Materials Physics and Mechanics. 2022;50(1): 158-165
Research article
http://dx.doi.org/10.18149/MPM.5012022_12

Submitted: May 24, 2022
Approved: July 14, 2022
Accepted: August 3, 2022

Instability of plastic deformation in crystalline alloys: the Portevin-Le Chatelier effect

G.F. Sarafanov[1]✉, Yu.G. Shondin[2]

[1]Mechanical Engineering Research Institute of the Russian Academy of Sciences – Branch of Federal Research Center "Institute of Applied Physics of the RAS", Nizhny Novgorod

[2]Kozma Minin Nizhny Novgorod State Pedagogical University, Minina 1, 603000 Nizhny Novgorod, Russia

✉ gf.sarafanov@yandex.ru

Abstract. A mechanism of the plastic deformation instability of crystalline alloys is considered in an autowave model of the Portevin-Le Chatelier effect. The model is defined by a system of differential equations for deforming stress, dislocation velocity, the concentration of dissolved impurity atoms interacting with moving dislocations, and forming an "atmosphere" of atoms around them, which provides braking of dislocations. In the model, the distribution of impurity atoms at a certain dislocation rate is considered to be stationary, which is typical for elevated temperatures. In this case, it is shown that the braking force at a dislocation velocity above the critical one has a region of negative sensitivity to the deformation rate, as a result of which the Portevin-Le Chatelier effect is realized. The numerical solution of the original system under the assumptions made showed that the effect manifests itself in the form of relaxation self-oscillations of the deforming stress and the rate of plastic deformation. An expression for the oscillation period is obtained, which is inversely proportional to a given rate of plastic deformation and temperature.

Keywords: Portevin-Le Chatelier effect, alloys, serrated deformation, high temperatures, relaxation self-oscillation

Acknowledgements. *The Russian Science Foundation supported this work, project No. 22-22-00749.*

Citation: Sarafanov GF, Shondin YuG. Instability of plastic deformation in crystalline alloys: the Portevin-Le Chatelier effect. *Materials Physics and Mechanics*. 2022;50(1): 158-165. DOI: 10.18149/MPM.5012022_12.

1. Introduction

The mechanics of crystalline alloy deformation in a certain interval of elevated temperatures reveals experimentally features associated with serrated deformation [1,2]. In the literature, this phenomenon is called the Portevin-Le Chatelier effect after the French researchers, who first observed this effect in 1923.

The serrated deformation is especially clearly manifested in crystalline alloys with a BCC structure, where atoms of a solute substance are embedded in the internodes of an octahedral structure. In solid substitution solutions, the Portevin-Le Chatelier effect is somewhat weaker distinguished from the ordinary background deformation hardening [3].

The effect is the repeated occurrence of localized or spreading deformation bands, causing short-term jumps of deformation in the crystal. When deformation occurs at a constant rate, this manifests itself macroscopically in the form of jumps of the deforming stress due to the elastic response of the machine-sample system. Sometimes a fairly smooth change in the load is observed (usually for FCC crystals), but more often the dynamics of jumps in the form of the relaxation oscillations take place.

The Portevin-Le Chatelier effect, as a manifestation of the instability of plastic flow, can be observed for a wide class of alloys under arbitrary types of loading in certain ranges of deformation rates and temperatures. In experiments on uniaxial loading, the effect is manifested at low deformation rates and elevated temperature: the loading-deformation diagram acquires a sawtooth shape (Fig. 1).

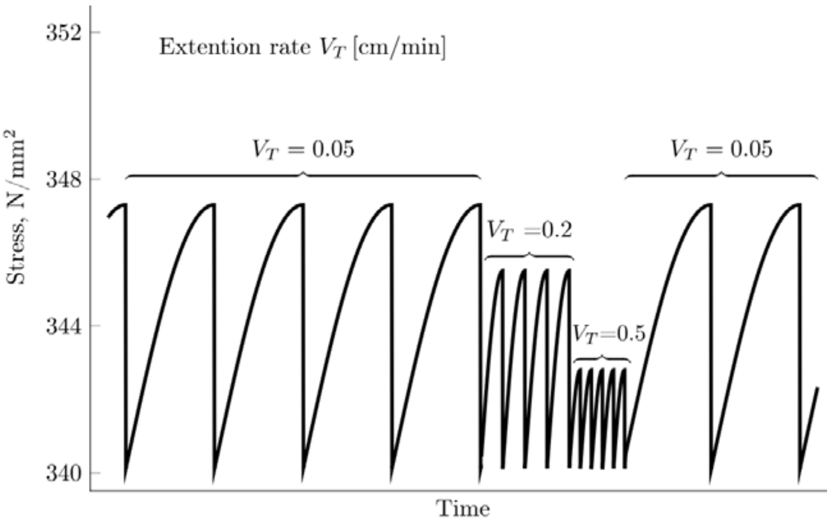

Fig. 1. Experimental dependence of the applied to the sample stresses on time at different values of the deformation rate. Tensile tests were carried out (V_T is the rod velocity) for the alloy $Al - Mg5 - Mn0.8$ at the temperature $T = 395K$ [4]

Usually, there are three main types of manifestation of the Portevin-Le Chatelier effect [5,6,7]: type A – the appearance and movement of a single (solitary) Luders deformation band along the axis of the sample, which can occur repeatedly; type B – deformation bands appear and disappear in an oscillating or intermittent mode, spreading along the sample; type C – Luders bands occur randomly along the length of the sample.

The analysis of experimental data allows us to conclude that the manifestation of the Portevin-Le Chatelier effect [8-11] is directly related to the formation of "atmospheres" of dissolved atoms [1, 3] on mobile dislocations. This leads to a negative velocity sensitivity of the flow stress and, as a consequence, to the instability of plastic deformation in the form of Luders bands and the Portevin-Le Chatelier effect.

Due to the recent increase in interest in this effect [12-15], a large number of various models [10,16-18] have been proposed in the literature. However, a complete theory has not yet been built.

First of all, this is due to the phenomenological nature of many models and, accordingly, the lack of rigor in the formulation of the problem. For example, when analyzing from a mathematical point of view the serrated deformation characteristic to the Portevin-Le Chatelier effect, one should take into account the inertial terms (the effective mass of dislocations) in the original system of equations. This makes the model mathematically more correct. In this situation, it is advisable to formulate a model for mesoscopic variables: deforming stress and dislocation velocity, within the framework of the model [19], taking into account the dynamics of clouds of impurity atoms in the vicinity of moving dislocations.

In this article, an autowave model is formulated, which allows us to give a possible explanation of the Portevin -Le Chatelier effect and the formation of deformation bands (Luders bands)from a single position. Within the framework of the model under consideration, the article assumes obtaining the dependence of the deforming stress and dislocation velocity on time in the form of relaxation oscillations. Other unstable modes of plastic deformation based on the proposed model will be investigated in subsequent works.

2. The model of unstable deformation in alloys

Let us consider the behavior of an ensemble of dislocations in a slip band of width w. Let us choose the $0x$ axis in the direction of a given dislocation sliding system at some angle to the axis of the tensile test specimen. Let the distribution of dislocations in the slip band be characterized by their densities $\rho_+(\mathbf{r},t)$ and $\rho_-(\mathbf{r},t)$, and $\rho_+^0 = \rho_-^0 = \rho_0/2$ in equilibrium. Denote the velocity of dislocations by $v(\mathbf{r},t)$. Accordingly, the rate of plastic deformation in the slip strip is defined as $\dot{\varepsilon} = b\rho_0 v$ (b is the module of the Burgers dislocation vector).

In general, the movement of dislocations with an atmosphere of dissolved atoms (for example, Cottrell's atmosphere) is a complex two-dimensional problem. For this reason, we first consider the one-dimensional case to clarify the features of the nonlinear dynamics of dislocations and impurity atoms. We assume that the deviation of the dislocation density from the stationary value is insignificant, then the process of plastic deformation in the shear band in the loading regime with a given rate $\dot{\varepsilon}_0$ of plastic deformation can be described by the following system of equations

$$m^*(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}) = b(\tau + \tau^{int}) - F(v), \tag{1}$$

$$\frac{\partial \tau^{int}}{\partial t} = -\frac{\tau^{int}}{t_a} + \gamma_1 \frac{\partial^2 \dot{\varepsilon}}{\partial x^2}, \tag{2}$$

$$\frac{\partial \tau}{\partial t} = G^*[\dot{\varepsilon}_0 - \frac{b\rho_0}{L_p}\int_0^{L_p} v(x,t)dx], \tag{3}$$

$$\frac{\partial c}{\partial t} = D_c \frac{\partial^2 c}{\partial x^2} + \frac{D_c}{kT}\frac{\partial}{\partial x}(c\frac{\partial W}{\partial x}) + v\frac{\partial c}{\partial x}, \tag{4}$$

$$F(v) = -\int_{-\infty}^{\infty} c\frac{\partial W}{\partial x}dx, \tag{5}$$

where equation (1) is the equation of dislocation motion, m^* is its effective mass, $\tau = \tau_e - \tau_i$ is the shear deforming stress in the current sliding system, τ_e is the external stress component in the current sliding system, τ_i is the dry friction stress (the internal friction from randomly located dislocations and Hall-Petch stress), τ^{int} is the field of internal stresses from a system of dislocation charges at the grain boundaries. τ^{int} is defined by equation (2), which takes into account the fact that elastic fields induced at the boundaries (in accordance with the continuum limit of the Ballou-Bilby formula $\tau^{int} = \gamma_1 \partial_{xx}^2 \varepsilon$) relax due to accommodative adjustments. The parameter $\gamma_1 \approx \alpha_g G d^2$ serves as a measure of elastic grain correlation ($\alpha_g \approx 1$, d is grain size), and t_a is a characteristic time of plastic accommodation [19].

The equation (3) is the Gilman-Johnston equation for the active loading regime [19,20], which takes into account the dynamics of load changes under the condition of the stretching rate constancy of the crystal sample, G^* is the effective modulus of elasticity, L_p is the length of the plastic deformation zone.

The diffusion and drift of solute atoms in one-dimensional approximation are determined by the equation (4), where $c(t,x)$ is the number of impurity atoms per unit area, D_c is the diffusion coefficient of dissolved atoms. $F(v)$ is the nonlinear braking force of dislocation per unit length is due to the interaction of impurity atoms with dislocation, which is determined by the interaction energy

$$W = \frac{\beta}{(b^2 + x^2)^{1/2}}, \tag{6}$$

where β is the interaction constant in the dimensional effect [3]. The energy $W(x)$ in the one-dimensional approximation creates a cloud of impurity atoms in the vicinity of the dislocation similar to the Cottrell atmosphere in the two-dimensional case [1,3].

The original system (1)-(5) is quite complex and contains a large variety of possible solutions. For this reason, in this paper, we restrict ourselves by the following approximations: (i) let the time of plastic accommodation t_a in the equation (3) is significantly less than the characteristic time scale of changing the variable τ^{int}, then $\tau^{int} \propto \partial^2_{xx}\dot{\varepsilon}$ [7,19], (ii) we neglect in the diffusion equation (4) by the time derivative (i.e. we consider the stationary case, when the formation time of the atmosphere of impurity atoms is less than the time of characteristic changing of the variables τ and v). As a result, we have

$$m^*(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}) = b(\tau + \tau^{int}) - F(v), \tag{7}$$

$$\tau^{int} \approx t_a \gamma_1 \frac{\partial^2 \dot{\varepsilon}}{\partial x^2} = \eta \frac{\partial^2 v}{\partial x^2}, \tag{8}$$

$$\frac{\partial \tau}{\partial t} = G^*[\dot{\varepsilon}_0 - \frac{b\rho_0}{L_p}\int_0^{L_p} v(x,t)dx], \tag{9}$$

$$D_c \frac{\partial^2 c}{\partial x^2} + \frac{D_c}{kT}\frac{\partial}{\partial x}(c\frac{\partial W}{\partial x}) + v\frac{\partial c}{\partial x} = 0, \tag{10}$$

$$F(v) = -\int_{-\infty}^{\infty} c\frac{\partial W}{\partial x}dx, \tag{11}$$

where $\eta = b\rho_0 t_a \gamma_1$.

In these approximations, since equations (10) and (11) determine the behavior of the system (7)-(9), we consider first the diffusion problem and find the braking force $F(v)$. Solving (10) and (11) numerically, we obtain the following results, shown in Fig. 2.

The dependence of the braking force $F(v)$ on dislocation velocity has a characteristic nonlinear appearance with a descending part corresponding to the negative velocity sensitivity of the flow stress (Fig. 2a). The function $F(v)$ has a maximum at some velocity $v = v_a$, which can be evaluated as follows.

From a physical point of view, the mobility of dislocations is determined by the diffusion of dissolved atoms, if the relaxation time x_1/v for displacement during dislocation drift over a certain distance x_1 is less than the relaxation time x_1^2/D_c for diffusion over the same distance, where $x_1 = \beta/kT$ is the effective width of the dislocation potential (6).

At the maximum point, the nature of the dislocation movement changes from diffusion to drift i.e. $x_1/v \approx x_1^2/D_c$ must be performed, or, taking into account $x_1 = \beta/kT$, we get

$$v_a \approx \frac{D_c kT}{\beta} = v_t. \tag{12}$$

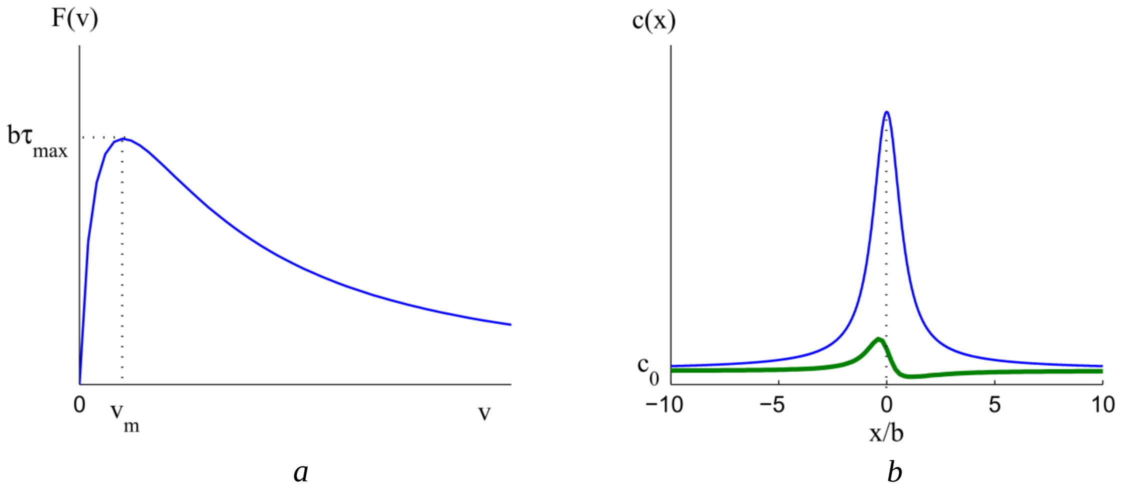

Fig. 2. The dependence of the braking force $F(v)$ on the dislocation velocity (a) in the case of braking by the atmosphere determined by the solution of the equation (10). In the figure (b) the symmetric curve corresponds to the atmosphere $c = c_0 \exp(W(x)/kT)$ at $v = 0$ and the asymmetric curve to the atmosphere at $v >> v_a$ (T is temperature, k is the Boltzmann constant, c_0 is the impurity concentration at $x = \pm\infty$)

It was observed in [11], that with diffusion dynamics ($v < v_a$), the distribution of dissolved impurity atoms around a moving dislocation is approximately the same as with $v = 0$, i.e.

$$c \approx c_0 \exp(W/kT). \tag{13}$$

In this case, using the theorem about dissipation [3]

$$Fv = \frac{kT}{D_c}\int_{-\infty}^{\infty} \frac{v^2(c-c_0)^2}{c}dx, \tag{14}$$

which follows from (11), and considering (13) for $\beta < 2bkT$ and $v = v_t$ we get

$$b\tau_{max} = \frac{4kTc_0 v}{D_c}\int_{-\infty}^{\infty} \mathrm{sh}^2 \frac{\beta}{2kT\sqrt{b^2+x^2}} dx \approx 6\pi c_0\beta/b. \tag{15}$$

Then, at low velocities, the deforming stress depends linearly on the velocity, i.e. $\tau = Bv$, where the mobility is defined as

$$B = \frac{\tau_{max}}{v_t} = \frac{6\pi c_0 \beta^2}{D_c b^2 kT}, \tag{16}$$

which, up to a numerical coefficient, coincides with the Cottrell-Jesvon[11] formula.

3. The Portevin-Le Chatelier effect

From the analysis of the solution for the braking force $F(v)$, it follows that at $v > v_a$, the braking force decreases with increasing velocity. This is due to the fact that, as the velocity increases, the dislocation gradually loses the atmosphere of dissolved atoms (Fig. 2b), at some critical velocity [3]

$$v_c = D_c\beta/kTb^2 \tag{17}$$

the atmosphere disappears and at $v > v_c$ the braking is caused only by the statistically distributed atoms of the solute. For this domain ($v > v_c$), we consider the mechanism proposed by Hirt and Lote in the field of solute atoms [3]. They found that the flow stress is determined in this case by the ratio

$\tau = 2vc_0 kT/D_c b.$ (18)

For further analysis, we introduce the function

$f(v) = F(v)/b + 2vc_0 kT/D_c b,$ (19)

which is the sum of the function $F(v)/b$ with the stress τ (18). The graph of $f(v)$ has the form of a nonlinear N-shaped figure, shown in Fig. 3.

Depending on the specified loading conditions of the sample ($\dot{\varepsilon} = \dot{\varepsilon}_0$), the straight line $v = v_0 = \dot{\varepsilon}_0/b\rho_0$ corresponding to the homogeneous stationary solution of the systems (7)-(9) can intersect the curve $\tau = f(v)$ differently.

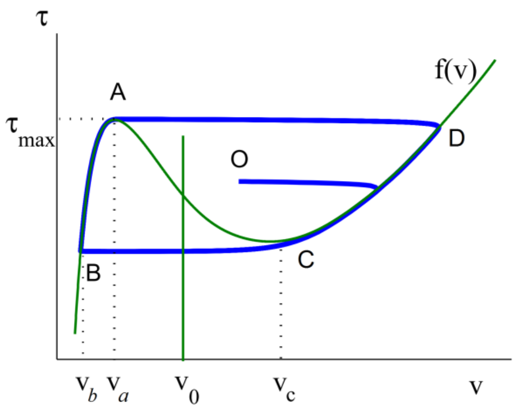

Fig. 3. Phase diagram for the variables τ and v. The straight line $v - v_0 = 0$ located on the descending branch of the function $f(v)$

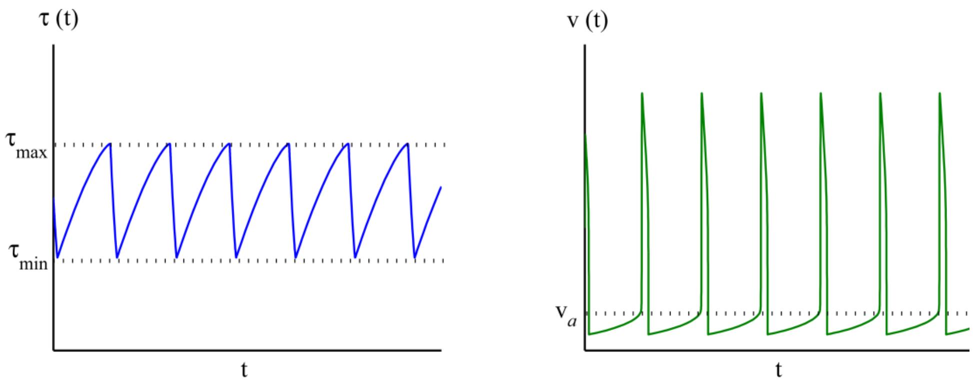

Fig. 4. The form of the obtained relaxation self-oscillations of the deforming stress $\tau(t)$ and the dislocation velocity $v(t)$ in the slip band as a function of time t

If it intersects $f(v)$ in the stable region ($f'(v)|_{v=v_0} > 0$), then the plastic deformation process develops through the propagation of the excitation fronts of the deformation rate (Luders bands) [19], if in an unstable ($f'(v)|_{v=v_0} < 0$), then, additionally with the propagation of the wavefronts, a jump-like self-oscillating mode of the unstable flow is established, which manifests itself as the Portevin-Le Chatelier effect. Let us consider this particular case.

We assume that the time of spontaneous formation of deformation bands at the lateral boundary of the sample is significantly less than the time of propagation of the bands through the cross-section. In this case, the load change does not have time to adjust to the specified regime of plastic deformation and the deformation bands fill the entire sample space almost instantly.

We consider for this case solution of the system of equations (7)-(9) with the replacement $F(v)$ by $bf(v)$. Numerical analysis of this system has shown that the flow stress $\tau(t)$ and the dislocation velocity $v(t)$ change periodically over time.

A self-oscillating process on the plane of variables (τ, v) corresponds to the phase trajectory $\tau(v)$, shown in Fig. 3. Under any initial conditions (for example, corresponding to the point O), the system goes to the limit cycle $ABCD$ regime, consisting of segments of fast and slow movements. Meanwhile, the system performs relaxation self-oscillations (Fig. 4).

4. Conclusions

It can be noted, that the system (7)-(9) has a small parameter with the time derivative of velocity, the smallness of which is due to the extreme smallness of the effective dislocation mass ($m^* \approx 10^{-15}$g/cm). Then the analysis of the solution of the system (7)–(9) can be carried out by standard asymptotic methods, dividing the system into the subsystem of fast movements

$$m^*(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}) = \eta\frac{\partial^2 v}{\partial x^2} + b\tau - bf(v), \quad \tau = const, \tag{20}$$

and the subsystem of slow movements

$$\frac{\partial \tau}{\partial t} = G^*[\dot{\varepsilon}_0 - \frac{b\rho_0}{L_p}\int_0^{L_p} v(x,t)dx], \quad \tau = f(v). \tag{21}$$

The subsystem of fast movements actually coincides with that considered in [19] and is a mathematical model of the formation and propagation of Luders bands.

From equation (21), it is possible to determine the oscillation period T_0 by calculating the time of movement along the limit cycle. Meanwhile, the time of fast movements can be neglected.

The main contribution to the oscillation period is made by the segment $A-B$ of slow movements (see Fig. 4), on which the average dislocation velocity

$$\bar{v} = \frac{1}{L_p}\int_0^{L_p} v(x,t)dx \tag{22}$$

coincides approximately with the velocity of dislocations in the slip band ($\bar{v} \approx v$). Then from (21), we find

$$T_0 = \frac{1}{G^* b\rho_0}\int_{v_b}^{v_a} \frac{f'(v)}{v_0 - v}dv \approx \frac{6\pi c_0 \beta^2}{G^*\dot{\varepsilon}_0 D_c b^2 kT}. \tag{23}$$

Expression for the oscillation period (23) is obtained under the assumption of an approximation of $f(v)$ on the segment $A-B$ by a linear function for $v_0 > v_a$, and using also relations (12) and (15). Thus, the oscillation period is T_0 inversely proportional to the rate of plastic deformation $\dot{\varepsilon}_0$ [4] (see Fig. 1) and decreases with increasing temperature T [6].

References

1. Cottrell AH. *Dislocations and Plastic Flow in Crystals*. Oxford Univ. Press, London; 1953.
2. Friedel J. *Dislocations*. Oxford: Pergamon; 1964.
3. Hirth JP, Lothe J. *Theory of dislocations*. New York: John Wiley; 1968.
4. Korbel A, Dybiec H. The problem of the negative strain - rate sensitivity of metals under the Portevin-Le Chatelier deformation conditions. *Acta Metall.* 1981;29(1): 89-93.
5. Maj P, Zdunek J, Mizera J, Kurzydlowski KJ. The effect of a notch on the Portevin-Le Chatelier phenomena in an Al-3Mg model alloy. *Mater. Charact.* 2014;96: 46-53.

6. Kubin LP, Estrin Y. Evolution of dislocation densities and the critical conditions for the Portevin-Le Chatelier effect. *Acta Metall. Mater.* 1990;38(5): 679-695.
7. Lebedkin MA, Brecher Y, Estrin Y, Kubin LP. Statistical behavior and regularities of localization of deformations in the Portevin-LeChatelier effect. *Phys. Rev. Lett.* 1995;74(23): 4758-4761.
8. Brechet Y, Estrin Y. On the influence of precipitation on the Portevin-Le Chatelier effect. *Acta Metal. Mater.* 1995;43(3): 955-963.
9. Halmer P. On the physics of the Portevin-Le Chatelier effect part 1: The statistics of dynamic strain ageing. *Materials Science and Engineering A.* 1996;207(2): 208-215.
10. MacCormick PG. The Portevin-Le Chatelier effect in an Al-Mg-Si alloy. *Acta Metall.* 1971;19(5): 463-471.
11. Cottrell AH. A note on the Portevin-Le Chatelier effect. *Philosophical Magazine.* 1953;44(335): 829-832.
12. Chen H, Chen Z, Wang C et al. The effect of TiB2 ceramic particles on Portevin–Le Chatelier behavior of TiB2/Al-Mg metal. *Journal of Materials Research and Technology.* 2021;14: 2302-2311.
13. Cui CY, Zhang R, Zhou YZ. Portevin-Le Chatelier effect in wrought Ni-based superalloys: experiments and mechanisms. *J. Mater. Sci. Technol.* 2020;51: 16-21.
14. GengYX, Zhang D, Zhang JS, Zhuang L. Zn/Cu regulated critical strain and serrated flow behavior in Al-Mg alloys. *Mater. Sci. Eng. A.* 2020;795: 139991-139997.
15. Wang, XG, Wang L, Huang MX. Kinematic and thermal characteristics of Luders and Portevin-Le Chatelier bands in a medium Mn transformation-induced plasticity steel. *Acta Materialia*. 2017;124: 17-29.
16. Kubin LP, Estrin Y. The Portevin Le Chatelier effect in deformation with constant stress rate. *Acta Metall.* 1985;33(3): 397-407.
17. MacCormick PG. Theory of flow localisation due to dynamic strain ageing. *Acta Metall.* 1989;36(12): 3061-3067.
18. Hahner P. Modelling of propagative plastic instabilities. *Scripta Metall.* 1993;29: 1171-1176.
19. Sarafanov GF, Shondin YuG. Deformation instability in crystalline alloys: Luders bands. *Materials Physics and Mechanics*. 2021;47(3): 431-437.
20. Johnston WG, Gilman J. Dislocation velocities, dislocation densities and plastic flow in lithium fluoride crystals. *J. Appl Phys.* 1959;30(2): 129-143.

THE AUTHORS

Sarafanov G.F.
e-mail: gf.sarafanov@yandex.ru
ORCID: -

Shondin Yu.G.
e-mail: yu.shondin@yandex.ru
ORCID: 0000-0002-1246-8503