$$
\begin{gathered}
s^2 G_{1kl}^L = \frac{\partial^2 G_{1kl}^L}{\partial x_1^2} + \mu \frac{\partial^2 G_{1kl}^L}{\partial x_2^2} + \mu k_T^2 \left(\frac{\partial G_{3kl}^L}{\partial x_1} - G_{1kl}^L \right) + (\lambda + \mu) \frac{\partial^2 G_{2kl}^L}{\partial x_1 \partial x_2} + \sum_{j=1}^N \alpha_j \frac{\partial G_{j+3,kl}^L}{\partial x_1}, \\
s^2 G_{2kl}^L = \mu \frac{\partial^2 G_{2kl}^L}{\partial x_1^2} + \frac{\partial^2 G_{2kl}^L}{\partial x_2^2} + \mu k_T^2 \left(\frac{\partial G_{3kl}^L}{\partial x_2} - G_{2kl}^L \right) + (\lambda + \mu) \frac{\partial^2 G_{1kl}^L}{\partial x_1 \partial x_2} + \sum_{j=1}^N \alpha_j \frac{\partial G_{j+3,kl}^L}{\partial x_2}, \\
s^2 G_{3kl}^L = \mu k_T^2 \left(\frac{\partial^2 G_{3kl}^L}{\partial x_1^2} - \frac{\partial G_{1kl}^L}{\partial x_1} + \frac{\partial^2 G_{3kl}^L}{\partial x_2^2} - \frac{\partial G_{2kl}^L}{\partial x_2} \right), \\
s G_{q+3,kl}^L + \tau_q s^2 G_{q+3,kl}^L = D_q \left(\frac{\partial^2 G_{q+3,kl}^L}{\partial x_1^2} + \frac{\partial^2 G_{q+3,kl}^L}{\partial x_2^2} \right) + \Lambda_q \left(\frac{\partial^3 G_{1kl}^L}{\partial x_1^3} + \frac{\partial^3 G_{2kl}^L}{\partial x_1^2 \partial x_2} + \frac{\partial^3 G_{1kl}^L}{\partial x_1 \partial x_2^2} + \frac{\partial^3 G_{2kl}^L}{\partial x_2^3} \right);
\end{gathered} \tag{20}
$$

$$
\begin{gathered}
\left. \left(\frac{\partial G_{1kl}^L}{\partial x_1} + \lambda \frac{\partial G_{2kl}^L}{\partial x_2} + \sum_{j=1}^N \alpha_j G_{j+3,kl}^L \right) \right|_{x_1=0} = \delta_{1k} \delta_{1l} \delta(x_2 - \zeta), \\
\left. \left(\lambda \frac{\partial G_{1kl}^L}{\partial x_1} + \frac{\partial G_{2kl}^L}{\partial x_2} + \sum_{j=1}^N \alpha_j G_{j+3,kl}^L \right) \right|_{x_2=0} = \delta_{1k} \delta_{2l} \delta(x_1 - \xi), \\
\left. \left(\frac{\partial G_{1kl}^L}{\partial x_1} + \lambda \frac{\partial G_{2kl}^L}{\partial x_2} + \sum_{j=1}^N \alpha_j G_{j+3,kl}^L \right) \right|_{x_1=l_1} = 0, \quad \left. \left(\lambda \frac{\partial G_{1kl}^L}{\partial x_1} + \frac{\partial G_{2kl}^L}{\partial x_2} + \sum_{j=1}^N \alpha_j G_{j+3,kl}^L \right) \right|_{x_2=l_2} = 0, \\
\left. G_{2kl}^L \right|_{x_1=0} = \delta_{2k} \delta_{1l} \delta(x_2 - \zeta), \quad \left. G_{2kl}^L \right|_{x_1=l_1} = 0, \quad \left. G_{1kl}^L \right|_{x_2=0} = \delta_{2k} \delta_{2l} \delta(x_1 - \xi), \quad \left. G_{1kl}^L \right|_{x_2=l_2} = 0, \\
\left. G_{3kl}^L \right|_{x_1=0} = \delta_{3k} \delta_{1l} \delta(x_2 - \zeta), \quad \left. G_{3kl}^L \right|_{x_1=l_1} = 0, \quad \left. G_{3kl}^L \right|_{x_2=0} = \delta_{3k} \delta_{2l} \delta(x_1 - \xi), \quad \left. G_{3kl}^L \right|_{x_2=l_2} = 0, \\
\left. G_{q+1,kl}^L \right|_{x_1=0} = \delta_{q+3,k} \delta_{1l} \delta(x_2 - \zeta), \quad \left. G_{q+1,kl}^L \right|_{x_1=l_1} = 0, \\
\left. G_{q+1,kl}^L \right|_{x_2=0} = \delta_{q+3,k} \delta_{2l} \delta(x_1 - \xi), \quad \left. G_{q+1,kl}^L \right|_{x_2=l_2} = 0.
\end{gathered} \tag{21}
$$

Multiplying the first equation in (20) by $\cos\lambda_n x_1 \sin\mu_m x_2$, the second equation by $\sin\lambda_n x_1 \cos\mu_m x_2$, the third and fourth by equation $\sin\lambda_n x_1 \sin\mu_m x_2$, $\lambda_n = \pi n / l_1$, $\mu_m = \pi m / l_2$ and integrating into the rectangle $D = [0, l_1] \times [0, l_2]$ taking into account conditions (21), we obtain

$$
\begin{gathered}
k_{1nm}(s) G_{1klnm}^{LF}(\xi, \zeta, s) + K_{nm} G_{2klnm}^{LF}(\xi, \zeta, s) - \\
- \mu k_T^2 \lambda_n G_{3klnm}^{LF}(\xi, \zeta, s) - \lambda_n \sum_{j=1}^N \alpha_j G_{j+3,klnm}^{LF}(\xi, \zeta, s) = F_{1klnm}(\xi, \zeta) \\
K_{nm} G_{1klnm}^{LF}(\xi, \zeta, s) + k_{2nm}(s) G_{2klnm}^{LF}(\xi, \zeta, s) - \\
- \mu k_T^2 \mu_m G_{3klnm}^{LF}(\xi, \zeta, s) - \mu_m \sum_{j=1}^N \alpha_j G_{j+3,klnm}^{LF}(\xi, \zeta, s) = F_{2klnm}(\xi, \zeta), \\
- \mu k_T^2 \left[\lambda_n G_{1klnm}^{LF}(\xi, \zeta, s) + \mu_m G_{2klnm}^{LF}(\xi, \zeta, s) \right] + k_{3nm}(s) G_{3klnm}^{LF}(\xi, \zeta, s) = F_{3klnm}(\xi, \zeta), \\
- M_{1qnm} G_{1klnm}^{LF}(\xi, \zeta, s) - M_{2qnm} G_{2klnm}^{LF}(\xi, \zeta, s) + k_{q+3,nm}(s) G_{q+3,klnm}^{LF}(\xi, \zeta, s) = F_{q+3,klnm}(\xi, \zeta),
\end{gathered} \tag{22}
$$

where

$$
\begin{gathered}
k_{1nm}(s) = s^2 + \mu k_T^2 + \lambda_n^2 + \mu \mu_m^2, \quad k_{2nm}(s) = s^2 + \mu k_T^2 + \mu \lambda_n^2 + \mu_m^2, \quad \Lambda_q^* = \Lambda_q (\lambda - 1), \\
k_{3nm}(s) = s^2 + \mu k_T^2 \left(\lambda_n^2 + \mu_m^2 \right), \quad k_{q+3,nm}(s) = s + \tau_q s^2 + D_q \left(\lambda_n^2 + \mu_m^2 \right), \\
M_{1qnm} = \Lambda_q \lambda_n \left(\lambda_n^2 + \mu_m^2 \right), \quad M_{2qnm} = \Lambda_q \mu_m \left(\lambda_n^2 + \mu_m^2 \right), \quad K_{nm} = (\lambda + \mu) \lambda_n \mu_m,
\end{gathered}
$$

$$F_{1klnm}(\xi,\zeta) = -\frac{4}{l_1 l_2}\left(\delta_{1k} + \mu k_T^2 \delta_{3k}\right)\delta_{1l}\sin\mu_m\zeta + \frac{4\mu\mu_m}{l_1 l_2}\left(\delta_{1l}\cos\mu_m\zeta + \delta_{2l}\cos\lambda_n\xi\right)\delta_{2k},$$

$$F_{2klnm}(\xi,\zeta) = -\frac{4}{l_1 l_2}\left(\delta_{1k} + \mu k_T^2 \delta_{3k}\right)\delta_{2l}\sin\lambda_n\xi + \frac{4\mu\lambda_n}{l_1 l_2}\left(\delta_{1l}\cos\mu_m\zeta + \delta_{2l}\cos\lambda_n\xi\right)\delta_{2k},$$

$$F_{3klnm}(\xi,\zeta) = \frac{4\mu k_T^2}{l_1 l_2}\left(\lambda_n\delta_{1l}\sin\mu_m\zeta + \mu_m\delta_{2l}\sin\lambda_n\xi\right)\delta_{3k},$$

$$F_{q+3,klnm}(\xi,\zeta) = \frac{4\Lambda_q}{l_1 l_2}\left(\lambda_n\delta_{1l}\sin\mu_m\zeta + \mu_m\delta_{2l}\sin\lambda_n\xi\right)\delta_{1k} + \frac{4\lambda_n\mu_m\Lambda_q^*}{l_1 l_2}\left(\delta_{1l}\cos\mu_m\zeta + \delta_{2l}\cos\lambda_n\xi\right)\delta_{2k} +$$
$$+\frac{4\lambda_n}{l_1 l_2}\left(D_q\delta_{q+3,k} - \Lambda_q\sum_{j=1}^{N}\alpha_j\delta_{j+3,k}\right)\delta_{1l}\sin\mu_m\zeta + \frac{4\mu_m}{l_1 l_2}\left(D_q\delta_{q+3,k} - \Lambda_q\sum_{j=1}^{N}\alpha_j\delta_{j+3,k}\right)\delta_{2l}\sin\lambda_n\xi;$$

$$G_{1klnm}^{LF}(\xi,\zeta,s) = \frac{4}{l_1 l_2}\int_0^{l_1}\int_0^{l_2} G_{1kl}^{L}(x_1,x_2,\xi,\zeta,s)\cos\lambda_n x_1\sin\mu_m x_2\,dx_2\,dx_1,$$

$$G_{2klnm}^{LF}(\xi,\zeta,s) = \frac{4}{l_1 l_2}\int_0^{l_1}\int_0^{l_2} G_{2kl}^{L}(x_1,x_2,\xi,\zeta,s)\sin\lambda_n x_1\cos\mu_m x_2\,dx_2\,dx_1, \tag{23}$$

$$G_{pklnm}^{LF}(\xi,\zeta,s) = \frac{4}{l_1 l_2}\int_0^{l_1}\int_0^{l_2} G_{pkl}^{L}(x_1,x_2,\xi,\zeta,s)\sin\lambda_n x_1\sin\mu_m x_2\,dx_2\,dx_1,$$

$$G_{1kl}^{L}(x_1,x_2,\xi,\zeta,\tau) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} G_{1klnm}^{LF}(\xi,\zeta,\tau)\cos\lambda_n x_1\sin\mu_m x_2,$$

$$G_{2kl}^{L}(x_1,x_2,\xi,\zeta,\tau) = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} G_{2klnm}^{LF}(\xi,\zeta,\tau)\sin\lambda_n x_1\cos\mu_m x_2,$$

$$G_{pkl}^{L}(x_1,x_2,\xi,\zeta,\tau) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} G_{pklnm}^{LF}(\xi,\zeta,\tau)\sin\lambda_n x_1\sin\mu_m x_2, \quad p \ge 3.$$

The solution of the system (22) has the form $\left(q,p=\overline{1,N},\ \ k=\overline{1,N+3},\ \ l=1,2\right)$

$$G_{iklnm}^{LF}(\xi,\zeta,s) = \frac{P_{iklnm}(\xi,\zeta,s)}{P_{nm}(\xi,\zeta,s)} \quad (i=1,2), \tag{24}$$

$$G_{3klnm}^{LF}(\xi,\zeta,s) = \frac{4\mu k_T^2}{l_1 l_2}\frac{\left(\lambda_n\delta_{1l}\sin\mu_m\zeta + \mu_m\delta_{2l}\sin\lambda_n\xi\right)\delta_{3k}}{k_{3nm}(s)} + \frac{P_{3klnm}(\xi,\zeta,s)}{Q_{3nm}(\xi,\zeta,s)},$$

$$G_{q+3,klnm}^{LF}(\xi,\zeta,s) = \frac{P_{q+3,klnm}(\xi,\zeta,s)}{Q_{q+3,nm}(\xi,\zeta,s)} + \frac{4}{l_1 l_2}\frac{\Lambda_q\left(\lambda_n\delta_{1l}\sin\mu_m\zeta + \mu_m\delta_{2l}\sin\lambda_n\xi\right)\delta_{1k}}{k_{q+3,nm}(s)} +$$
$$+\frac{4\lambda_n\mu_m\Lambda_q^*}{l_1 l_2}\frac{\left(\delta_{1l}\cos\mu_m\zeta + \delta_{2l}\cos\lambda_n\xi\right)\delta_{2k}}{k_{q+3,nm}(s)} +$$
$$+\frac{4}{l_1 l_2}\frac{\lambda_n\left(D_q\delta_{q+3,k} - \Lambda_q\sum_{j=1}^{N}\alpha_j\delta_{j+3,k}\right)\delta_{1l}\sin\mu_m\zeta + \mu_m\left(D_q\delta_{q+3,k} - \Lambda_q\sum_{j=1}^{N}\alpha_j\delta_{j+3,k}\right)\delta_{2l}\sin\lambda_n\xi}{k_{q+3,nm}(s)},$$

where

$$P_{nm}(s)=\left[k_{1nm}(s)k_{2nm}(s)-K_{nm}^2\right]k_{3nm}(s)\Pi_{nm}(s)-$$
$$-\mu^2k_T^4\left[\mu_m^2k_{1nm}(s)+\lambda_n^2k_{2nm}(s)-2K_{nm}\mu_m\lambda_n\right]\Pi_{nm}(s)-$$
$$-k_{3nm}(s)\sum_{j=1}^{N}\alpha_j\left\{\left[\mu_mk_{1nm}(s)-\lambda_nK_{nm}\right]M_{2jnm}+\left[\lambda_nk_{2nm}(s)-\mu_mK_{nm}\right]M_{1jnm}\right\}\Pi_{jnm}(s),$$
$$Q_{pnm}(s)=k_{pnm}(s)P_{nm}(s),\quad p\geq 3,$$
$$\Pi_{nm}(s)=\prod_{j=1}^{N}k_{j+3,nm}(s),\quad \Pi_{jnm}(s)=\prod_{r=1,r\neq j}^{N}k_{r+3,nm}(s),\quad \Pi_{ijnm}(s)=\prod_{r=1,r\neq i,j}^{N}k_{r+3,nm}(s).$$
$$S_{1nm}(s)=k_{2nm}(s)k_{3nm}(s)-\mu^2k_T^4\mu_m^2,\quad S_{2nm}(s)=k_{1nm}(s)k_{3nm}(s)-\mu^2k_T^4\lambda_n^2,$$
$$S_{3nm}(s)=K_{nm}k_{3nm}(s)-\mu^2k_T^4\mu_m\lambda_n.$$

The rest of the polynomials $P_{iklnm}(\xi,\zeta,s)$, included in the solution in solution (24) are defined as:

$$P_{111nm}(\xi,\zeta,s)=-\frac{4}{l_1l_2}\left[S_{1nm}(s)\Pi_{nm}(s)-\mu_mk_{3nm}(s)\sum_{j=1}^{N}\alpha_jM_{2jnm}\Pi_{jnm}(s)\right]\sin\mu_m\zeta+$$
$$+\frac{4\lambda_n}{l_1l_2}\left[\lambda_nS_{1nm}(s)-\mu_mS_{3nm}(s)\right]\sum_{j=1}^{N}\alpha_j\Lambda_j\Pi_{jnm}(s)\sin\mu_m\zeta,$$
$$P_{112nm}(\xi,\zeta,s)=\frac{4}{l_1l_2}\left[S_{3nm}(s)\Pi_{nm}(s)-\lambda_nk_{3nm}(s)\sum_{j=1}^{N}\alpha_jM_{2jnm}\Pi_{jnm}(s)\right]\sin\lambda_n\xi+$$
$$+\frac{4\mu_m}{l_1l_2}\left[\lambda_nS_{1nm}(s)-\mu_mS_{3nm}(s)\right]\sum_{j=1}^{N}\alpha_j\Lambda_j\Pi_{jnm}(s)\sin\lambda_n\xi,$$
$$P_{121nm}(\xi,\zeta,s)=\frac{4}{l_1l_2}\mu\left[\mu_mS_{1nm}(s)-\lambda_nS_{3nm}(s)\right]\Pi_{nm}(s)\cos\mu_m\zeta+$$
$$+\frac{4\mu}{l_1l_2}k_{3nm}(s)\left(\lambda_n^2-\mu_m^2\right)\sum_{j=1}^{N}\alpha_jM_{2jnm}\Pi_{jnm}(s)\cos\mu_m\zeta+$$
$$+\frac{4\lambda_n\mu_m}{l_1l_2}\left[\lambda_nS_{1nm}(s)-\mu_mS_{3nm}(s)\right]\sum_{j=1}^{N}\alpha_j\Lambda_j^*\Pi_{jnm}(s)\cos\mu_m\zeta,$$
$$P_{122nm}(\xi,\zeta,s)=\frac{4}{l_1l_2}\mu\left[\mu_mS_{1nm}(s)-\lambda_nS_{3nm}(s)\right]\Pi_{nm}(s)\cos\lambda_n\xi+$$
$$+\frac{4\mu}{l_1l_2}k_{3nm}(s)\left(\lambda_n^2-\mu_m^2\right)\sum_{j=1}^{N}\alpha_jM_{2jnm}\Pi_{jnm}(s)\cos\lambda_n\xi+$$
$$+\frac{4\lambda_n\mu_m}{l_1l_2}\left[\lambda_nS_{1nm}(s)-\mu_mS_{3nm}(s)\right]\sum_{j=1}^{N}\alpha_j\Lambda_j^*\Pi_{jnm}(s)\cos\lambda_n\xi,$$
$$P_{131nm}(\xi,\zeta,s)=\frac{4}{l_1l_2}\mu k_T^2\left[\mu k_T^2\lambda_n\left(\lambda_nk_{2nm}(s)-\mu_mK_{nm}\right)-S_{1nm}(s)\right]\Pi_{nm}(s)\sin\mu_m\zeta+$$
$$+\frac{4}{l_1l_2}\mu k_T^2\mu_mk_{3nm}(s)\sum_{j=1}^{N}\alpha_jM_{2jnm}\Pi_{jnm}(s)\sin\mu_m\zeta,$$
$$P_{132nm}(\xi,\zeta,s)=\frac{4}{l_1l_2}\mu k_T^2\left[S_{3nm}(s)+\mu k_T^2\mu_m\left(\lambda_nk_{2nm}(s)-\mu_mK_{nm}\right)\right]\Pi_{nm}(s)\sin\lambda_n\xi+$$
$$-\frac{4}{l_1l_2}\mu k_T^2\lambda_nk_{3nm}(s)\sum_{j=1}^{N}\alpha_jM_{2jnm}\Pi_{jnm}(s)\sin\lambda_n\xi,$$

$$
\begin{gathered}
P_{1,q+3,1nm}(\xi,\zeta,s)=\frac{4\alpha_q\lambda_n}{l_1l_2}\left[\lambda_nS_{1nm}(s)-\mu_mS_{3nm}(s)\right]\left[D_q\Pi_{qnm}(s)-\Lambda_q\sum_{j=1}^{N}\alpha_j\Pi_{jnm}(s)\right]\sin\mu_m\zeta,\\
P_{1,q+3,2nm}(\xi,\zeta,s)=\frac{4\alpha_q\mu_m}{l_1l_2}\left[\lambda_nS_{1nm}(s)-\mu_mS_{3nm}(s)\right]\left[D_q\Pi_{qnm}(s)-\Lambda_q\sum_{j=1}^{N}\alpha_j\Pi_{jnm}(s)\right]\sin\lambda_n\xi,\\
P_{211nm}(\xi,\zeta,s)=\frac{4}{l_1l_2}\left[S_{3nm}(s)\Pi_{nm}(s)-\mu_mk_{3nm}(s)\sum_{j=1}^{N}\alpha_jM_{1jnm}\Pi_{jnm}(s)\right]\sin\mu_m\zeta+\\
+\frac{4\lambda_n}{l_1l_2}\left[\mu_mS_{2nm}(s)-\lambda_nS_{3nm}(s)\right]\sum_{j=1}^{N}\alpha_j\Lambda_j\Pi_{jnm}(s)\sin\mu_m\zeta,\\
P_{212nm}(\xi,\zeta,s)=-\frac{4}{l_1l_2}\left[S_{2nm}(s)\Pi_{nm}(s)-\lambda_nk_{3nm}(s)\sum_{j=1}^{N}\alpha_jM_{1jnm}\Pi_{jnm}(s)\right]\sin\lambda_n\xi+\\
+\frac{4\mu_m}{l_1l_2}\left[\mu_mS_{2nm}(s)-\lambda_nS_{3nm}(s)\right]\sum_{j=1}^{N}\alpha_j\Lambda_j\Pi_{jnm}(s)\sin\lambda_n\xi,\\
P_{221nm}(\xi,\zeta,s)=\frac{4}{l_1l_2}\mu\left[\lambda_nS_{2nm}(s)-\mu_mS_{3nm}(s)\right]\Pi_{nm}(s)\cos\mu_m\zeta+\\
+\frac{4\mu}{l_1l_2}\left(\mu_m^2-\lambda_n^2\right)k_{3nm}(s)\sum_{j=1}^{N}\alpha_jM_{1jnm}\Pi_{jnm}(s)\cos\mu_m\zeta+\\
+\frac{4\lambda_n\mu_m}{l_1l_2}\left[\mu_mS_{2nm}(s)-\lambda_nS_{3nm}(s)\right]\sum_{j=1}^{N}\alpha_j\Lambda_j^*\Pi_{jnm}(s)\cos\mu_m\zeta,\\
P_{222nm}(\xi,\zeta,s)=\frac{4\mu}{l_1l_2}\left[\lambda_nS_{2nm}(s)-\mu_mS_{3nm}(s)\right]\Pi_{nm}(s)\cos\lambda_n\xi+\\
+\frac{4\mu}{l_1l_2}\left(\mu_m^2-\lambda_n^2\right)k_{3nm}(s)\sum_{j=1}^{N}\alpha_jM_{1jnm}\Pi_{jnm}(s)\cos\lambda_n\xi+\\
+\frac{4\lambda_n\mu_m}{l_1l_2}\left[\mu_mS_{2nm}(s)-\lambda_nS_{3nm}(s)\right]\sum_{j=1}^{N}\alpha_j\Lambda_j^*\Pi_{jnm}(s)\cos\lambda_n\xi,\\
P_{231nm}(\xi,\zeta,s)=\frac{4\mu}{l_1l_2}k_T^2\left[S_{3nm}(s)+\mu k_T^2\lambda_n\left(\mu_mk_{1nm}(s)-\lambda_nK_{nm}\right)\right]\Pi_{nm}(s)\sin\mu_m\zeta-\\
-\frac{4\mu}{l_1l_2}k_T^2\mu_mk_{3nm}(s)\sum_{j=1}^{N}\alpha_jM_{1jnm}\Pi_{jnm}(s)\sin\mu_m\zeta,\\
P_{232nm}(\xi,\zeta,s)=\frac{4\mu}{l_1l_2}k_T^2\left[\mu k_T^2\mu_m\left(\mu_mk_{1nm}(s)-\lambda_nK_{nm}\right)-S_{2nm}(s)\right]\Pi_{nm}(s)\sin\lambda_n+\\
+\frac{4\mu}{l_1l_2}k_T^2\lambda_nk_{3nm}(s)\sum_{j=1}^{N}\alpha_jM_{1jnm}\Pi_{jnm}(s)\sin\lambda_n\xi,\\
P_{2,q+3,1nm}(\xi,\zeta,s)=\frac{4\alpha_q}{l_1l_2}\lambda_n\left[\mu_mS_{2nm}(s)-\lambda_nS_{3nm}(s)\right]\left[D_q\Pi_{qnm}(s)-\Lambda_q\sum_{j=1}^{N}\alpha_j\Pi_{jnm}(s)\right]\sin\mu_m\zeta,\\
P_{2,q+3,2nm}(\xi,\zeta,s)=\frac{4\alpha_q}{l_1l_2}\mu_m\left[\mu_mS_{2nm}(s)-\lambda_nS_{3nm}(s)\right]\left[D_q\Pi_{qnm}(s)-\Lambda_q\sum_{j=1}^{N}\alpha_j\Pi_{jnm}(s)\right]\sin\lambda_n\xi,\\
P_{3klnm}(\xi,\zeta,s)=\mu k_T^2\left[\lambda_nP_{1klnm}(\xi,\zeta,s)+\mu_mP_{2klnm}(\xi,\zeta,s)\right],\\
P_{q+3,klnm}(\xi,\zeta,s)=M_{1qnm}P_{1klnm}(\xi,\zeta,s)+M_{2qnm}P_{2klnm}(\xi,\zeta,s).
\end{gathered}
\tag{25}
$$

The Laplace originals of the functions in (24) have the form [21] ($i,l=1,2$, $q=\overline{1,N}$, $j=\overline{1,2N+6}$, $k=\overline{1,N+3},$)

$$G_{iklnm}^{F}(\xi,\zeta,\tau)=\sum_{j=1}^{2N+6}A_{1klnm}^{(j)}(\xi,\zeta)e^{s_{jnm}\tau},$$

$$G_{3klnm}^{F}(\xi,\zeta,\tau)=\sum_{j=1}^{2N+8}A_{3klnm}^{(j)}(\xi,\zeta)e^{s_{jnm}\tau}+\frac{4\mu k_T^2}{l_1l_2}\sum_{j=2N+7}^{2N+8}\frac{(\lambda_n\delta_{1l}\sin\mu_m\zeta+\mu_m\delta_{2l}\sin\lambda_n\xi)\delta_{3k}}{k'_{3nm}(s_{jnm})}e^{s_{jnm}\tau},$$

$$G_{q+3,1lnm}^{F}(\xi,\zeta,\tau)=\sum_{j=1}^{2N+6}A_{q+3,1lnm}^{(j)}(\xi,\zeta)e^{s_{jnm}\tau}+\sum_{j=1}^{2}A_{q+3,1lnm}^{(2N+6+j)}(\xi,\zeta)e^{\gamma_{jnm}^{(q)}\tau}+$$
$$+\frac{4\Lambda_q}{l_1l_2}\sum_{j=1}^{2}\frac{\lambda_n\delta_{1l}\sin\mu_m\zeta+\mu_m\delta_{2l}\sin\lambda_n\xi}{k'_{q+3,nm}\left(\gamma_{jnm}^{(q)}\right)}e^{\gamma_{jnm}^{(q)}\tau},$$

$$G_{q+3,2lnm}^{F}(\xi,\zeta,\tau)=\sum_{j=1}^{2N+6}A_{q+3,2lnm}^{(j)}(\xi,\zeta)e^{s_{jnm}\tau}+\sum_{j=1}^{2}A_{q+3,2lnm}^{(2N+6+j)}(\xi,\zeta)e^{\gamma_{jnm}^{(q)}\tau}+$$
$$+\frac{4\Lambda_q^*\lambda_n\mu_m}{l_1l_2}\sum_{j=1}^{2}\frac{\delta_{1l}\cos\mu_m\zeta+\delta_{2l}\cos\lambda_n\xi}{k'_{q+3,nm}\left(\gamma_{jnm}^{(q)}\right)}e^{\gamma_{jnm}^{(q)}\tau}, \tag{26}$$

$$G_{q+1,p+3,lnm}^{F}(\xi,\zeta,\tau)=\sum_{j=1}^{2N+6}A_{q+3,p+3,lnm}^{(j)}(\xi,\zeta)e^{s_{jnm}\tau}+\sum_{j=1}^{2}A_{q+3,p+3,lnm}^{(2N+6+j)}(\xi,\zeta)e^{\gamma_{jnm}^{(q)}\tau}+$$
$$+\frac{4}{l_1l_2}(D_q\delta_{qp}-\alpha_p\Lambda_q)(\delta_{1l}\lambda_n\sin\mu_m\zeta+\delta_{2l}\mu_m\sin\lambda_n\xi)\sum_{j=1}^{2}\frac{e^{\gamma_{jnm}^{(q)}\tau}}{k'_{q+3,nm}\left(\gamma_{jnm}^{(q)}\right)},$$

$$A_{iklnm}^{(j)}(\xi,\zeta)=\frac{P_{iklnm}(\xi,\zeta,s_{jnm})}{P'_{nm}(s_{jnm})},\quad A_{3klnm}^{(r)}(\xi,\zeta)=\frac{P_{iklnm}(\xi,\zeta,s_{rnm})}{Q'_{3nm}(s_{rnm})},\quad r=\overline{1,2N+8},$$

$$A_{q+3,kl}^{(j)}(\lambda_n,\mu_m,\xi,\zeta)=\frac{P_{q+1,klnm}(\xi,\zeta,s_{jnm})}{Q'_{q+3,nm}(s_{jnm})},\quad A_{q+3,klnm}^{(2N+6+i)}(\xi,\zeta)=\frac{P_{q+1,klnm}\left(\xi,\zeta,\gamma_{jnm}^{(q)}\right)}{Q'_{q+3,nm}\left(\gamma_{jnm}^{(q)}\right)},$$

where s_{jnm}, $j=\overline{1,2N+6}$ are zeros of the polynomial $P_{nm}(s)$, $s_{2N+6+i,nm}$ are additional zeroes of the polynomial $Q_{3nm}(s)$, $\gamma_{jnm}^{(q)}$ are additional zeroes of the polynomial $Q_{q+3,nm}(s)$

$$s_{2N+7,nm}=-ik_T\sqrt{\mu\left(\lambda_n^2+\mu_m^2\right)},\quad s_{2N+8,nm}=ik_T\sqrt{\mu\left(\lambda_n^2+\mu_m^2\right)},$$

$$\gamma_{1nm}^{(q)}=\frac{-1-\sqrt{1-4\tau_qD_q\left(\lambda_n^2+\mu_m^2\right)}}{2\tau_q},\quad \gamma_{2nm}^{(q)}=\frac{-1+\sqrt{1-4\tau_qD_q\left(\lambda_n^2+\mu_m^2\right)}}{2\tau_q}.$$

6. Example

For calculation, we consider a rectangular three-component ($N=3$) plate (independent components zinc and copper, which diffuse in duralumin), with the following characteristics [22].

$$\lambda^*=6.93\cdot10^{10}\,\frac{N}{m^2},\ \mu^*=2.56\cdot10^{10}\,\frac{N}{m^2},\ \rho=2700\frac{kg}{m^3},\ D_{11}^{*(2)}=D_{22}^{*(2)}=2.89\cdot10^{-14}\,\frac{m^2}{sec},\ T_0=700\,K,$$

$$\alpha_{11}^{*(1)}=\alpha_{22}^{*(1)}=1.55\cdot10^{7}\,\frac{J}{m^3},\ \alpha_{11}^{*(2)}=\alpha_{22}^{*(2)}=6.14\cdot10^{7}\,\frac{J}{m^3},\ D_{11}^{*(1)}=D_{22}^{*(1)}=2.62\cdot10^{-12}\,\frac{m^2}{sec},$$

$$n_0^{(1)}=0.0084,\ n_0^{(2)}=0.045,\ m^{(1)}=0.065\,\frac{kg}{mol},\ m^{(2)}=0.064\,\frac{kg}{mol},\ l=h^*=5\cdot10^{-4}\,m.$$

We assume that the plate has the following dimensions: $l_1^*=0.01\,m$, $l_2^*=0.01\,m$, $h^*=0.0005\,m$.

Let us set the following load parameters in the boundary conditions (15) (other $f_{mkl} = 0$ in (19))

$$f_{111}(x_2,\tau) = -\frac{12}{h^3} M_1^{(1)}(x_2,\tau) = H(\tau)\sin(\mu_1 x_2),$$
$$f_{112}(x_2,\tau) = -\frac{12}{h^3} M_2^{(1)}(x_2,\tau) = H(\tau)\sin(\mu_1 x_2). \qquad (27)$$

Calculating convolutions (16), taking into account (23) - (26), we obtain

$$\chi_1(x_1,x_2,\tau) = \sum_{k=1}^{N+2}\int_0^{l_2}\int_0^{\tau}\left[G_{111}(x_1,x_2,\zeta,\tau-t) - G_{111}(l_1-x_1,x_2,\zeta,\tau-t)\right]H(t)\sin(\mu_1\zeta)\,dt\,d\zeta =$$
$$= l_2\sin(\mu_1 x_2)\sum_{n=1}^{\infty}\sin\frac{\lambda_n}{2}\cos\lambda_n\left(\frac{1}{2}-x_1\right)\sum_{j=1}^{2N+6}\frac{e^{s_{jn1}\tau}-1}{s_{jn1}}\frac{\tilde{P}_{111n1}(s_{jn1})}{P'_{n1}(s_{jn1})},$$

$$\chi_2(x_1,x_2,\tau) = \sum_{k=1}^{N+2}\int_0^{l_2}\int_0^{\tau}\left[G_{211}(x_1,x_2,\zeta,\tau-t) - G_{211}(l_1-x_1,x_2,\zeta,\tau-t)\right]H(t)\sin\mu_1\zeta\,dt\,d\zeta =$$
$$= l_2\sin(\mu_1 x_2)\sum_{n=1}^{\infty}\sin\frac{\lambda_n}{2}\cos\lambda_n\left(\frac{1}{2}-x_1\right)\sum_{j=1}^{2N+6}\frac{e^{s_{jn1}\tau}-1}{s_{jn1}}\frac{\tilde{P}_{211n1}(s_{jn1})}{P'_{n1}(s_{jn1})},$$

$$w(x_1,x_2,\tau) = \sum_{k=1}^{N+2}\int_0^{l_2}\int_0^{\tau}\left[G_{311}(x_1,x_2,\zeta,\tau-t) + G_{311}(l_1-x_1,x_2,\zeta,\tau-t)\right]H(t)\sin(\mu_1\zeta)\,dt\,d\zeta =$$
$$= l_2\sin(\mu_1 x_2)\sum_{n=1}^{\infty}\sin\frac{\lambda_n}{2}\cos\lambda_n\left(\frac{1}{2}-x_1\right)\sum_{j=1}^{2N+8}\frac{e^{s_{jn1}\tau}-1}{s_{jn1}}\frac{\tilde{P}_{311n1}(s_{jn1})}{Q'_{3n1}(s_{jn1})},$$

$$H_q(x_1,x_2,\tau) = \sum_{k=1}^{N+2}\int_0^{l_2}\int_0^{\tau}\left[G_{q+3,11}(x_1,x_2,\zeta,\tau-t) + G_{q+3,11}(l_1-x_1,x_2,\zeta,\tau-t)\right]H(t)\sin(\mu_1\zeta)\,dt\,d\zeta =$$
$$= l_2\sin(\mu_1 x_2)\sum_{n=1}^{\infty}\sin\frac{\lambda_n}{2}\cos\left[\lambda_n\left(\frac{1}{2}-x_1\right)\right]\sum_{j=1}^{2N+6}\frac{e^{s_{jn1}\tau}-1}{s_{jn1}}\frac{\tilde{P}_{q+1,11n1}(s_{jn1})}{Q'_{q+3,n1}(s_{jn1})}+$$
$$+l_2\sin(\mu_1 x_2)\sum_{n=1}^{\infty}\sin\frac{\lambda_n}{2}\cos\left[\lambda_n\left(\frac{1}{2}-x_1\right)\right]\sum_{j=1}^{2}\frac{e^{\gamma_{jn1}^{(q)}\tau}-1}{\gamma_{jn1}^{(q)}}\frac{\tilde{P}_{q+1,11n1}(\gamma_{jn1}^{(q)})}{Q'_{q+3,n1}(\gamma_{jn1}^{(q)})}+$$
$$+\frac{4\Lambda_q}{l_1}\sin(\mu_1 x_2)\sum_{n=1}^{\infty}\lambda_n\sin\frac{\lambda_n}{2}\cos\left[\lambda_n\left(\frac{1}{2}-x_1\right)\right]\sum_{j=1}^{2}\frac{e^{\gamma_{jn1}^{(q)}\tau}-1}{k'_{q+3,n1}(\gamma_{jn1}^{(q)})\gamma_{jn1}^{(q)}},$$

where according to (3.8)

$$\tilde{P}_{m11n1}(s) = \frac{P_{m11n1}(\xi,\zeta,s)}{\sin\mu_m\zeta},\quad \tilde{P}_{q+1,11n1}(s) = M_{1qn1}\tilde{P}_{111n1}(s) + M_{2qn1}\tilde{P}_{211n1}(s)\quad (m=1,2,3).$$

The calculation results are shown in Figs. 2-9. 20 series terms are used for calculation. Further increasing does not lead to visible changes in the results. E.g., the difference between 10 and 20 terms of series for deflections and concentration increments is less than 1%, and for rotations of normal fibres it is about 3%.

Figures 2, and 3 show the rotations of a normal fibre, and Figs. 4, 5 show the deflections of the plate. The result obtained in terms of deflections is in satisfactory agreement with the result obtained for the Kirchhoff-Love plate [18,19].

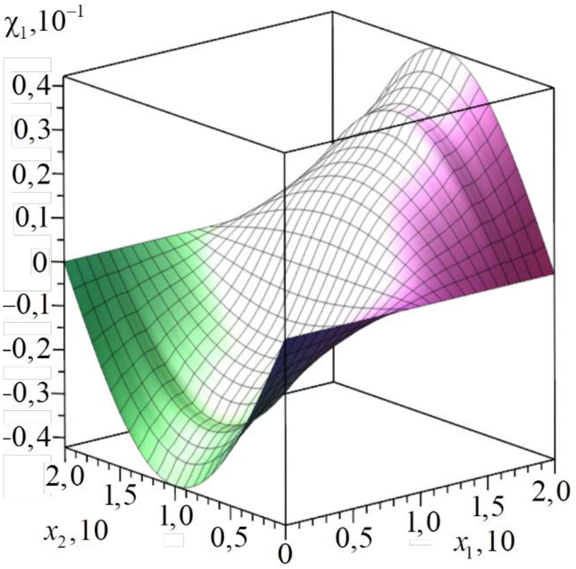

Fig. 2. The angle of rotation $\chi_1\left(x_1, x_2, 6.5\cdot 10\right)$

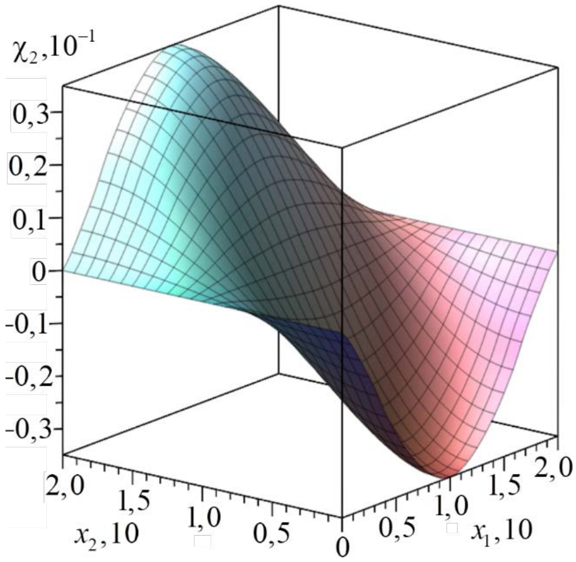

Fig. 3. The angle of rotation $\chi_2\left(x_1, x_2, 6.5\cdot 10\right)$

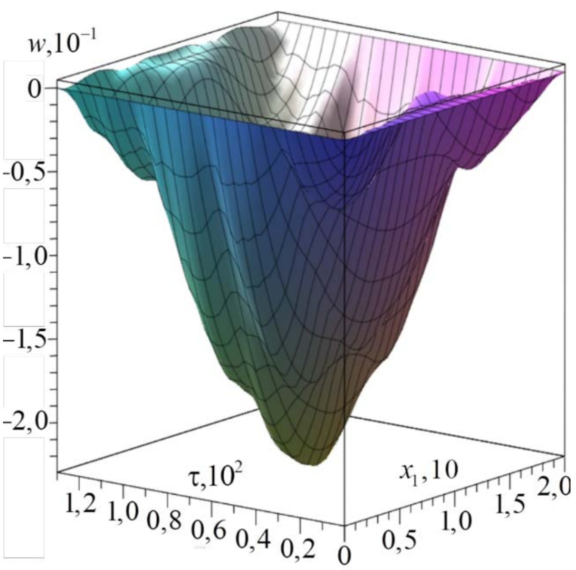

Fig. 4. The plate deflection $w\left(x_1, 0.5 l_2, \tau\right)$

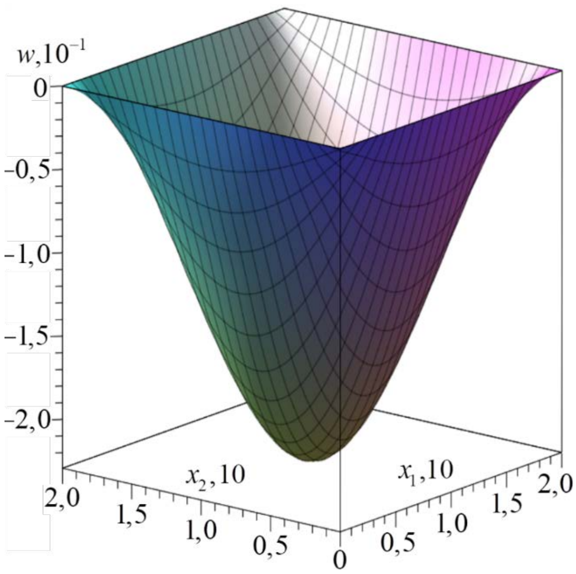

Fig. 5. The plate deflection $w\left(x_1, x_2, 6.5\cdot 10\right)$

A comparison of the results for the elastic diffusion model and elastic model (at $\alpha_{ij}^{*(q)} = 0$) shows that the influence of diffusion on the mechanical field at given boundary conditions is negligibly small.

Figures 6, 7 show the surface density of the concentration increment of zinc (Fig. 6) and copper (Fig. 7), which are initiated by bending moments (27) applied to the edges of the plate $x_1 = 0$ and $x_1 = l_1$. These increments have small values, which is confirmed by experimental studies [23], according to which the effect of mechanical loads on the diffusion field begins to manifest itself significantly, mainly during plastic deformations. Thus, elastic deformations have little effect on the kinetics of mass transfer.

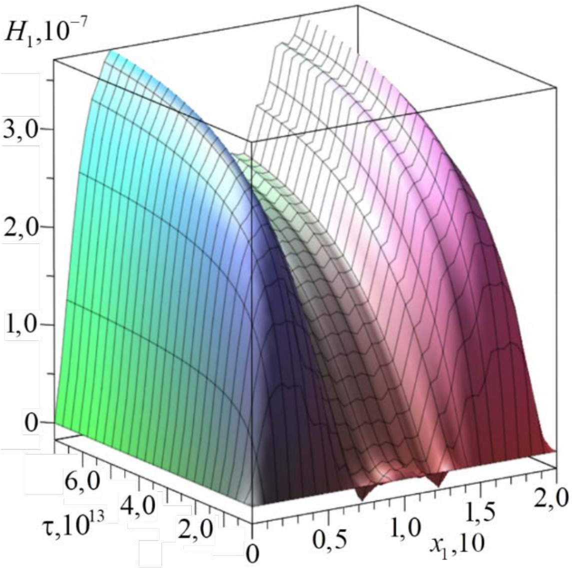

Fig. 6. Surface density of the concentration increment $H_1(x_1, 0.5l_2, \tau)$

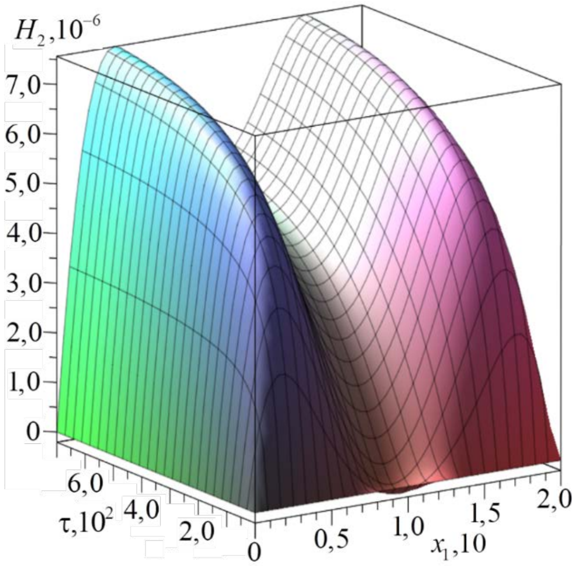

Fig. 7. Surface density of the concentration increment $H_2(x_1, 0.5l_2, \tau)$

Figures 8, and 9 show the effect of relaxation processes on the kinetics of mass transfer. It is shown that the effects associated with considering the finite velocity of propagation of diffusion fluxes are significantly manifested in a finite time interval, commensurate with the relaxation time $\tau^{(q)}$ and then decay (Fig. 9).

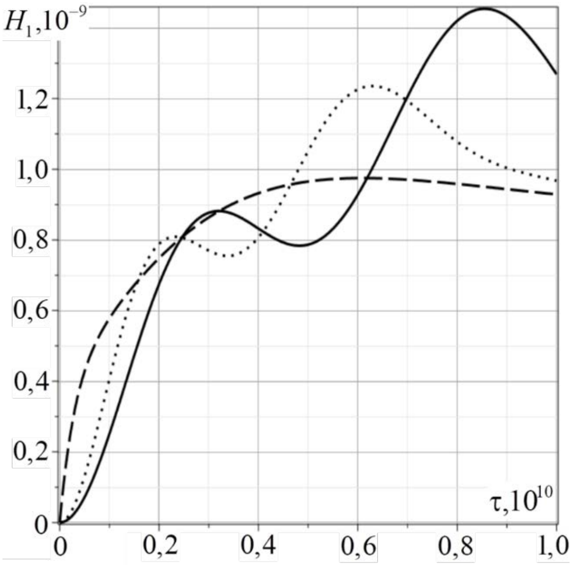

Fig. 8. Surface density of the concentration increment $H_1(0.1l_1, 0.5l_2, \tau)$, $\tau \in [0, 10^{10}]$ at different relaxation times. The solid line corresponds to $\tau^{(q)} = 200\ sec.$, the dotted line to $\tau^{(q)} = 100\ sec.$, the dashed line $\tau^{(q)} = 0$

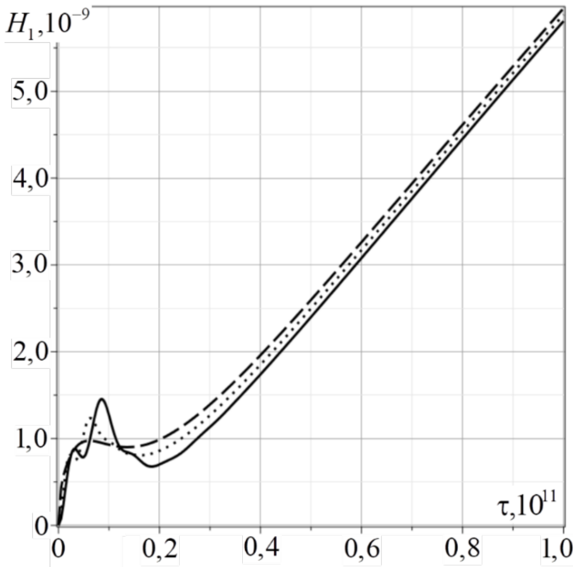

Fig. 9. Surface density of the concentration increment $H_1(0.1l_1, 0.5l_2, \tau)$, $\tau \in [0, 10^{11}]$ at different relaxation times. The solid line corresponds to $\tau^{(q)} = 200\ sec.$, the dotted line to $\tau^{(q)} = 100\ sec.$, the dashed line $\tau^{(q)} = 0$

6. Conclusions

Using the d'Alembert variational principle, a mathematical model of unsteady elastic diffusion transverse vibrations of a rectangular isotropic Timoshenko plate was constructed, which describes the relationship between mechanical and diffusion fields in a continuum. Furthermore, the model considers the relaxation diffusion effects, which determine the final velocity of diffusion disturbance propagation.

A method for finding Green's functions in the problem of unsteady elastic diffusion transverse vibrations of a simply supported plate is proposed. This method applies the integral Laplace transform and the expansion in the double trigonometric Fourier series. Found Green's functions made it possible to obtain an analytical solution to the problem of a simply supported plate bending under the action of moments suddenly applied to the plate's edges.

The calculation performed for a three-component plate makes it possible to modeling the nature of the mechanical and diffusion fields' interaction in a bent plate and the influence of relaxation processes on the kinetics of mass transfer.

It should be mentioned that the algorithm suggested is not a universal method of coupled initial boundary value problems solution, and in particular of mechanical diffusion problems for plates. In the work [14] it is shown that a solution in the form of Fourier series is possible only under certain boundary conditions. These include simple support. In other cases, for example, for cantilever restraint, the method of equivalent boundary conditions is used. It is tested on the example of one-dimensional problems of mechanical diffusion [14], as well as for cantilever Bernoulli-Euler beams taking diffusion into account [25]. Solution of analogous problems for plates using the method mentioned transcends the scope of this work because of its awkwardness.

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THE AUTHORS

Vestyak V.A.
e-mail: v.a.vestyak@mail.ru
ORCID: 0000-0002-9434-5477

Zemskov A.V.
e-mail: azemskov1975@mail.ru
ORCID: 0000-0002-2653-6378

Tarlakovskii D.V.
e-mail: tdvhome@mail.ru
ORCID: 0000-0002-5694-9253