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Quasi-static thermal response of a circular plate due to the influence of memory-dependent derivatives

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ABSTRACT

In this paper, the thermal deformation response of a circular plate due to the influence of memory-dependent derivatives (MDD) is analyzed using a quasi-static approach. The top, bottom, and curved surfaces of the plate experience convective boundaries with heat flow on the outer curved radii, and additional cross-sectional heating is prescribed on the top and bottom plate surfaces. Integral transformation methods are used to solve the memory-dependent heat transfer model. Due to the complex nature of the analytical analyses, the Laplace transform is numerically inverted. The rate of change in temperature and thermal deflection is dependent on past changes, making it more suitable for studying physical problems. Numerical calculations of the obtained thermal results are performed for a copper plate and presented graphically.

KEYWORDS

memory-dependent derivatives • circular plate • temperature • thermal deflection • integral transform

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Introduction

The fractional order theory of thermoelasticity is an integral part of fractional calculus, which falls well under the branch of mathematics. Fractional-order thermoelastic problems have been a hot topic for mathematicians and researchers in recent decades due to their practical application. The results of various investigations based on modelling fractional thermoelasticity have been successfully studied and illustrated by many renowned researchers [1–10].

Wang and Li [11] first proposed the concept of memory-dependent deductions in 2011, and compared to Caputo derivations, it proves to be more suitable for modelling problems based on memory. Since MDD may represent memory-dependent derivatives in a variety of physical processes, it has emerged as a new area of fractional calculus that is constantly growing. The fractional derivative (FD) mostly reflects local change, even though it is stated on an interval. Compared to FD, MDD's physical significance is noticeably more apparent. The kernel function reflects the memory-dependent weight, and the time delay shows how long the memory effect lasts. For temporal modeling, which is helpful in explaining the thermal effect of solid bodies, the memory-related derivative is more appropriate, according to the research that is currently available. In the domains of thermoelasticity, thermoelectricity, particle physics, vibration mechanics, etc., memory-dependent derivatives can serve as a helpful substitute for fractional derivatives. Karamany and Ezzat [12] developed a new generalized concept of thermoelasticity with the effect of time delay and applied it to the solution of one-dimensional half-spaces with free choice of kernel function. Memory-based differentiation was utilized by Purkait et al. [13] to investigate the issue in an infinite space. Sun and Wang [14] recreate the MDD heat transfer model and use first-order memory-dependent advance differentiation to analyze the one-dimensional heat transmission problem. The temporal thermal stress problem associated with a hollow cylinder with underlying surface cracking subjected to a temperature shock at its interior was resolved by Xue et al. [15]. Ma and Gao [16] investigated the dynamic response of a generalized thermoelastic problem in an infinite cylindrical body due to thermal shock and the memory effect.

Sur et al. [17] investigated the innovative mathematical framework for generalized thermoelasticity in the background of memory-related heat transfer. Karamany and Ezzat [18] developed fundamental equations for thermoelastic diffusion in various solids. Qi et al. [19] investigated nonclassical continuum mechanics in a micro/nano-scale system with a memory-based effect. From a mathematical perspective, Verma et al. [20] developed the hygrothermoelasticity theory with fractional order theory.

Othman and Mondal [21] introduced phase-lag models and calculated the displacement and stress functions for generalized thermoelasticity. Awwad et al. [22] studied the thermoelastic response with temperature-dependent properties for a cylindrical hole. Mondal [23] discussed transient phenomena in a rod considering a moving heat source with memory response in generalised thermoelasticity. Abouelregal et al. [24] successfully investigated MDD's response with time delay by constructing a new thermal model and studying its effect graphically. Using the Laplace transform, Abouelregal et al. [25] determined the solution for temperature, bending moment, and displacement for a thermomechanically rotating size-dependent nanobeam. A dynamically bar was the subject of an investigation into the memory phenomenon and discussion of thermo-mechanical performance by Abouelregal et al. [26]. To understand the memory phenomenon in solid objects under thermoelasticity, Lamba [27] recently studied the memory effects of an internal heat source by taking a cylindrical, thick shape under the impact of radiation boundaries. Also, some other renowned authors contributed their work to the field, as reflected in [28–36]. Lamba and Deshmukh [37] conducted a recent analytical and numerical study to examine the impact of time delay on the temperature, displacement, and thermal heat transfer stress histories in an infinitely long thermoelastic solid circular cylinder.

Compared to fractional-order derivatives, the concept of MDD proves to be superior and suitable for describing the memory effect. This inspired the author to create a mathematical model of a solid object to study the thermal effect.

In the present work, a two-dimensional boundary value problem of a circular plate with ranges $0 \le r \le b; -h \le z \le h$ is considered to investigate the thermal response due to the effects of MDD on the temperature and deflection distribution (the geometry of the problem is as shown in Fig. 1). The top, bottom and curved surfaces of the plate are subject to convective heat transfer with heat flux on the outer curved boundary, and additional cross-sectional heating is prescribed on the top and bottom plate surfaces. The integral transform method is used to solve the governing heat equation with memory-based derivative.

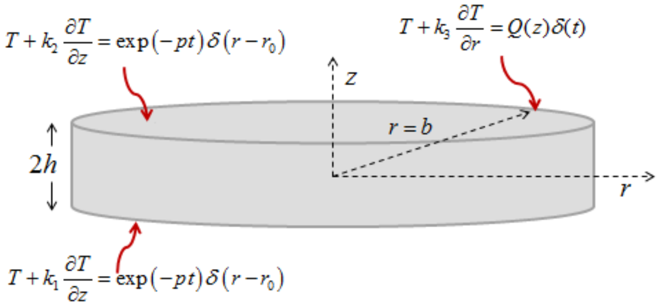

Fig. 1. Geometrical shape of the memory-based circular plate

Investigators working on the development and design of the novel structural material may find the present work to be helpful as it adds to the collection of knowledge in the subject of thermoelasticity.

Modelling of thermoelastic problem

Heat conduction with MDD

The new concept of memory-based Cattaneo and Vernotte (CV) modelling is developed by introducing first-order MDD as in [28]:

$$q + \tau D_{\omega} q = -k \nabla T. \tag{1}$$

If there is no heat inside the body, the equation of thermal equilibrium is expressed as [31]:

$$\nabla q = -\rho_m c_E \frac{\partial T}{\partial t} \tag{2}$$

where ρ_m and c_E are, respectively, mass density and specific heat capacity.

For an axially symmetric circular plate in cylindrical coordinates with influence of memory dependent derivative, the governing equation of heat conduction is obtained by transforming Eq. (1) into Eq. (2) as:

$$k \nabla^2 T = \rho_m c_E (1 + \tau D_{\omega}) \frac{\partial T}{\partial t}. \tag{3}$$

A list of variables is shown below in without dimensions form for ease of use: $r' = \frac{r(t',\tau',\omega')}{r_0} = \frac{1}{\rho_m c_E r_0^2}(t,\tau,\omega)$, $z' = \frac{z(t',\tau',\omega')}{z_0} = \frac{1}{\rho_m c_E r_0^2}(t,\tau,\omega)$, $T' = \frac{T}{T_0}$. Using the above dimensionless variables, Eq. (3) takes the following form (omitting the prime numbers for simplicity):

$$\nabla^2 T = (1 + \tau D_{\omega}) \frac{1}{k} \frac{\partial T}{\partial t}, \tag{4}$$

where the function's memory dependent derivative is a weighted integral of its common integer-order derivative on an interval that slips, which is denoted by [11]:

$$D_{\omega} T(t) = \frac{1}{\omega} \int_{t-\omega}^{t} K(t-\xi) \frac{\partial T(\xi)}{\partial \xi} d\xi. \tag{5}$$

For any function $T(t)$ that is m times differentiable with respect to t, the memory-dependent derivative of order m of $T(t)$ is:

$$D_{\omega}^{m} T(t) = \frac{\partial^{m-1}}{\partial t^{m-1}} D_{\omega} T(t) = \frac{1}{\omega} \int_{t-\omega}^{t} K(t-\xi) \frac{\partial^{m} T(\xi)}{\partial \xi^{m}} d\xi, \tag{6}$$

where the time delay ω and kernel function $K(t-\xi)$ are arbitrary choices made to reflect the actual behaviours of the materials.

Kernel functions, in particularly, can be selected as:

$$K(t-\xi) = 1 - \frac{2l_2}{\omega}(t-\xi) + \frac{{l_1}^2}{\omega^2}(t-\xi)^2, \quad (7)$$

where l_1 and l_2 are constants.

The function of the kernel usually falls between 0 and 1. Also $\xi \in [t-\omega, t]$ and $|D_\omega T(r,z,t)| \leq \left|\frac{\partial T(r,z,t)}{\partial t}\right|$.

Quasi-static deflection based on thermal moment

The deflection function's differential formulation is given in [29].

$$\nabla^4 W = \frac{-\nabla^2 M_T}{\Omega(1-v')}, \quad (8)$$

where M_T is the plate's thermal moment, which is stated as:

$$M_T = a_t E' \int_{-h}^{h} T(r,z,t) z \, dz, \quad (9)$$

where Ω is the disc's rigidity,

$$\Omega = E' h^3 / 12(1 - v'^2), \quad (10)$$

where a_t, E' and v' represents the disc's material linear thermal expansion coefficient, the Young's modules, and Poisson's ratio, respectively.

For an annular disc's edge to be fixed and clamped, one must write:

$$W(b,z,t) = \frac{\partial W(b,z,t)}{\partial r} = 0, \quad (11)$$

$$W(t=0) = 0. \quad (12)$$

Boundary and initial constraints

The differential form of heat transfer Eq. (4) of a circular plate under the impact of MDD is subjected to the following constraints:

$$\left[T + k_3 \frac{\partial T}{\partial r}\right]_{(r=b)} = Q(z)\delta(t), \quad (13)$$

$$\left[T + k_1 \frac{\partial T}{\partial z}\right]_{(z=-h)} = exp(-pt)\,\delta(r-r_0), \quad (14)$$

$$\left[T + k_2 \frac{\partial T}{\partial z}\right]_{(z=h)} = exp(-pt)\,\delta(r-r_0), \quad (15)$$

$$T(r,\ z,t) \ = \ 0, t = 0, \quad (16)$$

where, $exp(-pt)\,\delta(r-r_0)$ denotes the additional sectional heating applied at the bottom and top surfaces of plate and $Q(z)\delta(t)$ is the heat flux at outer radii. Also, k_1 and k_2 denotes the radiation constants on the plate plane surfaces and k_3 on the outer curved surface, respectively.

The problem under examination is mathematically formulated in Eq. (4) through Eq. (16).

Solution of the modeling

Evaluation of temperature function

To determine the integral of the heat transfer memory-related differential Eq. (4), first we write the formula of the finite Marchi-Fasulo transform and its inverting formula for any function $F(r,z,t)$ as [30].

$$\overline{F}(r,\Lambda_n,t) = \int_{z=-h}^{h} F(r,z,t) L_n(z) dz, \quad (17)$$

$$F(r,z,t)=\sum_{n=1}^{\infty}\frac{\overline{F}(\Lambda_n)}{\lambda_n}L_n(z), \tag{18}$$

where

$L_n(z)=M_n\cos(\Lambda_n z)-N_n\sin(\Lambda_n z)$,

$M_n=\Lambda_n(\gamma_1+\gamma_2)\cos(\Lambda_n h)+(\eta_1-\eta_2)\sin(\Lambda_n h)$,

$N_n=(\eta_1+\eta_2)\cos(\Lambda_n h)+(\gamma_2-\gamma_1)\Lambda_n\sin(\Lambda_n h)$,

$\lambda_n=\int_{z=-h}^{h}L_n^2(z)dz=h\left[{M_n}^2+{N_n}^2\right]+\frac{\sin(2\Lambda_n h)}{2\Lambda_n}\left[{M_n}^2-{N_n}^2\right]$,

where the solutions to the equation below are satisfied by the Eigen values Λ_n.

$$[\eta_1\sin(\Lambda h)+\gamma_1\Lambda\cos(\Lambda h)]\times[\gamma_2\Lambda\sin(\Lambda h)+\eta_2\cos(\Lambda h)]= \\ =[-\eta_2\sin(\Lambda h)+\gamma_2\Lambda\cos(\Lambda h)]\times[-\gamma_1\Lambda\sin(\Lambda h)+\eta_1\cos(\Lambda h)], \tag{19}$$

where γ_1, γ_2, η_1 and η_2 are the constants.

On utilizing Eq. (17) to Eq. (4) using Eqs. (14) and (15), one obtains:

$$\frac{\partial^2\bar{T}}{\partial r^2}+\frac{1}{r}\frac{\partial\bar{T}}{\partial r}-\Lambda_n^2\bar{T}+\left[\frac{L_n(h)}{k_2}-\frac{L_n(-h)}{k_1}\right]exp(-pt)\,\delta(r-r_0)=(1+\tau D_\omega)\frac{1}{k}\frac{\partial\bar{T}}{\partial t}, \tag{20}$$

where $\bar{T}$ represents the integral transform of T.

With transformed boundaries as:

$$\left[\bar{T}(r=b)+k_3\frac{\partial\bar{T}(r=b)}{\partial r}\right]=\overline{Q}(\Lambda_n)\delta(t),\ \ t>0, \tag{21}$$

$$\bar{T}(r,\ \Lambda_n,t)\ =\ 0,\ t=0\,, \tag{22}$$

here $\overline{Q}(\Lambda_n)=\int_{z=-h}^{h}Q(z)L_n(z)dz$.

Secondly, we state the finite Hankel transform formula and its inversion for $\bar{F}$ which satisfies convective boundary conditions [31]:

$$\bar{\bar{F}}(\mu_m,n,t)=\int_0^b r\bar{F}(r,n,t)K_0(\mu_m,r)dr, \tag{23}$$

$$\bar{F}(r,n,t)=\sum_{m=1}^{\infty}\bar{\bar{F}}(\mu_m,n,t)K_0(\mu_m,r), \tag{24}$$

where $K_0(\mu_m,r)=\frac{\sqrt{2}}{b}\frac{\mu_m k_3}{\left[1+k_3^2\mu_m^2\right]^{\frac{1}{2}}}\frac{J_0(\mu_m,r)}{J_0(\mu_m,b)}$, here μ_m denotes the root of the below equation:

$$k_3\mu J_0^{'}(\mu,b)+J_0(\mu,b)=0, \tag{25}$$

where $\bar{\bar{T}}$ represents $\bar{T}$'s Hankel transform.

Now, applying the integral method of transformation defined above in Eq. (23) to Eq. (20) with transformed boundary conditions (21), one obtains:

$$-(\mu_m^2+\Lambda_n^2)\bar{\bar{T}}+\frac{bK_0(\mu_m,b)}{k_3}\delta(t)\overline{Q}(\Lambda_n)+\left[\frac{L_n(h)}{k_2}-\frac{L_n(-h)}{k_1}\right]exp(-pt)\,r_0 f_0(\mu_m,r_0)=(1+\tau D_\omega)\frac{1}{k}\frac{\partial\bar{\bar{T}}}{\partial t}. \tag{26}$$

and transformed initial condition:

$$\bar{\bar{T}}(\mu_m,\ \Lambda_n,t)\ =\ 0,\ \ t=0. \tag{27}$$

Next, taking Laplace transformation of Eq. (26) and utilizing transformed initial boundary (27), one get:

$$\bar{\bar{T}}^*(\mu_m,\ \Lambda_n,s)=\frac{bkK_0(\mu_m,b)}{k_3\{(1+G)s+k(\mu_m^2+\Lambda_n^2)\}}\overline{Q}(\Lambda_n)+\frac{k\psi_1(h)\psi_2(r_0)}{(s+p)\{(1+G)s+k(\mu_m^2+\Lambda_n^2)\}}, \tag{28}$$

where $\left[\frac{L_n(h)}{k_2}-\frac{L_n(-h)}{k_1}\right]=\psi_1(h)$, $r_0 f_0(\mu_m,r_0)=\psi_2(r_0)$ and $G=\frac{\tau}{\omega}\left\{(1-e^{-s\omega})\left(1-\frac{2l_2}{\omega s}+\right.\right.$ $\left.\left.+\frac{2l_1^2}{\omega^2 s^2}\right)-\left(l_1^2-2l_2+\frac{2l_1^2}{\omega s}\right)e^{-s\omega}\right\}$.

Finally, inverting the integral transforms in Eq. (28) by using inversion formula defined in Eqs. (24) and (18), one obtains the expression of temperature distribution in Laplace transform domain as below:

$$T^*=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{\lambda_n}K_0(\mu_m,r)\,L_n(z)\left[\frac{bkK_0(\mu_m,b)}{k_3\{(1+G)s+k(\mu_m^2+\Lambda_n^2)\}}\overline{Q}(\Lambda_n)+\frac{k\psi_1(h)\psi_2(r_0)}{(s+p)\{(1+G)s+k(\mu_m^2+\Lambda_n^2)\}}\right]. \tag{29}$$

Determination of thermal deflection

Equations (8), (9) and (11) can be rewriting in the Laplace transform domain as:

$$\nabla^4 W^* = \frac{-\nabla^2 M_T^*}{\Omega(1-v')}, \tag{30}$$

$$M_T^* = a_t E' \int_{z=-h}^{h} T^*(r,z,s) z\, dz, \tag{31}$$

$$W^* = \frac{\partial W^*}{\partial r} = 0, \text{ at } r = b. \tag{32}$$

Equation (31), which incorporates the value of the temperature (29), yields the equation for heat based moments in the Laplace transform domain as:

$$M_T^* = a_t E' \frac{\sqrt{2}}{b} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\mu_m k_3}{[1+k_3^2\mu_m^2]^{\frac{1}{2}}} \frac{J_0(\mu_m r)}{J_0(\mu_m b)} \left[\frac{bkK_0(\mu_m b)}{k_3\{(1+G)s+k(\mu_m^2+\Lambda_n^2)\}} \left(\int_{z=-h}^{h} Q(z) L_n(z) dz \right) + \right.$$
$$\left. + \frac{k\psi_1(h)\psi_2(r_0)}{(s+p)\{(1+G)s+k(\mu_m^2+\Lambda_n^2)\}} \right] \times \int_{-h}^{h} \frac{z L_n(z)}{\lambda_n} dz. \tag{33}$$

Assuming that Eq. (30) has a solution that satisfies condition (32) in the domain of the Laplace transform:

$$W^*(r,s) = \sum_{m=1}^{\infty} c_n^*(s) [2bJ_0(\mu_m r) - 2bJ_0(\mu_m b) + \mu_m(r^2 - b^2) J_1(\mu_m b)]. \tag{34}$$

As a result, the condition (32) is satisfied by the solution (34):

$$\nabla^4 W^* = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} \right)^2 \sum_{m=1}^{\infty} c_n^*(s) [2bJ_0(\mu_m r) - 2bJ_0(\mu_m b) + \mu_m(r^2 - b^2) J_1(\mu_m b)]. \tag{35}$$

Using well-known result $\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r} \right) \right] J_0(\mu_m r) = -\mu_m^2 J_0(\mu_m r)$, Eq. (35) can be rewritten as:

$$\nabla^4 W^* = \sum_{m=1}^{\infty} c_n^*(s) [2b\mu_m^4 J_0(\mu_m r) + 2\mu_m J_1(\mu_m b)] \tag{36}$$

Also,

$$\nabla^2 M_T^* = -a_t E' \frac{\sqrt{2}}{b} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{k_3}{[1+k_3^2\mu_m^2]^{\frac{1}{2}}} \frac{\mu_m^3 J_0(\mu_m r)}{J_0(\mu_m b)} \left[\frac{bkK_0(\mu_m b)}{k_3\{(1+G)s+k(\mu_m^2+\Lambda_n^2)\}} \left(\int_{z=-h}^{h} Q(z) L_n(z) dz \right) + \right.$$
$$\left. + \frac{k\psi_1(h)\psi_2(r_0)}{(s+p)\{(1+G)s+k(\mu_m^2+\Lambda_n^2)\}} \right] \times \int_{-h}^{h} \frac{z L_n(z)}{\lambda_n} dz \tag{37}$$

Equations (36) and (37) combined with Eq. (30) yield:

$$c_n^*(s) = a_t E' \frac{\sqrt{2}}{b\Omega(1-v')} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{k_3}{[1+k_3^2\mu_m^2]^{\frac{1}{2}}} \frac{\mu_m^3 J_0(\mu_m r)}{J_0(\mu_m b)[2b\mu_m^4 J_0(\mu_m r) + 2\mu_m J_1(\mu_m b)]} \times$$
$$\times \left[\frac{bkK_0(\mu_m b)}{k_3\{(1+G)s+k(\mu_m^2+\Lambda_n^2)\}} \left(\int_{z=-h}^{h} Q(z) L_n(z) dz \right) + \frac{k\psi_1(h)\psi_2(r_0)}{(s+p)\{(1+G)s+k(\mu_m^2+\Lambda_n^2)\}} \right] \int_{-h}^{h} \frac{z L_n(z)}{\lambda_n} dz. \tag{38}$$

Substituting Eq. (38) into Eq. (34), we get:

$$W^*(r,s) = a_t E' \frac{\sqrt{2}}{b\Omega(1-v')} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{k_3 \mu_m^3 J_0(\mu_m r)[2bJ_0(\mu_m r) - 2bJ_0(\mu_m b) + \mu_m(r^2-b^2)J_1(\mu_m b)]}{[1+k_3^2\mu_m^2]^{\frac{1}{2}} J_0(\mu_m b)[2b\mu_m^4 J_0(\mu_m r) + 2\mu_m J_1(\mu_m b)]} \times$$
$$\times \left[\frac{bkK_0(\mu_m b)}{k_3\{(1+G)s+k(\mu_m^2+\Lambda_n^2)\}} \left(\int_{z=-h}^{h} Q(z) L_n(z) dz \right) + \frac{k\psi_1(h)\psi_2(r_0)}{(s+p)\{(1+G)s+k(\mu_m^2+\Lambda_n^2)\}} \right] \int_{-h}^{h} \frac{z L_n(z)}{\lambda_n} dz. \tag{39}$$

In the Laplace transform domain, the mathematical formula for temperature and thermal deflection has thus been found. So, for the purpose of numerical inversion algorithm proposed by Brancik [32,33] is adopted.

Numerical results

Dimension

Let the radius of circular plate varies from $r = 0$ to $r = 1$ m and having thickness $h = 0.1$ m. Fixing $Q(z) = z^2 \times (z^2 - h^2)^2$.

Material properties

Following material properties of copper metal plate is considered for the purpose of numerical computations.

Table 1. Thermo-mechanical properties

$k = 112.34 \times 10^{-6} m/s^2$	$v' = 0.35$
$a_t = 16.5 \times 10^{-6} K$	$\mu = 26.67 GPa$
$\rho = 8954 kg/m^3$	$c_p = 383 J/(kgK)$

Graphical presentation

This part is primarily concerned with the time delay's influence on how temperatures are distributed and thermal deflection fluctuates in a circular plate. All the plots presented below are made considering dimensionless quantities. For the graphical computation, the dimensionless thickness of the plate is chosen as $z = 0.2$ and the dimensionless time is chosen as $t = 0.5$. Figures 2 and 3 show the graphically plotted temperature and deflection flow radially under the influence of the time delay parameter $\omega = 0, 0.01, 0.02, 0.03$ for thin copper plates.

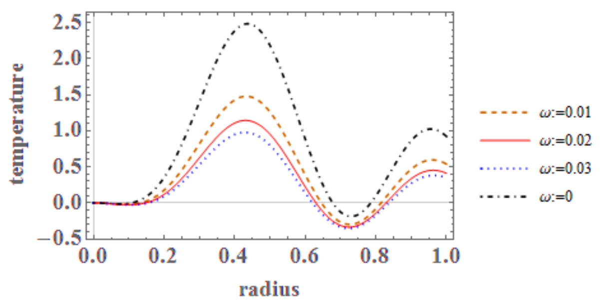

Fig. 2. Temperature distribution with impact of time delay

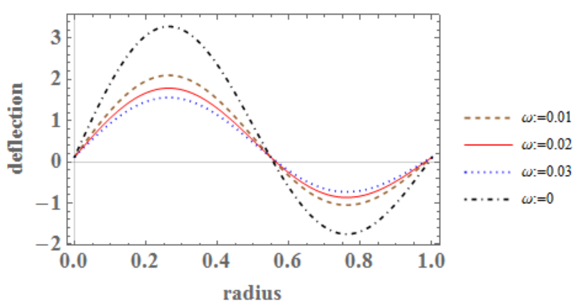

Fig. 3 Deflection distribution with impact of time delay

For various time delay parameters $\omega = 0, 0.01, 0.02, 0.03$, Fig. 2 shows the dimensionless temperature behaviour along radii at $t = 0.5$. At the inner radii, the temperature is initially zero, while at the outer radii it is nonzero due to the applied heat flux. The maximum temperature distribution is found in the middle of the radial direction, which may be due to the effect of additional cross-sectional heating on the bottom and top plate surfaces. When the time delay is shortened and $K(t - \xi) = 1$, the present heat transfer model reduces to the model of Cattaneo and Vernotte (CV). Further observations show that the temperature distribution changes smoothly for large values of the time delay values, implying that the temperature flow depends on the values of the time delay variation. Also, depending on the time delay parameters, the thermal waves vary continuously, uniformly, and strongly. As a result, the time delay factor can be crucial in the classification and design of new structural materials.

Figure 3 shows the deflection function along radii at $t = 0.5$ for different $\omega = 0, 0.01, 0.02, 0.03$. It is observed that the variation in thermal deflection is zero at the internal and external radii, which fulfills the applied boundary condition mathematically defined in Eq. (11). Also, the variations in the deflection curve are observed at the transition from inner to outer radii, which may be due to the effect of the additional cross-sectional heating at the top and bottom surfaces of the plate. A smooth and continuous variation of the deflection curves is also observed for large time delay parameters. Moreover, the curve shows finite wave propagation characteristics compared to the diffusive characteristics of the Fourier model.

It follows that the temperature and deflection behaviour in a circular body is considerably affected by the time delay and depends on the past changes, which makes this study more suitable for the study of the physical problems associated with the development of novel materials.

Conclusions

The governing equation of the memory-dependent heat transmission equation for a circular plate with certain boundary conditions is solved analytically by employing the Marchi-Fasulo, Hankel, and Laplace transformations. The effect of the time delay parameters on the temperature distribution and thermal deflection (based on thermal moment) is successfully investigated.

The following important findings are highlighted from the graphical investigations:

1. The change in the past affects the instantaneous rate of temperature change and thermal deflection, which is more suitable for the study of physical problems and has applications in the real world.
2. The rate of finite wave propagation can be seen from the change in temperature and deflection curves.
3. For different time delay parameters, a significant difference in the temperature and deflection curves is observed.
4. For large time delays, a uniform distribution of temperature and deflection is observed. It can be concluded that temperature and deflection are considerably affected by the time delay parameter.

As a result, the time delay factor plays an important role in both the development and categorisation of new structural materials. Moreover, the present work is useful for mathematicians and researchers working on the development of fractional and memory theory by considering mathematical modelling of various solids.

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