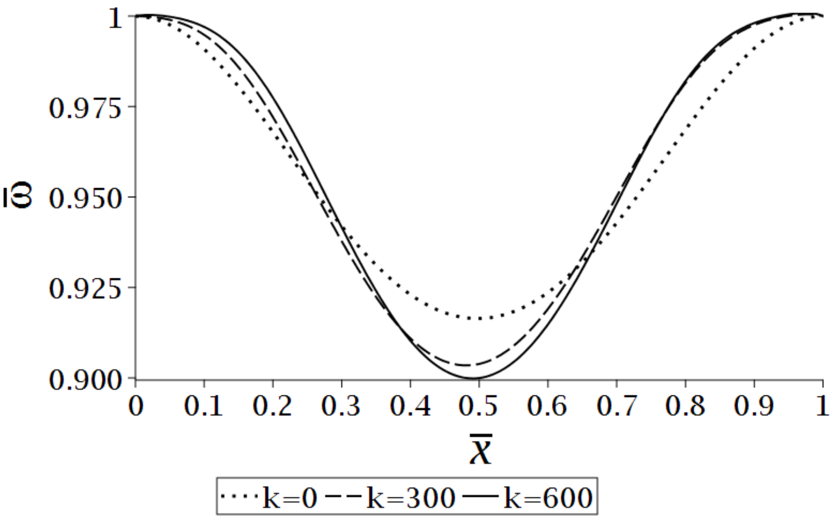

Fig. 4. The effect of rotational stiffness on fundamental frequency of BDFGM with attached lumped mass damper ($\bar{k}_{\theta r} = 2\bar{k}_{\theta l} = k$)

Figures 2 and 4 show that the shape of the fundamental frequency reduction curve is similar to the first mode shape of vibration of the BDFGM beam, schematically. The first three mode shapes of the BDFGM beam rested on semi-rigid supports with lumped mass damper are presented in Fig. 5.

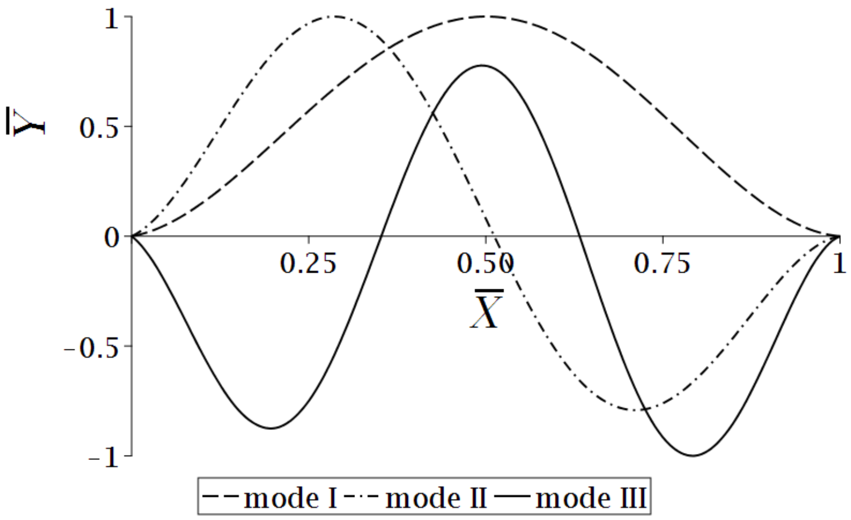

Fig. 5. First three mode shapes of the BDFGM beam with semi-rigid supports and lumped mass

4. Conclusion

The effect of the position of the attached lumped mass on the frequency reduction of the BDFGM beam with rectangular cross section and bidirectional material distributions is investigated. The equivalent linear mass density and equivalent bending stiffness of the BDFGM beam are calculated. The results for different end conditions are presented. The dimensionless fundamental frequency decreases as the amount of connected lumped mass increases. It is shown that the decrease in natural frequency is a function of the amount and position of the lumped mass. The shape of the frequency decrease diagram is schematically similar to the mode shape of the BDFGM beam. However, both the elasticity modulus and the mass density are affected by the axial material gradation, but by increasing the parameter of material gradation in the axial direction, the fundamental frequency will be increased. The effect of the mass damper on frequency reduction at vicinity of mid- span is increased by increasing the torsional stiffness of the torsion springs. The position of the attached lumped mass can be used to control the vibration of the BDFGM beam. In the present work, the position of the attached lumped mass is proposed for frequency reduction for the first time. This new concept can be used as a new device for vibration control of beams such as the cantilever beams.

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