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# Integral equations of deformation of cylindrical workpieces in axisymmetric matrices of complex shape

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**Abstract.** The article considers the process of deformation of thin-walled pipes using complex-shaped tooling. The article solves the actual problem of elastic and plastic deformation of pipe blanks in the stamping process, taking into account physical nonlinearity since the power law of hardening is taken into account, as well as the compressibility of the material at the stage of elasticity. When determining the stress and strain state during the deformation of thin-walled pipe blanks using axisymmetric tooling, the method of variable elasticity parameters was used, which allows taking into account not only the change in thickness during deformation but also the compressibility and nonlinearity of the hardening of the material. Integral equations are obtained for various processes: crimping and drawing, distribution, and broaching of a pipe billet. The described processes differ in the way the external load is applied. For all processes, two sections with different directions of curvature in the meridional section can be distinguished. The solution for determining the stress and strain state of the pipe, in accordance with the method of variable elasticity parameters, is proposed to be carried out by the method of successive approximations according to the constructed recurrent scheme.

**Keywords:** integral equations of deformation, stamping, workpieces of complex curvature, stresses, compressibility of the material, physical nonlinearity

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## Introduction

One of the most important tasks in the development of technological processes of deformation of thin-walled pipes, such as distribution and broaching, crimping, and drawing, is to determine the stress-strain state taking into account the nonlinear plasticity of the material. In most known solutions of similar processes, ideally rigid plastic material or a material with linear hardening is considered [1-4], which leads to significant errors.

In this article, when determining the stress-strain state during the deformation of thin-walled pipes using axisymmetric tooling, we will use the method of variable elasticity


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parameters [4,5], which allows us to take into account not only the change in thickness during the deformation but also the compressibility and nonlinearity of the hardening of the material.

The most profound issues of deformation of shell elements of complex shapes in the processes of shaping were considered in [1-9]. New technological methods of metal processing by pressure using broaching, and issues of roughness during broaching and drawing are considered in the works [10,11]. The use of stretching models for shell-type elements is considered in [12]. The problems of distribution and crimping, and the peculiarities of stress distribution during rolling are investigated in the works [13,14]. The issues of stamping blanks of complex shapes are considered in [15]. Numerical methods for estimating the stress-strain state of shell elements are considered in articles [16-19].

**Methods**

To implement the method of variable elasticity parameters, it is necessary to have an analytical expression defining the deformation diagram. Analysis of existing methods of approximation of deformation diagrams proposed by N.N. Malinin [4], M.I. Lysov [6], and other researchers [7,8] showed that usually, this curve is well approximated either by a power dependence $\sigma_i = Ae_i{}^n$, or linear-power dependence

$$\sigma_i = \begin{cases} 3Ge_i \text{ при } e_i \le e_{iT} \\ Ae_i{}^n \text{ при } e_i > e_{iT} \end{cases},$$

where $G = E/2(1+\mu)$ – modulus of elasticity of the second kind, $e_i -$ intensity of logarithmic strains, $e_{i\text{T}} -$ intensity of logarithmic strains corresponding to the yield strength, $A, n -$ approximating coefficients of the power function, $E -$ Young's module, $\mu -$ Poisson's ratio.

Let us consider the process of deformation of thin-walled pipes using complex-shaped tooling. Depending on the loading scheme, when increasing the diameter of the pipe using a punch, these operations will be called distribution and broaching, when reducing the diameter using a matrix – crimping (compression) and drawing [4]. In Figure 1(a) and Figure 2 with the application of forces, the distribution, and crimping schemes are shown from above, respectively, with the application of forces from below – broaching and drawing.

In the general case, the equilibrium equations of a thin-walled axisymmetric shell (the shell can be both concave and convex), taking into account the specific friction force acting from the side of the tooling and being under pressure (pressure can be directed both inward and outward) in relation to the tangent and normal to the surface of the element in question, can be represented in the following form [3,4,8]:

$$\left.\begin{aligned} \frac{d}{d\rho}(\sigma_m \rho S) - \sigma_\theta S \pm \frac{q_{fr}\rho}{\sin\alpha} = 0; \\ \pm\frac{\sigma_m}{R_m} \pm \frac{\sigma_\theta}{R_\theta} = \frac{q}{S}\ , \end{aligned}\right\} \qquad (1)$$

where $\sigma_m$ is the meridional main normal stress; $\sigma_\theta$ is the circumferential main normal stress; $q$ is the specific pressure acting from the side of the tooling along the normal to the shell element in question; $R_m$ is the meridional radius of curvature of the median surface of the shell (the sign (+) is set if the direction of the normal curve in the meridional section coincides with the direction of the specific pressure, otherwise we put the sign (–));$R_\theta$ is the circumferential radius of curvature of the median surface of the shell (we put the sign (+) if the direction of the normal of the curve in the circumferential section coincides with the direction of the specific pressure, otherwise we put the sign (–));$q_{fr} = fq$ is the specific friction force acting on the part of the tooling on the element in question shells (we put the sign (+) if the given friction force is directed in the direction of increasing $\rho$, otherwise we put the sign (-));$f$ is the coefficient of friction; S is the thickness of the shell; $\alpha$ is the angle between the tangent to the shell element and its axis of symmetry; $\rho$ is the radius of the circle of the median surface of the shell in a section perpendicular to the axis of the shell.

The circumferential radius of curvature of the median surface of the shell is determined by the formula

$R_\theta = \rho / \cos\alpha$ .

To define $R_m$, we express the curvature of the shell in the meridional direction by $\alpha$ and $\rho$ [9]:

$$\kappa_m = \frac{1}{R_m} = \left|\frac{d\alpha}{dl}\right| = \left|\frac{d\alpha}{d\rho}\right| \sin\alpha \, .$$

Having determined the value of the radii, we write down an equation that allows us to determine the specific pressure:

$$q = \left(\pm\sigma_\theta \frac{\cos\alpha}{\rho} \pm \sigma_m \left|\frac{d\alpha}{d\rho}\right| \sin\alpha\right) S \, . \quad (2)$$

Let us consider the processes of distribution and broaching of a pipe billet (Fig. 1(a)).

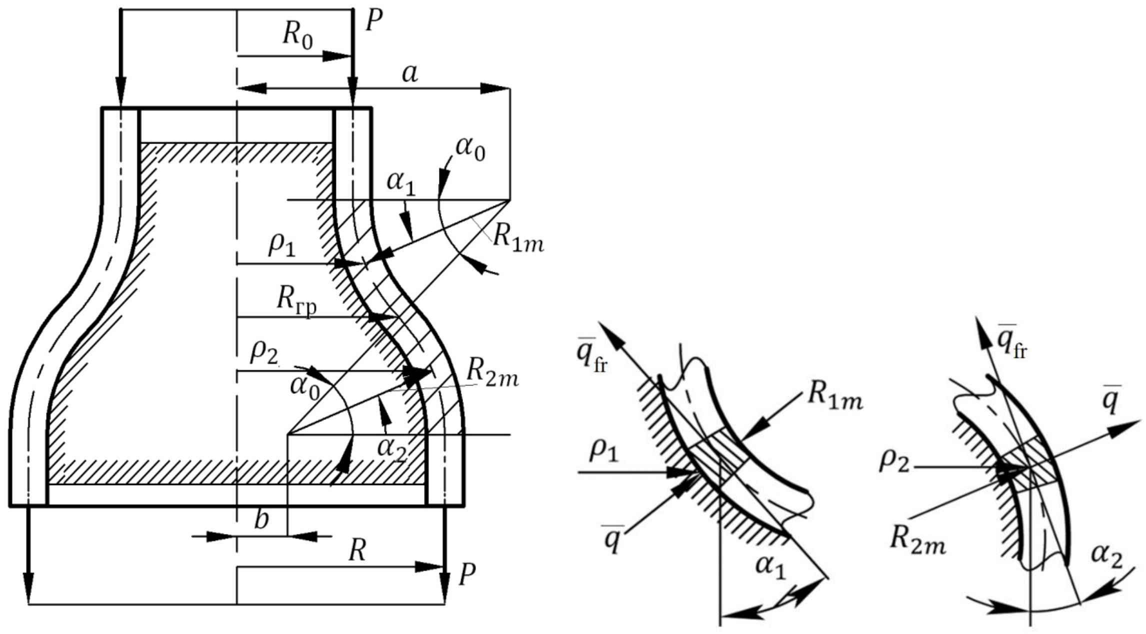



**Fig. 1.** The general case of distribution and broaching of a pipe billet

In general, it is possible to distinguish two sections with different directions of curvature in the meridional section (Fig. 1(b,c)).

For the first section (Fig. 1b), the system of equations (1) can be represented as:

$$\left.\begin{aligned} &\frac{d}{d\rho}(\sigma_m \rho S) - \sigma_\theta S - \frac{q_{fr}\rho}{\sin\alpha} = 0; \\ &q = \left(\sigma_\theta \frac{\cos\alpha}{\rho} - \sigma_m \left|\frac{d\alpha}{d\rho}\right| \sin\alpha\right) S \, . \end{aligned}\right\} \quad (3)$$

Substituting the second equation of the system (3) into the first, and performing simple transformations, it is possible to obtain an equilibrium equation for the first part in the process of distribution and broaching:

$$\frac{d(\sigma_m S)}{d\rho} = \frac{\sigma_\theta (1 + f ctg\, \alpha) - \sigma_m (1 + f\rho \left|\frac{d\alpha}{d\rho}\right|)}{\rho} S \, . \quad (4)$$

For the second section (Fig. 1(c)), the system of equations (3) will have the form:

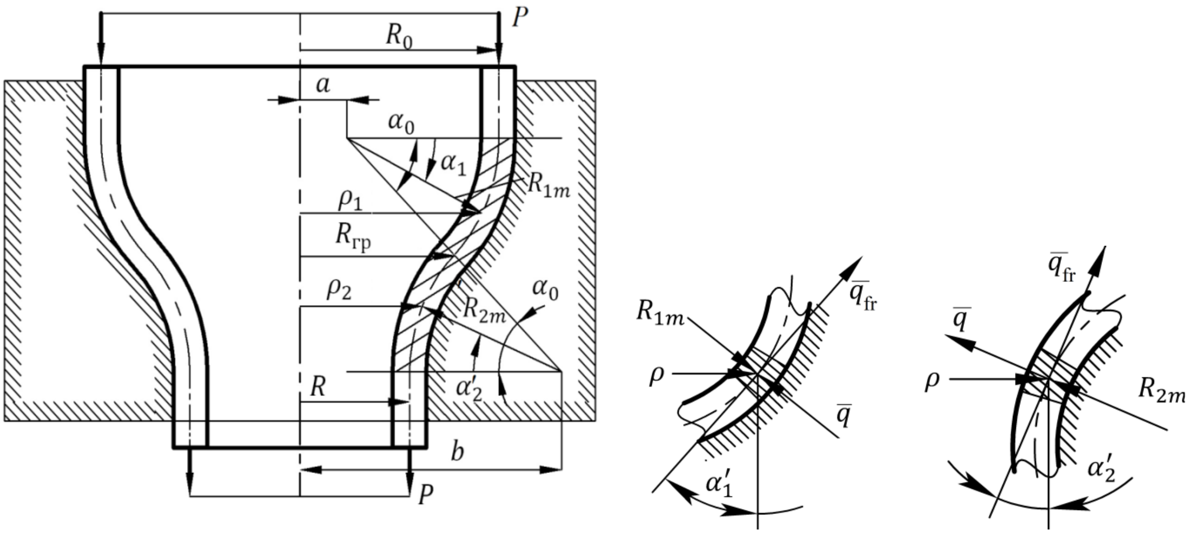
$$\left.\begin{aligned} &\frac{d}{d\rho}(\sigma_m \rho S) - \sigma_\theta S - \frac{q_{fr}\rho}{\sin\alpha} = 0; \\ &q = \left(\sigma_\theta \frac{\cos\alpha}{\rho} + \sigma_m \left|\frac{d\alpha}{d\rho}\right| \sin\alpha\right) S, \end{aligned}\right\} \tag{5}$$

and the equation of equilibrium, respectively

$$\frac{d(\sigma_m S)}{d\rho} = \frac{\sigma_\theta(1 + f ctg\,\alpha) - \sigma_m\left(1 - f\rho\left|\frac{d\alpha}{d\rho}\right|\right)}{\rho} S. \tag{6}$$

Let us consider the process of crimping and drawing a pipe billet (Fig. 2(a)). When applying forces to the billet from above, we have a crimping scheme, and when applied from below, drawing.

As in the case of distribution and broaching, when crimping and drawing, two sections with different directions of curvature in the meridional section can be distinguished (Fig. 2(b,c)).




a) b) c)

**Fig. 2.** The general case of crimping and drawing of a pipe billet

For the first section (Fig. 2(b)), the system of equations (1) can be represented as:

$$\left.\begin{aligned} &\frac{d}{d\rho}(\sigma_m \rho S) - \sigma_\theta S + \frac{q_{fr}\rho}{\sin\alpha'} = 0; \\ &q = \left(-\sigma_\theta \frac{\cos\alpha'}{\rho} - \sigma_m \left|\frac{d\alpha'}{d\rho}\right| \sin\alpha'\right) S. \end{aligned}\right\} \tag{7}$$

Substituting the second equation of the system (7) into the first, and performing simple transformations, it is possible to obtain the equilibrium equation for the first section in the process of crimping and drawing:

$$\frac{d(\sigma_m S)}{d\rho} = \frac{\sigma_\theta(1 + f ctg\,\alpha') - \sigma_m\left(1 - f\rho\left|\frac{d\alpha'}{d\rho}\right|\right)}{\rho} S. \tag{8}$$

For the second section (Fig. 2(c)), the system of equations (7) will have the form (9):

$$\left.\begin{aligned} &\frac{d}{d\rho}(\sigma_m \rho S) - \sigma_\theta S + \frac{q_{fr}\rho}{\sin\alpha'} = 0; \\ &q = \left(-\sigma_\theta \frac{\cos\alpha'}{\rho} + \sigma_m \left|\frac{d\alpha'}{d\rho}\right| \sin\alpha'\right) S, \end{aligned}\right\} \tag{9}$$

and the equation of equilibrium, respectively

$$\frac{d(\sigma_m S)}{d\rho} = \frac{\sigma_\theta(1 + f ctg\,\alpha') - \sigma_m\left(1 + f\rho\left|\frac{d\alpha'}{d\rho}\right|\right)}{\rho} S. \tag{10}$$

Analysis of equations (4), (6), (8), and (10) shows that for all cases of distribution and broaching, crimping, and drawing, one equilibrium equation can be written as

$$\frac{d(\sigma_m S)}{d\rho} = \frac{\sigma_\theta(1+fctg\,\alpha)-\sigma_m(1\pm f\rho\left|\frac{d\alpha}{d\rho}\right|)}{\rho}S\,, \tag{11}$$

where $\alpha$ is the angle between the tangent to the shell element and its axis of symmetry (for distribution and broaching, the angle is counted from the axis of symmetry counterclockwise, for crimping and drawing – clockwise); a sign (–) is placed in the bracket of the second layer if the shell is convex in the meridional section and a sign (+) if concave (Figs. 1, 2).

Considering that for a convex hull always $\frac{d\alpha}{d\rho} < 0$, and for the concave $-\frac{d\alpha}{d\rho} > 0$, the equilibrium equation for all cases of deformation of thin-walled pipes using axisymmetric tooling can be written as follows:

$$\frac{d(\sigma_m S)}{d\rho} = \frac{\sigma_\theta(1+fctg\,\alpha)-\sigma_m(1+f\rho\frac{d\alpha}{d\rho})}{\rho}S\,.$$

To solve this equation by the method of variable elasticity parameters, it is advisable to switch to integral equations [5].

Let us write equation (11) in the form:

$$\frac{d(\sigma_m S)}{d\rho} + \frac{(1+f\rho\frac{d\alpha}{d\rho})}{\rho}(\sigma_m S) = \frac{\sigma_\theta S(1+fctg\,\alpha)}{\rho}\,.$$

This equation can be considered a linear inhomogeneous equation of the first degree

$$Y' + A(\rho)Y = B(\rho), \tag{12}$$

where $Y = \sigma_m S,\ A(\rho) = \frac{(1+f\rho\frac{d\alpha}{d\rho})}{\rho},\ B(\rho) = \frac{\sigma_\theta S(1+fctg\,\alpha)}{\rho}$.

Such an equation can be solved by the Bernoulli method

$$Y = U(\rho)V(\rho),$$

where $U(\rho) = \exp[\int -A(\rho)d\rho\,]\,, V(\rho) = \int \frac{B(\rho)}{U(\rho)}d\rho + C$.

Thus, the general solution of equation (12) can be written as:

$$Y = U(\rho)\left[\int \frac{B(\rho)}{U(\rho)}d\rho + C\right],$$

or

$$\sigma_m S = U(\rho)\left[\int_{\rho_0}^{\rho} \frac{\sigma_\theta S(1+fctg\,\alpha)}{\rho\cdot U(\rho)}d\rho + C\right]. \tag{13}$$

Using equation (13) and the stress-strain coupling equations, in the form:

$$\left.\begin{aligned} \sigma_m &= \frac{E^*}{(1-\mu^{*2})}(e_m+\mu^* e_\theta); \\ \sigma_\theta &= \frac{E^*}{(1-\mu^{*2})}(e_\theta+\mu^* e_m), \end{aligned}\right\}$$

where $E^*$ and $\mu^*$ – variable elasticity parameters.

In this case, taking into account the boundary conditions, it is possible to write the integral equation of equilibrium in deformations:

$$e_m = -\mu^* e_\theta + \\ + \frac{(1-\mu^{*2})U(\rho)}{E^* S}\left[\int_{\rho_0}^{\rho}\frac{E^* S(1+fctg\,\alpha)}{(1-\mu^{*2})\rho U(\rho)}(e_\theta+\mu^* e_m)\,d\rho + \sigma_{m\rho_0}S_{\rho_0}\right] \tag{14}$$

where $\sigma_{m\rho_0}$ and $S_{\rho_0}$ – the meridional stress and thickness of the deformable pipe at one of the boundaries, and the function $U(\rho)$ defined by the equation:

$$U(\rho) = exp\left[-\int \frac{(1 + f\rho\frac{d\alpha}{d\rho})}{\rho} d\rho\right] = \frac{1}{\rho} exp\,(-f\alpha).$$

Substituting the obtained expression into equation (14), we write down the integral equation of equilibrium in deformations for all cases of deformation of cylindrical pipes in axisymmetric matrices of complex shape:

$$e_m = -\mu^* e_\theta + \frac{(1-\mu^{*2})}{E^* S\, \rho \exp(f\alpha)} *$$
$$* \left[\int_{\rho_0}^{\rho} \frac{E^* S(1+fctg\,\alpha)\exp(f\alpha)}{(1-\mu^{*2})} (e_\theta + \mu^* e_m)\, d\rho + \sigma_{m\rho_0} S_{\rho_0}\right] \tag{15}$$

Let us consider the order of solving the problem of determining the stress-strain state during the deformation of cylindrical pipes in matrices of complex shape. To solve the integral equation (15), it is necessary to know the function of the angle change α depending on $\rho$: $\alpha = \alpha(\rho)$.

In the case of the deformation of cylindrical pipes using axisymmetric tooling, circumferential deformations can be considered known and depend only on the coordinate of the point in question:

$$e_\theta = \ln\left(\frac{\rho}{R_0}\right), \tag{16}$$

where $R_0$ – is the initial radius of the median surface of the pipe.

The boundary conditions are determined at the pipe edge opposite to the force application.

The solution for determining the stress-strain state of the pipe, in accordance with the method of variable elasticity parameters, is carried out by the method of successive approximations according to the recurrent scheme using equation (14) for the given boundary conditions:

$$e_m^{(k+1)} = -\mu^{*(k)} \ln\left(\frac{\rho}{R_0}\right) + \frac{\left(1-\mu^{*(k)2}\right)}{E^{*(k)} S^{(k)}\, \rho} *$$
$$* \left[\int_{\rho_0}^{\rho} \frac{E^{*(k)} S^{(k)}\left(1 + fctg\,\alpha(\rho)\right)\exp(f\alpha(\rho))}{\left(1-\mu^{*(k)2}\right)} \left(\ln\left(\frac{\rho}{R_0}\right) + \mu^{*(k)} e_m^{(k)}\right) d\rho + \sigma_{m\rho_0} S_{\rho_0}\right],$$

where the values with index ($k$) and ($k+1$) denote, respectively, their values in the k-th and (k+1)-th approximations.

Numerical integration during crimping and distribution is carried out from $R$ to $R_0$, and during drawing and broaching from $R_0$ to $R$. If the matrix or punch is of variable curvature, integration is carried out in two sections.

As the calculations have shown, the results of the calculations do not depend on the choice of the values of the initial approximation, therefore, in the initial approximation we take:

$$e_m^{(0)} = 0;\ \ S^{(0)} = S_0;\ \ E^{*(0)} = 3G;\ \ \mu^{*(0)} = \mu,$$

where $S_0$ – initial pipe thickness.

Then the deformations are calculated by the thickness of the pipe:

$$e_z^{(k+1)} = \frac{\mu^{*(k)}}{(\mu^{*(k)} - 1)}\left(\ln\left(\frac{\rho}{R_0}\right) + e_m^{(k+1)}\right).$$

This strain is necessary to determine the intensity of strains

After determining the strain state, the stress state of the pipe is determined using the equations of the relationship between stresses and deformations:

$$\left.\begin{aligned}\sigma_m{}^{(k+1)} &= \frac{E^{*(k)}}{\left(1-\mu^{*(k)2}\right)}\left(e_m{}^{(k+1)}+\mu^{*(k)}e_\theta{}^{(k+1)}\right);\\ \sigma_\theta{}^{(k+1)} &= \frac{E^{*(k)}}{\left(1-\mu^{*(k)2}\right)}\left(e_\theta{}^{(k+1)}+\mu^{*(k)}e_m{}^{(k+1)}\right).\end{aligned}\right\}$$

Next, the intensity of stresses and the intensity of deformations are determined and the value is specified $E_{\text{sec}}$, using the equation of approximation of the deformation diagram by a power function:

$$E_{\text{sec}}{}^{(k+1)} = \frac{A\left(e_i{}^{(k+1)}\right)^n}{e_i{}^{(k+1)}}.$$

Then the value of the variable elasticity parameters is specified [4]:

$$E^{*(k+1)} = \frac{E_{\text{sec}}{}^{(k+1)}}{1+\frac{1-2\mu}{3E}E_{\text{sec}}{}^{(k+1)}};$$

$$\mu^{*(k+1)} = \frac{\frac{1}{2}-\frac{1-2\mu}{3E}E_{\text{sec}}{}^{(k+1)}}{1+\frac{1-2\mu}{3E}E_{\text{sec}}{}^{(k+1)}}.$$

To control the convergence of the process, the values of stress intensities are compared: $\left|\sigma_i{}^{(k+1)}-\sigma_i{}^{(k)}\right| \le \Delta\sigma_i$.

The calculation is continued until the specified accuracy is reached.

## Research results

To verify the reliability of the obtained integral equations, a calculation and comparison were carried out with the well-known analytical solution for crimping a pipe billet in a curved matrix with a constant radius in the meridional direction (Fig. 3), described in [3, p. 387].

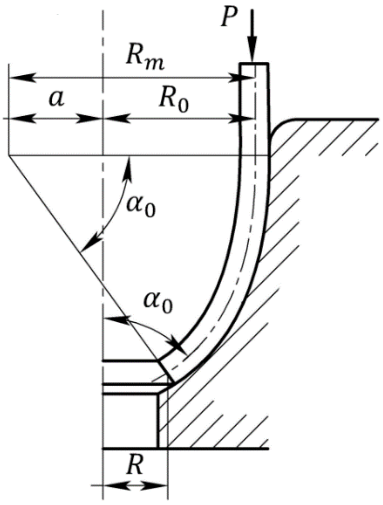



**Fig. 3.** Scheme of crimping a pipe billet in a curved matrix with a constant radius in the meridional direction

The analytical calculation was carried out for absolutely plastic material and the Tresca-Saint-Venant plasticity condition was used.

Using the designations we have adopted and the deformation scheme (Fig. 2), we obtain the following relations:

$$a = R_0 - R_m\,;\ \rho = R_m\cos\alpha + a\,;\ d\rho = -R_m\sin\alpha\, d\alpha. \tag{17}$$

Taking into account the accepted ratios, the analytical solution determining the dependence of stresses on the angle α can be represented as:

$$\sigma_m = -\sigma_{\mathrm{T}}\beta\big((1+f\alpha)\cos\alpha - (1+f\alpha_0)\cos\alpha_0 + 2f(\sin\alpha_0 - \sin\alpha)\big) * \frac{R_m\, exp\,(-f\alpha)}{(R_m\cos\alpha + a)}\ ;$$

$$\sigma_\theta = -\sigma_{\mathrm{T}}\beta,$$

where $\sigma_m$ – meridional stresses, $\sigma_\theta$ – circumferential stresses, $\beta$ – coefficient taking into account the influence of the average main stress ($1 \le \beta \le 2/\sqrt{3}$).

In order to obtain by the method of variable elasticity parameters of the solution depending on the angle α, it is necessary for equations (15), (16) to replace the current radius $\rho$ with the current angle α, using the given relations (16) and to integrate along the angle α:

$$e_m = -\mu^* e_\theta + \frac{(1-\mu^{*2})\, exp(-f\alpha)}{E^*S\,(R_m\cos\alpha + a)} * \left[\int_{\alpha_0}^{\alpha} -\frac{E^*SR_m(\sin\alpha + f\cos\alpha)\exp(f\alpha)}{(1-\mu^{*2})}(e_\theta + \mu^* e_m)\, d\alpha\right]; \qquad (18)$$

$$e_\theta = \ln\left(\frac{R_m \cos\alpha + a}{R_0}\right). \qquad (19)$$

Further determination of the stress-strain state is carried out according to the algorithm described above.

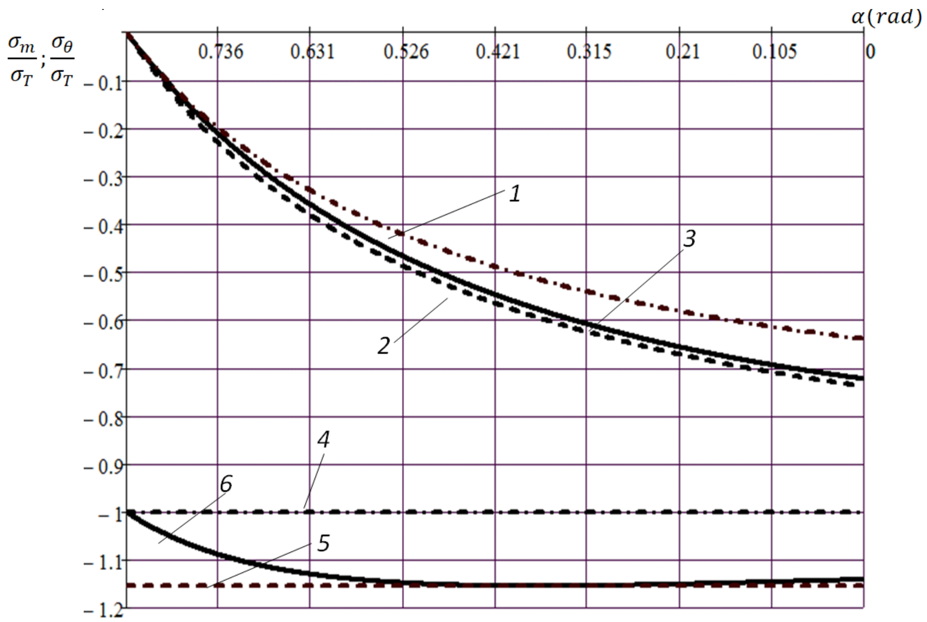



**Fig. 4.** Comparison of the results of calculations obtained by the method of variable elasticity parameters without taking into account changes in thickness with an analytical solution without taking into account changes in thickness:
1 – analytical solution for $\beta = 1$ for $\sigma_m/\sigma_T$; 2 – analytical solution for $\beta = 2/\sqrt{3}$ for $\sigma_m/\sigma_T$ ; 3 – solution by the method of variable elasticity parameters for $\sigma_m/\sigma_T$; 4 – analytical solution for $\beta = 1$ for $\sigma_\theta/\sigma_T$; 5 – analytical solution for $\beta = 2/\sqrt{3}$ for $\sigma_\theta/\sigma_T$ ; 6 – analytical solution for $\sigma_\theta/\sigma_T$

Figure 4 shows the results of calculations for an absolutely plastic material obtained analytically at two extreme values of $\beta$ and by the method of variable elasticity parameters without taking into account thickness changes.

As expected, the results obtained by the method of variable elasticity parameters using equations (18 and 19) lie between the results obtained for the extreme values of $\beta$. Moreover, the smallest difference between the two solutions is observed when $\beta = 2/\sqrt{3}$.

Figure 5 shows the results of calculations for an absolutely plastic material obtained analytically without taking into account changes in thickness and by the method of variable elasticity parameters taking into account changes in thickness.

As can be seen from the comparison of Figs. 4 and 5, calculations without taking into account the thickness change give an underestimated value of the required deformation force by more than 20 %.

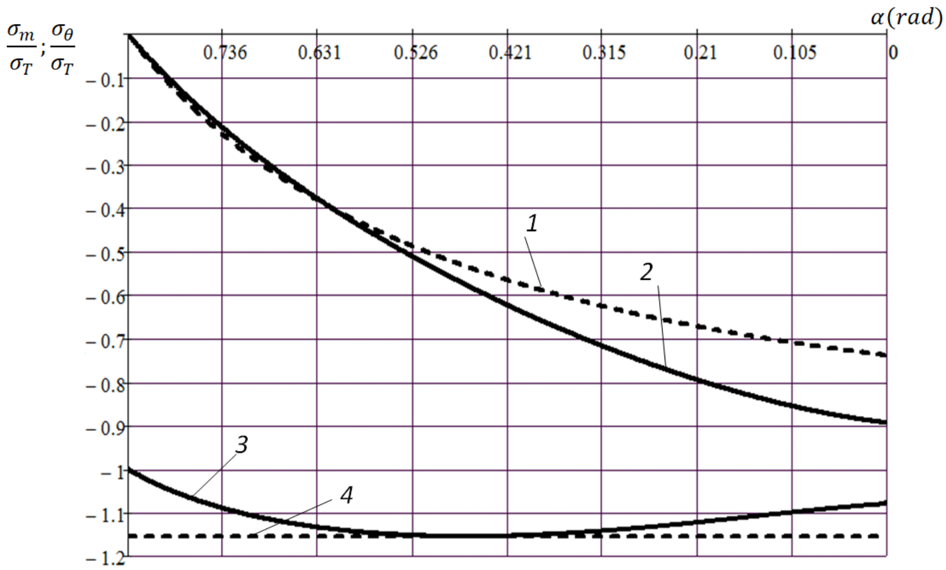



**Fig. 5.** Comparison of the results of calculations obtained by the method of variable elasticity parameters, taking into account the change in thickness, with an analytical solution without taking into account the change in thickness:

1 – analytical solution without taking into account the change in thickness at $\beta = 2/\sqrt{3}$ for $\sigma_m/\sigma_T$ ; 2 – solution by the method of variable elasticity parameters taking into account the change in thickness for $\sigma_m/\sigma_T$ ; 3 – analytical solution without taking into account the change in thickness at $\beta = 2/\sqrt{3}$ for $\sigma_\theta/\sigma_T$ ; 4 – solution by the method of variable elasticity parameters taking into account the change in thickness for $\sigma_\theta/\sigma_T$

## Conclusion

A comparison of the results of calculations obtained by the method of variable elasticity parameters with the known analytical solution showed the reliability of the obtained integral equations. With the well-known equation of a curve forming the shape of a curved matrix or a curved punch, as well as with a well-known material deformation diagram, the obtained integral equations allow solving the problems of deformation of pipe blanks by the method of variable elasticity parameters, taking into account changes in thickness and nonlinear plasticity.

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