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MODELING ANTIFRICTION PROPERTIES OF COMPOSITE BASED ON DYNAMIC CONTACT PROBLEM FOR A HETEROGENEOUS FOUNDATION

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Abstract. Based on the solution of the dynamic contact problem of vibration of a rigid punch on a heterogeneous half-space, taking into account friction in the contact area, the tribological properties of an oil-filled composite material with a microstructure are modeled. The microstructure of the base is taken into account in the framework of the Biot-Frenkel model. The boundary-value problem is reduced to the integral equation, its approximate solution is constructed, which describes contact stresses, tangential displacements. The dependencies of the friction forces on the microstructure of the composite, the viscosity of the fluid filling the pores, and the degree of phase interaction are investigated.

Keywords: dynamic contact problem with friction, oil-filled composite

1. Introduction

Currently, there is a considerable amount of concepts, hypotheses, and research on diverse friction and wear-related issues. Many attempts have been made to study the properties, structure, and state of the surface layers of tribological conjugations at the atomic and molecular levels. The core problem of surface engineering (for example, metal friction units) is the synthesis of coating technologies and materials with specified wear-resistant properties. The most remarkable result to emerge from our study [1] is that the methods of vacuum ion-plasma treatment (PVD-method) and atomic modifications of diamond-like coatings (DLC) are the most appropriate among the wide array of methods aimed at hardening surface and improving tribological properties. However, along with increasing the strength properties of the materials, such technologies contribute to the formation of a damping layer with residual compressive stresses inside and a large number of stress micro-concentrators at the boundary with the main material. The above-listed aspects predetermine the formulation and solution of the dynamic contact problem, taking into account the friction forces on tribocontact. This problem is attracting an increasing interest due to the widespread application of polymer composite materials [2]. Recently, oil-filled nanocomposites, whose components are characterized with viscoelastic properties and the properties of viscous-fluid filler, have become extensively used in units and parts of tribotechnical purposes [3-5]. Constructing new composite materials with given physical and mechanical properties poses an actual, yet practically unexplored task to study the influence of dynamic effects that are caused by vibration on the antifriction properties of these materials. We undertook this study to investigate the patterns of changes in the stress-strain state of a composite material depending on its composition and dynamic loading conditions by solving a dynamic contact problem. The latest takes into account friction in the contact area for a base with a microstructure. The

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equations of the heterogeneous Biot medium are used to describe the microstructure of a base consisting of an isotropic viscoelastic matrix and a filler fluid possessing the properties of a viscous amorphous liquid [6]. The mechanical properties of the matrix material were determined experimentally by nanoindentation. The change in the mechanical modules of the medium while varying porosity was studied by the differential scheme of the self-consistency method. It should be noted that contact problems in a quasi-static formulation for homogeneous viscoelastic media that simulate surface phenomena in tribology are considered in [7-9]. The properties of the contacting surfaces significantly affect the friction force. Much work on the influence of microgeometry of contacting surfaces on the friction force was carried out by [9]. Consideration of the base microstructure as a whole for contact stresses was presented in [10-12]. In these studies, contact problems in quasistatic and dynamic formulations were considered, and the microstructure of a heterogeneous medium was described in terms of the Biot-Frenkel model. In the study [12] a two-phase medium has been described as equivalent to a single-phase assuming that the velocities of solid and liquid phases are equal. The tribological properties of oil-filled composites were experimentally studied in [13]. Dynamic contact problems for elastic layered bases are presented in [14], including non-traditional for tribology formulations [15].

This paper outlines a new approach to the study of surface interaction in the contact area, taking into account the heterogeneity of the base within the model of two-phase Biot-Frenkel medium and dynamic effects. In this context, we tried to find out the dependencies of the friction forces on the composite microstructure, the viscosity of the pore-filling fluid, and the degree of phase interaction. Interestingly, the tribological process is also characterized by tangential displacements under the punch. The strength and wear resistance of the composite depends on the energy impact in the area of contact with friction. Moreover, it is necessary to take into account both normal and tangential displacements in the contact area.

2. Statement and solution of the contact problem for a heterogeneous half-space

The contact problem of oscillations of rigid flat punch on the surface of a heterogeneous half-space under the action of a force applied to it, which varies in harmonic law, is considered. Let a two-phase medium consisting of a viscoelastic porous matrix-skeleton and fluid filling pores occupy a flat region $|x_1| < \infty,\ x_2 \le 0$. On the front impenetrable boundary of the heterogeneous medium, a rigid punch oscillates with frequency ω under the action of the force. The width of the rigid punch is $2a$. Let that force be applied to the punch so as to ensure full contact with the surface. In the contact area $|x_1| \le a$, the normal and tangential stresses are connected by the Amonton-Coulomb law. To take into account the internal microstructure of the base, we use, as the most tested model, described by the equations of the heterogeneous two-phase Biot - Frenkel medium in terms of the displacements [6]:

$$\begin{gathered} A\nabla\cdot\nabla\mathbf{u} + 2N\nabla\nabla\cdot\mathbf{u} + Q\nabla\nabla\cdot\mathbf{v} = \\ = \rho_{11}\frac{\partial^2\mathbf{u}}{\partial t^2} + \rho_{12}\frac{\partial^2\mathbf{v}}{\partial t^2} + b\left(\frac{\partial\mathbf{u}}{\partial t} - \frac{\partial\mathbf{v}}{\partial t}\right), \\ Q\nabla\nabla\cdot\mathbf{u} + R\nabla\nabla\cdot\mathbf{v} = \\ = \rho_{12}\frac{\partial^2\mathbf{u}}{\partial t^2} + \rho_{22}\frac{\partial^2\mathbf{v}}{\partial t^2} - b\left(\frac{\partial\mathbf{u}}{\partial t} - \frac{\partial\mathbf{v}}{\partial t}\right), \end{gathered} \tag{1}$$

$A, N, Q, R, \rho_{11}, \rho_{12}, \rho_{22}$ are the mechanical characteristics of a two-phase medium [16], $e_{ij}, \varepsilon_{ij},\ \ i, j = 1, 2$ are the strain tensors corresponding to displacement vectors of the solid phase $\mathbf{u}\{u_1, u_2\}$ and liquid phase $\mathbf{v}\{v_1, v_2\}$, $b = m^2\eta k_o^{-1}$ and $\eta,\ k_o$ – fluid viscosity and permeability, $\Gamma_{ij}=\sigma_{ij}^{s}+\delta_{ij}\sigma^{f}$, $i, j=1,2$ is a full stress tensor acting on a poroelastic medium, $\sigma_{ij}^{s}=Ae\delta_{ij}+2Ne_{ij}+Q\varepsilon\delta_{ij}$ is a stress tensor acting on a viscoelastic skeleton, $\sigma^{f}=Qe+R\varepsilon$ is a pore pressure. The viscosity of the composite matrix is taken into account in the framework of the model of frequency-independent internal friction. According to this approach, the shear modulus has the form $N(1+i\beta)$, where the value of β is proportional to the loss coefficient of a viscoelastic material [17] and can be determined experimentally [18]. As a result of this, a small complex component is present in the coefficients of equation (1) N, A, Q, R [19]. The oscillation mode is steady. Separate the time factor and the presentation will be carried out for the dimensionless amplitude values of the corresponding functions, while the linear dimensions are assigned to the half-width of the punch, and the stress to the shear modulus N of the matrix. The boundary conditions in this case are:

$$\begin{aligned}
&u_2(x_1,0)=v_2(x_1,0),\quad |x_1|<\infty,\\
&\Gamma_{21}(x_1,0)=\Gamma_{22}(x_1,0)=0,\quad |x_1|>1, \qquad (2)\\
&\Gamma_{21}(x_1,0)=\mu\Gamma_{22}(x_1,0),\ u_2(x_1,0)=\delta,\quad |x_1|\le 1,
\end{aligned}$$

where δ is the punch upsetting, μ is the coefficient of friction. The statement of the boundary value problem (1) – (2) closes the condition for the emission of waves at infinity. The contact pressure and horizontal displacement $u_1(x_1,0)$ under the punch must be found. We represent the displacements in the form of two scalar and vector potentials. As a result, equations (1) are split into three wave equations, and the potentials correspond to three types of waves propagating in a heterogeneous medium:

$$\begin{aligned}
&\mathbf{u}(\boldsymbol{x})=\nabla\left(L_1(\boldsymbol{x})+L_2(\boldsymbol{x})\right)+\nabla\times\boldsymbol{\Psi}(\boldsymbol{x}),\\
&\mathbf{v}(\boldsymbol{x})=\nabla\left(m_1L_1(\boldsymbol{x})+m_2L_2(\boldsymbol{x})\right)+\nabla\times\boldsymbol{\Psi}_1(\boldsymbol{x}),\ \boldsymbol{x}=(x_1,x_2), \qquad (3)\\
&\Delta L_j(\boldsymbol{x})+\theta_j^2L_j(\boldsymbol{x})=0,\quad \theta_j=\zeta_j\vartheta,\quad j=1,2,
\end{aligned}$$

m_j,ζ_j, $j=1,2$ are the roots of the following equation:

$$\begin{vmatrix} q_{12} & q_{11}\\ q_{22} & q_{12}\end{vmatrix}\zeta^2+\left(\begin{vmatrix} \gamma_{11} & \gamma_{12}\\ q_{12} & q_{22}\end{vmatrix}+\begin{vmatrix} \gamma_{22} & \gamma_{12}\\ q_{12} & q_{11}\end{vmatrix}\right)\zeta+\begin{vmatrix} \gamma_{12} & \gamma_{11}\\ \gamma_{22} & \gamma_{12}\end{vmatrix}=0.$$

$$\begin{vmatrix} \gamma_{12} & \gamma_{22}\\ q_{12} & q_{22}\end{vmatrix}m^2+\begin{vmatrix} \gamma_{11} & \gamma_{22}\\ q_{11} & q_{22}\end{vmatrix}m+\begin{vmatrix} \gamma_{11} & \gamma_{12}\\ q_{11} & q_{12}\end{vmatrix}=0,$$

$$\begin{aligned}
&\gamma_{11}=(\rho_{11}+ib/\omega)/\rho_{11},\quad \gamma_{12}=(\rho_{12}-ib/\omega)/\rho_{11},\\
&\gamma_{22}=(\rho_{22}+ib/\omega)/\rho_{11},\quad \vartheta^2=\rho\omega^2a^2/N,\\
&q_{11}=\left(A+2N\right)/N,\ q_{12}=Q/N,\ q_{22}=R/N.
\end{aligned}$$

The vector potential satisfies the following equation:

$$\begin{aligned}
&\Delta\boldsymbol{\Psi}(\boldsymbol{x})+\theta_3^2\boldsymbol{\Psi}(\boldsymbol{x})=0,\\
&\theta_3=\varsigma_3\vartheta,\ \zeta_3=(\gamma_{11}\gamma_{22}-\gamma_{12}^2)/\gamma_{22},\\
&\boldsymbol{\Psi}_1(\boldsymbol{x})=-\gamma_{12}/\gamma_{22}\boldsymbol{\Psi}(\boldsymbol{x}).
\end{aligned}$$

The velocities of longitudinal waves and shear wave are determined $V_i=\omega\, a/\theta_i$, $i=1,2,3$.

After applying the Fourier transform with respect to the variable x_1 to Eqs. (1) – (3) we construct Green's matrices. Further, using the inverse Fourier transform, we obtain the relationship of displacements and stresses of a two-phase medium:

$$\boldsymbol{u}(\boldsymbol{x})=\frac{1}{2\pi}\int_{\Upsilon}\boldsymbol{K}_u(\alpha,x_2)\mathbf{Q}(\alpha)e^{-i\alpha x_1}d\alpha,$$

$$\boldsymbol{v}(\boldsymbol{x})=\frac{1}{2\pi}\int_{\Upsilon}\boldsymbol{K}_v(\alpha,x_2)\mathbf{Q}(\alpha)e^{-i\alpha x_1}d\alpha,\qquad (4)$$

$$\mathbf{Q}(\alpha)=\int_{-\infty}^{\infty}\boldsymbol{q}(\boldsymbol{x})e^{i\alpha x_1}dx_1,$$

$$\boldsymbol{q}(\boldsymbol{x})=\{\Gamma_{21}(\boldsymbol{x}),\Gamma_{22}(\boldsymbol{x})\}.$$

The circuit Υ is selected in accordance with the conditions of radiation of waves at infinity, $\mathbf{K}_u(\alpha,x_2)$, $\mathbf{K}_v(\alpha,x_2)$ are Green's matrices for skeleton and fluid displacements, respectively [20].

Satisfying the boundary conditions (2) in relation (4), we have obtained the integral equation with a difference kernel with unknown normal contact pressure:

$$\int_{-1}^{1}K(x_1-\xi)q_2(\xi)d\xi=\delta_0,\ \ \delta_0=\delta/a.\qquad (5)$$

The kernel of the integral equation (4) has the form:

$$K(x_1-\xi)=\frac{1}{2\pi}\int_{\Upsilon}\left(\mu K_{21}(\alpha)+K_{22}(\alpha)\right)e^{i\alpha(x_1-\xi)}d\alpha,\qquad (6)$$

$$K_{11}(\alpha)=\left(\left(2g_3s_3+\gamma_{13}(g_{01}-g_{02})\right)\alpha^2+s_3(g_2-g_1)\right)/D(\alpha),$$

$$K_{12}(\alpha)=i\alpha\left(2g_0+2(s_1-s_2)\gamma_{13}\alpha^2+(\alpha^2+s_3^2)g_3\right)/D(\alpha),$$

$$K_{21}(\alpha)=i\alpha(2g_0+\gamma_{12}/\gamma_{22}g_1+g_2)/D(\alpha),$$

$$K_{22}(\alpha)=(m_1-m_2)s_1s_2\theta_3^2/D(\alpha),$$

$$D(\alpha)=2(2g_0+\gamma_{13}g_1)\alpha^2+(\alpha^2+s_3^2)(g_2-g_1),$$

$$g_1=\begin{vmatrix}g_{01} & g_{02}\\ s_1 & s_2\end{vmatrix},\ g_2=\begin{vmatrix}g_{01} & g_{02}\\ m_1s_1 & m_2s_2\end{vmatrix},\ g_3=\begin{vmatrix}s_2 & s_1\\ (1-m_1) & (1-m_2)\end{vmatrix},\ g_0=(m_1-m_2)s_1s_2s_3,$$

$$g_{0i}=2\alpha^2-\theta_i^2\left(q_{11}+q_{12}+(q_{12}+q_{22})(1-m_i)\right),\quad i=1,2,$$

$$\gamma_{13}=1+\gamma_{12}/\gamma_{22};\quad s_k=\sqrt{\alpha^2-\theta_k^2};\qquad k=1,2,3,$$

where α is the Fourier transform parameter, the functions $K_{ij}(\alpha),\ i,j=1,2$ are elements of the Green's matrix (4) for a heterogeneous medium. The functions $K_{ij}(\alpha),\ i,j=1,2$ have the following behavior at infinity:

$$\lim_{\alpha\to\infty}K_{ii}(\alpha)=d_{ii}/|\alpha|,\ \lim_{\alpha\to\infty}K_{ij}(\alpha)=(-1)^i d_{ij}/\alpha,\ i\neq j\ .$$

Next, we regularize the kernel (6) of the integral equation (5) to separate its logarithmic singularity. We use the function $d_{22}/\sqrt{\alpha^2+R^2}$ as a regularizer of the kernel $K(x_1-\xi)$ of the integral equation (5). This function coincides with the function $K_{22}(\alpha)$ at infinity and does not have poles in the complex plane. As a result, relation (6) has the form:

$$K(x_1-\xi)=I_1(x_1-\xi)+K_0(R|x_1-\xi|)/\pi,$$

$$I_1(x_1-\xi)=\frac{1}{2\pi}\int_{\Upsilon}L(\alpha)e^{i\alpha(x_1-\xi)}d\alpha,\qquad (7)$$

$$L(\alpha)=\mu K_{21}(\alpha)+K_{22}(\alpha)-d_{22}/\sqrt{\alpha^2+R^2},$$

where $K_0(z)$ is zero-order Macdonald function. The integral $I_1(x_1-\xi)$ in (7) is rapidly convergent. The choice of a parameter $R>>1$ minimizes the contribution of integrals over $iR\le\alpha<i\infty,\ -iR\ge\alpha>-i\infty$ cuts [12]. The Macdonald function zero-order has the expression in the form of a series [21]:

$$K_0(z)=-I_0(z)\ln(\gamma z/2)+\sum_{m=0}^{\infty}\left(\frac{z}{2}\right)^{2m}\frac{\psi\left(m+1)\right)}{(m!)^2},\ \lim_{z\to0}I_0(z)=1,$$

where $I_0(z)$ is modified Bessel function of the first kind, γ is the Euler's constant. The logarithmic singularity in the explicit form in relation (6) is separated.

After regularization of the integral equation (5), the representation of the kernel (7) makes it possible to efficiently apply the numerical solution scheme based on the boundary element method. We choose the partition points $x_{1i},\ i=\overline{1,N}$ that are uniformly distributed, with a step $h=2/N$ on a segment $[-1+h/2,1-h/2]$. The function $q_2(x_1)$ is constant within each of the elements of the partition: $q_2(x_1)\big|_{x_{1i}<x_1<x_{1i+1}}=q_2(x_{1i})=q_i,\ i=\overline{1,N}$. The midpoints of the corresponding segments of the partition $[x_{1i};x_{1i+1}],\ i=\overline{1,N}$ are selected as nodes. As a result, the solution of the integral equation (5) reduces to a finite system of linear N - order algebraic equations with respect to unknowns $q_i,\ i=\overline{1,N}$. The system has a quasi-diagonal matrix and converges quickly:

$$\sum_{m=1}^{N}r_{mn}q_n=\delta_0,\quad m,n=1,2,...,N,$$

$$r_{mn}=\begin{cases}I_1(x_{1m}-\xi_n)+\dfrac{\sqrt{2\pi}}{R}d_{22}sign(x_{1m+1}-\xi_n)\left(Erf(z_2)-Erf(z_1)\right)/h, & m\ne n\\ I_1(x_{1n}-\xi_n)+2d_{22}Erf(\sqrt{hR/2})/h, & m=n\end{cases}$$

$$z_1=\sqrt{(x_{1m}-\xi_n)R},\quad z_2=\sqrt{(x_{1m+1}-\xi_n)R},\tag{7}$$

$$d_{22}=(m_1-m_2)\theta_3/d_g,$$

$$d_g=2((\theta_1^2+g_{01})m_2-(\theta_2^2+g_{02})m_1-(\theta_2^2-\theta_1^2+g_{02}-g_{01})\gamma_{12}/\gamma_{22}),$$

where $Erf(z)$ is probability integral [22]. The integral $I_1(x_{1m}-\xi_n)$ is calculated by integration over the contour Υ in the complex plane. The integration contour is selected in accordance with the radiation conditions so that the displacements of the surface of the heterogeneous half-space decrease with distance from the vibrating punch. This choice is made after finding the poles and branch points of the integrands in (7) and analyzing them when the internal friction of the medium tends to zero. Note that to analyze the rate of convergence of the process, the residual elements were estimated for the number of partitions N and $3N$. Number of mesh elements chosen from the condition that the residual is less than 10^{-4}.

Horizontal movements under and outside the punch are determined through the elements of the Green's matrix of the heterogeneous medium $K_{11}(\alpha),\ K_{12}(\alpha)$ and contact pressure:

$$u_1(x_1,0)=\int_{-1}^{1}K_1(x_1-\xi)q_2(\xi)d\xi,$$

$$K_1(x_1-\xi)=\frac{1}{2\pi}\int_{\Upsilon}\left(\mu K_{11}(\alpha)+K_{12}(\alpha)\right)e^{i\alpha(x_1-\xi)}d\alpha.$$

Next, we perform the procedure for extracting a singularity in the function $K_1(x_1-\xi)$, similar to the algorithm described above. Then we discretize the contact area and determine the tangential displacement under the punch, taking into account contact pressures determined from relation (7):

$$u_1(x_1,0)=\sum_{n=1}^{N} q_n \tilde{r}_{nm}, \quad m=\overline{1,N}$$

$$\tilde{r}_{mn}=\begin{cases} I_2(x_{1m}-\xi_n)+\dfrac{\sqrt{2\pi}}{R}\mu\, d_{11} sign(x_{1m+1}-\xi_n)\left(Erf(z_2)-Erf(z_1)\right), & m\neq n,\\ I_2(x_{1n}-\xi_n)+\mu d_{11} Erf(\sqrt{hR/2}), & m=n, \end{cases}$$

$$d_{11}=(g_{01}m_2-g_{02}m_1+(g_{01}-g_{02})\gamma_{12}/\gamma_{22})/d_g.$$

3. Results of computational experiments

In accordance with the above method, normal and tangential contact stresses, tangential displacements under the punch were determined. The calculations were carried out for the mechanical characteristics corresponding to a two-component composite material with a Phenylone-based matrix modified with a nano-additive (magnesium aluminum spinel) and cylinder oil-contained filler.

Correct determination of the mechanical characteristics A,R,Q,N of a Biot medium for a heterogeneous composite is a multi-stage problem. The values of Young's modulus E_s and Poisson's ratio ν_s were determined during field experiments when compressing a Phenylone sample without a filler in a loading mode that provides purely elastic deformations of the sample [23]. Further, the mechanical properties of Phenylone and composites with a Phenylone matrix [24] were determined based on the nanoindentation method with the previously found Poisson's ratio. The next step was to study the effect of porosity on the bulk modulus drained porous medium K_b. The results obtained based on the self-consistent [25] scheme and generalized differential self-consistent scheme are compared. Note that at low porosity ($m<0.2$), the relative error of the two calculation methods did not exceed 4%. Thus, the coefficients of equations (1) for the known bulk modulus of a viscoelastic matrix K_s, drained porous medium K_b, fluid K_f , and porosity m were calculated by the formulas [10].

The calculations were carried out with the following data: $K_s=6.3$ GPa, $K_f=2$ GPa, $N=2.29$ GPa, $\rho_s=1.2\cdot10^3$ kg/m^3, $\rho_f=0.93\cdot10^3$ kg/m^3, $K_b(m=0.05)=5.32$ GPa, $K_b(m=0.1)=4.36$ GPa, $K_b(m=0.15)=3.40$ GPa, $K_b(m=0.2)=2.44$ GPa, $\omega=50$ Hz.

The Phenylone has a low tendency to creep under the action of stresses [26] and the use of nano-additives to modify the composite matrix allows one to suppress relaxation processes. The viscosity of the composite matrix was taken into account in the framework of the model of frequency-independent internal friction. The calculations were carried out for the range $10^{-3}<\beta<0.5\cdot10^{-1}$ [18]. The experimental determination of the coefficient of the interaction of phases, the permeability of the composite is a very laborious problem. We will rely on the known data for artificial media containing clay particles, they have a permeability coefficient in the range $10^{-14}<k_o<10^{-10}$ [27], and the tortuosity of the pore channels corresponds to the case of spherical particles [28].

Particular attention was paid to the analysis of the effect of porosity on the magnitude of contact pressures. The distribution of the real part of normal contact pressure $\mathrm{Re}\, q_2(x_1)$ with a change in porosity m and fluid saturation of the base is shown in Fig. 1 for values $\mu = 0.2, \delta_0 = 1, \beta = 0.05, b = 0$.

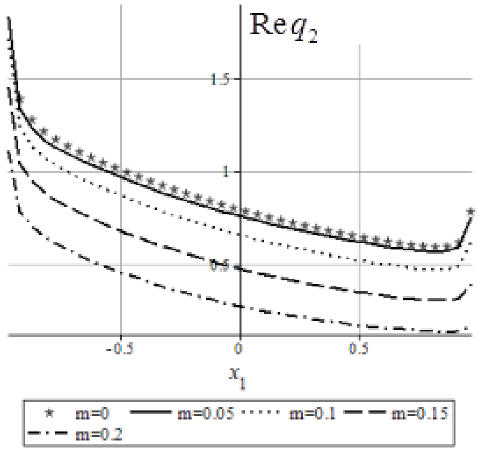

Fig. 1. Influence of porosity of a heterogeneous base on the distribution of the real part of normal contact pressure

Moreover, the curve marked with a marker «*» corresponds to zero porosity. This curve was obtained based on the solution of the dynamic problem of punch vibrations in a similar formulation for the case of an equivalent elastic medium [12].

It is established that the dependence of normal and tangential contact stresses on the porosity of a heterogeneous base is non-linear. Note that the contact normal and tangential stresses are largely dependent on the porosity and mass fraction of the filler fluid. Moreover, the stress distribution over the contact area is asymmetric, which is also characteristic of contact problems of the theory of elasticity with allowance for the friction forces. It is important to note that for the case of dynamic loading, the influence of the coefficient of friction on contact pressures is more pronounced than in problems in a quasistatic formulation [7,10].

It should be noted that the parameter $b = m^2\eta / k_0$ characterizing the interaction of the phases of the composite has a significant effect on contact pressures. With an increase in the viscosity of the fluid filling the pores and a decrease in the permeability coefficient of the porous elastic medium, the stress distribution under the punch changes, as illustrated in Fig. 2. This figure shows the distribution of the real parts of normal pressures under the punch, calculated with the same input data with increasing parameter η / k_0.

It has been experimentally shown [29] that the increase in the vibration frequency induces the decrease in the contact stresses. The results of the numerical analysis also confirm the given fact [11]. The change in the amplitude of the friction force during the period of vibration plays a key role. The wear resistance of the composite depends greatly on the energy impact in the friction area, taking into account both the normal and tangential displacements in the contact area [11]. An increase in the vibration frequency leads to a decrease in the contact stresses per unit time; however, a large amount of energy is generated in the contact area. Figure 3 presents the plot of the tangential contact stresses for a *T*-period under punch vibrations affected by the applied force that varies according to the $P = P_0 \cos \omega t$ law for $\eta / k_0 = 0.01 \cdot 10^{10}$. The tangential contact stresses Γ_{21} during the vibration period change sign, while its amplitude delineates a flat figure, the area of which is proportional to the energy of the friction force. The above information makes it possible to predict the composite's wear resistance.

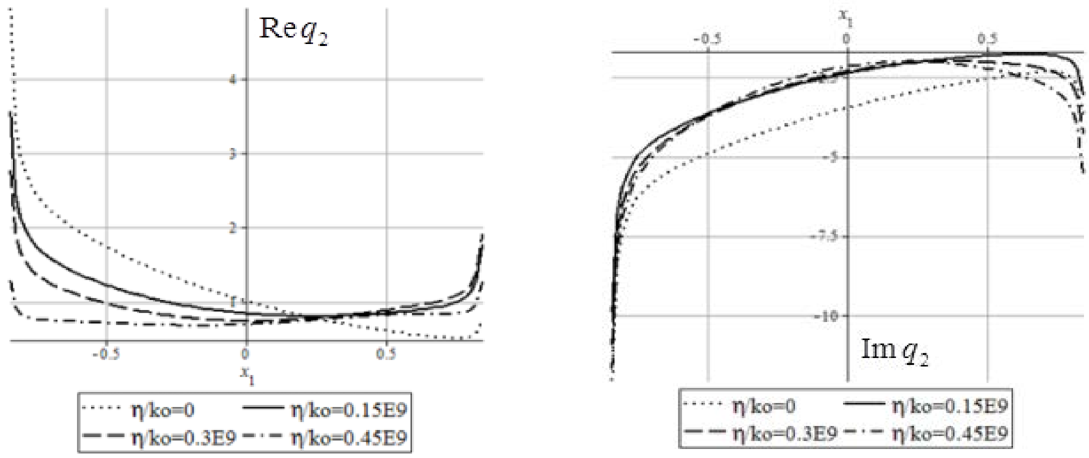

Fig. 2. The effect of fluid viscosity and permeability on the distribution of the real (left) part and imaginary (right) part of normal contact pressures

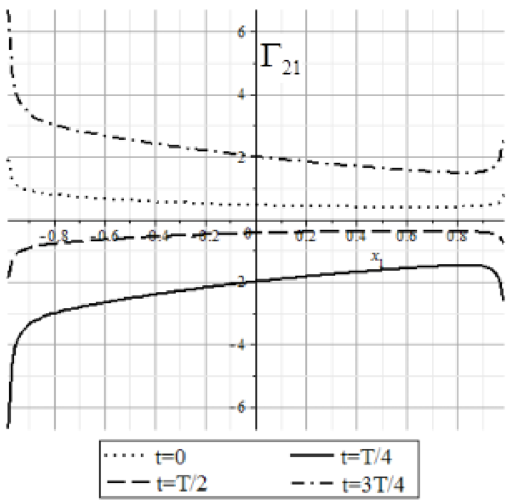

Fig. 3. Change in the tangential stresses during the vibration period T

4. Conclusions

Contact stresses and tangential displacements under the punch for a heterogeneous base depend not only on the friction coefficient in the contact area, but also substantially depend on the vibration frequency, porosity, permeability of the medium, and fluid viscosity. A change in the friction coefficient during vibration has a much greater effect on contact stresses than in a quasistatic problem when the punch moves. Based on the considered model, it is possible to evaluate the energy effect over a period of oscillation, on which the wear resistance of the composite material depends. In this case, it is necessary to take into account not only normal movements but also horizontal movements in the contact area. The studies that were performed allow us to find the optimal ratio of the mechanical properties of a composite material operating under dynamic loading.

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