

# DEFORMATION OF A RECTANGULAR PLATE MEDIUM THICKNESS FROM ORTHOTROPIC DIFFERENTLY RESISTANT MATERIAL

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**Abstract.** The construction of a physically nonlinear model of deformation of a rectangular orthotropic plate of average thickness loaded with a transverse uniform distributed load is considered. This model is limited by the scope of small deflections. In the formulation and solution, not only the orthotropy of the plate material was taken into account, but also the nonlinear differential resistance, which was described using the equations of state, constructed using the normalized stress space. The plate fastening is presented in two versions: hinged support and rigid fastening along the contour. An algorithm for solving this class of problems was developed and implemented. A practical solution was made using the MATLAB software package.

**Keywords:** rectangular plate, rigid clamping, hinged support, orthotropic material, nonlinear resistance to resistance

## 1. Introduction

The development of science and technology has given impetus to the design of more complex, improved, and unique buildings, structures, machine parts, and apparatus. An example of this is modern research centers, sports stadiums, military equipment, and the aviation industry. In all these industries, materials are used whose properties don't obey the linear laws of mechanics. All these objects require careful calculation since the slightest error at the initial design stage can lead to serious accidents and death of people later.

For the error-free design of such structures, various calculation theories are developed and simplified models of objects are proposed. Also, more and more technologically advanced materials are used, for the calculation of which conventional (classical) models aren't enough, for example, a structural material, a composite of carbon fiber AVCO Mod 3a [35]. The desire to reduce the weight of the structure while improving its quality makes it necessary to use modern calculation methods in the design process. That is why the development of new and modernization of old models is an urgent task of modern structural mechanics and mechanics of a deformable solid.

The issue of calculating material with different resistance, and specifically plates made of them, was dealt with by many researchers: S.A. Ambartsumyan [1-5], R.M. Jones [6-9], C.W. Bert [10-13], A.A. Zolochovsky [14-21], Lomakin E.V. [22-23], A.V. Berezin [24], N.M. Matchenko [26] and A.A. Treschev [25,27-32].

So S.A. Ambartsumyan in his works [1-3] proposed simple constitutive relations in the form of equations of state with tangential-linear dependencies between the principal stresses and deformations, and the question of the relations between shear stresses and shears wasn't

discussed. In his model, the field, the principal stresses are divided into regions of the first and second kind [4-5]. This model is similar in shape to the classical generalized Hooke's law of orthotropic material, but the elastic moduli and transverse strain coefficients in the directions of the principal axes are determined separately from the experiments on axial tension ( $E_k^+$ ,  $\nu_{km}^+$ ) and compression ( $E_k^-$ ,  $\nu_{km}^-$ ). Direct application of the proposed relations is possible only in those cases when the distribution of the principal stresses by their signs at different points of the body is known in advance, as well as subject to model constraints on the constants arising from the symmetry condition of the compliance tensor.

In the model of R.M. Jones [6-9] featured symmetric weighted compliance matrices. Their symmetry was achieved by different signs of the principal stresses due to the introduction of weight coefficients into the off-diagonal components. They were pairwise ratios of the modules of principal stresses ( $k_1 = |\sigma_1| / (|\sigma_1| + |\sigma_2|)$ ,  $k_2 = |\sigma_2| / (|\sigma_1| + |\sigma_2|)$ ).

One of the simplest models of equations of state for materials with different resistance was proposed by K.V. Bert (C.W. Bert) [10-13]. This model is applicable to fibrous materials, where the components of the compliance matrix depend on the sign of the normal stresses arising in the direction of the fibers. When equal to 0 along the fibers of normal stresses, this theory ceases to be valid.

The most complex and controversial model was proposed by A.A. Zolochovsky [14-21]. He introduced equivalent stress, the second degree of which determines the deformation potential. The potential constants are "hidden" in the expressions that make up the equivalent stress. Equivalent stress is determined by the sum of linear and quadratic joint stress invariants. Due to the presence of irrationality in the equations for the relationship between stresses and strains, it isn't possible to single out the compliance matrix in a general form. The complexity of this model lies in the experimental determination of a many numbers of constants (which can't always be isolated from experiments in sufficient quantities). In particular, for an orthotropic material in the quasi-linear approximation, it is necessary to determine thirty-two constants, and from the simplest reference experiments (uniaxial tension and compression in the direction of the main orthotropic axes and at an angle of 45 ° to them) only eighteen can be established.

## 2. Methods

It is obvious that even a detailed analysis of the most well-known models of constitutive relations for anisotropic materials with different resistances indicates that these models aren't free from serious shortcomings and are based on individual hypotheses, often unfounded by experimental facts. In particular, E.V. Lomakin in [22-23] formulates the strain potential for anisotropic materials in the form of an energy function of the ratio of average stress to stress intensity  $\xi = \sigma / \sigma_i$  (where  $\sigma = \sigma_{ij} \cdot \delta_{ij} / 3$  – average stresses,  $\sigma_i = \sqrt{1,5 S_{ij} S_{ij}}$  – average stresses;  $S_{ij} = \sigma_{ij} - \delta_{ij} \sigma$  – components of the stress deviator;  $\delta_{ij}$  – Kronecker symbol), multiplied by the convolution of the fourth rank compliance tensor with second rank stress tensors in the principal axes of material anisotropy. A serious drawback of the introduced relations is the emergence of uncertainties for the functional parameter of the  $\xi$  level  $\pm\infty$ , which was repeatedly pointed out in [24-26].

In the works of N.M. Matchenko and A.A. Trescheva [25-27] deformation potentials are constructed for anisotropic multi-resistive materials admitting quasi-linear approximation in nine-dimensional normalized stress space. In these works, equations of state of two levels of accuracy were obtained. Despite the rationality of this approach, the relations obtained are also not free from significant drawbacks, which for equations of the first level of accuracy are complex functional dependencies between uncorrelated constants of materials, and for

equations of the second level, there is an excessively large number of constants subject to experimental determination, which requires the involvement of experiments on complex stress conditions.

In subsequent works [28,31-32], a correcting formulation of the equations of state was carried out for a different class of anisotropic materials, both in quasilinear and nonlinear formulations. In the nonlinear model [29-30], equations of state are used, represented by the type of generalized Hooke's law for anisotropic materials by the type:

$$e_{km} = H_{kmpq}(\sigma_i, \alpha_{st}) \cdot \sigma_{pq}; \quad H_{kmpq} = H_{pqkm}; \quad k, m, q, p, s, t, = 1, 2, 3. \quad (1)$$

In particular, for an orthotropic material, these dependencies are presented as follows:

$$e_{11} = (A_{1111} + B_{1111} \cdot \alpha_{11}) \cdot \sigma_{11} + [A_{1122} + B_{1122} \cdot (\alpha_{11} + \alpha_{22})] \cdot \sigma_{22} + \quad (2)$$

$$+ [A_{1133} + B_{1133} \cdot (\alpha_{11} + \alpha_{33})] \cdot \sigma_{33};$$

$$e_{22} = [A_{1122} + B_{1122} \cdot (\alpha_{11} + \alpha_{22})] \cdot \sigma_{11} + (A_{2222} + B_{2222} \cdot \alpha_{22}) \cdot \sigma_{22} + \quad (3)$$

$$+ [A_{2233} + B_{2233} \cdot (\alpha_{22} + \alpha_{33})] \cdot \sigma_{33};$$

$$e_{33} = [A_{1133} + B_{1133} \cdot (\alpha_{11} + \alpha_{33})] \cdot \sigma_{11} + [A_{2233} + B_{2233} \cdot (\alpha_{22} + \alpha_{33})] \cdot \sigma_{22} + \quad (4)$$

$$+ (A_{3333} + B_{3333} \cdot \alpha_{33}) \cdot \sigma_{33};$$

$$2e_{12} = C_{1212}(\sigma_i) \cdot \tau_{12}; \quad (5)$$

$$2e_{23} = C_{2323}(\sigma_i) \cdot \tau_{23}; \quad (6)$$

$$2e_{13} = C_{1313}(\sigma_i) \cdot \tau_{13}; \quad (7)$$

where  $a_{ij} = \sigma_{ij} / S$ ; – normalized stresses in the principal axes of material anisotropy;

$S = (\sigma_{ij} \cdot \sigma_{ij})^{0.5} = \sqrt{\sigma_{11}^2 + \sigma_{22}^2 + \sigma_{33}^2 + 2(\tau_{12}^2 + \tau_{23}^2 + \tau_{31}^2)}$  – total stress modulus (stress space norm);

$A_{ijkm}, B_{ijkm}, C_{ijkm}$ , – nonlinear functions that determine the mechanical properties of the material.

For orthotropic bodies, the number of independent material functions reaches fifteen [31-32]. The representation of these functions that determine the properties of the material is carried out by approximating the experimental deformation diagrams under uniaxial tension and compression along the principal anisotropy axes and diagrams obtained for shear in three principal orthotropic planes by processing them in the Microcal Origin Pro 8.0 program (Microcal Software Inc.). In this case, for a structural orthotropic nonlinearly resistive composite material AVCO Mod 3a [35] are represented as follows:

$$A_{kkkk}(\sigma_i) = 0.5 \cdot [1/E_k^+(\sigma_i) + 1/E_k^-(\sigma_i)]; \quad (8)$$

$$B_{kkkk}(\sigma_i) = 0.5 \cdot [1/E_k^+(\sigma_i) - 1/E_k^-(\sigma_i)]; \quad (9)$$

$$A_{kkmm}(\sigma_i) = -0.5 \cdot \left[ \frac{\nu_{km}^+(\sigma_i)}{E_m^+(\sigma_i)} + \frac{\nu_{km}^-(\sigma_i)}{E_m^-(\sigma_i)} \right]; \quad (10)$$

$$B_{kkmm}(\sigma_i) = -0.5 \cdot \left[ \frac{\nu_{km}^+(\sigma_i)}{E_m^+(\sigma_i)} - \frac{\nu_{km}^-(\sigma_i)}{E_m^-(\sigma_i)} \right]; \quad (11)$$

$$C_{kmkm}(\sigma_i) = 1/G_{km}(\sigma_i); \quad (12)$$

$$E_k^\pm(\sigma_i) = a_k^\pm + m_k^\pm \cdot \sigma_i + n_k^\pm \cdot \sigma_i^2; \quad (13)$$

$$\nu_{km}^{\pm}(\sigma_i) = \lambda_{km}^{\pm} + \beta_{km}^{\pm} \cdot \sigma_i + \mu_{km}^{\pm} \cdot \sigma_i^2; \quad (14)$$

$$G_{km}(\sigma_i) = g_{km} + p_{km} \cdot \sigma_i + q_{km} \cdot \sigma_i^2; \quad (15)$$

where  $a_k^{\pm}$ ,  $m_k^{\pm}$ ,  $n_k^{\pm}$ ,  $\lambda_{km}^{\pm}$ ,  $\beta_{km}^{\pm}$ ,  $\mu_{km}^{\pm}$ ,  $g_{km}$ ,  $p_{km}$ ,  $q_{km}$  – constants of nonlinear material functions, determined by processing experimental deformation diagrams by the least-squares method and presented in Table 1.

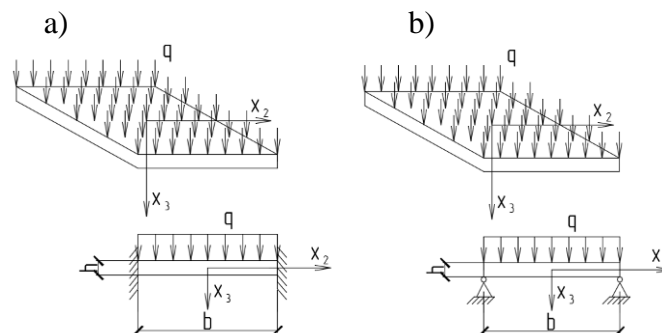
This model of a nonlinear orthotropic resistive material [28,31] is currently the least controversial, gives the results as close as possible to the experimental data, and therefore is used here as the basis for constructing a method for calculating plates.

The nonlinear properties of such materials manifest themselves already at the elastic stage of deformation, and this significantly affects the stress distribution with a further increase in the load. It isn't possible to describe the process of deformation of a plate made of similar materials by ordinary linear functions with the required degree of accuracy, and especially in a complex stress state, which is realized within the framework of transverse bending.

We consider the elastic equilibrium of a rectangular single-layer plate, referred to the Cartesian coordinate system (the  $X_1$  axis is along the long side of the plate, the  $X_2$  axis is along the short side of the plate, and the  $X_3$  axis is along the plate thickness). At an arbitrary point on the plate, one of the symmetry planes is parallel to the median surface, and the other two are perpendicular to the coordinate lines:  $x_1 = \text{const}$ ,  $x_2 = \text{const}$ .

Two options for supporting the plate along the contour are considered:

- plate with rigidly clamped contours in accordance with Figure 1a;
- the plate is hingedly supported along the contours in accordance with Fig. 1b.



**Fig. 1.** Design diagram of a rectangular plate with two types of support: a) with rigidly clamped contours; b) with hingedly supported contours

Table 1. Constants of composite material AVCO Mod 3a [35]

Sample test type	Technical parameter	1st element of nonlinear function	2nd element of nonlinear function	3rd element of nonlinear function
Uniaxial tension along the principal axes of orthotropy	$E_k^+(\sigma_i)$ , Pa	$\alpha_1^+$	$m_1^+$	$n_1^+$
		$1.058 \cdot 10^{-10}$	62.829	$1.535 \cdot 10^{-6}$
		$\alpha_2^+$	$m_2^+$	$n_2^+$
		$2.864 \cdot 10^{-10}$	-105.476	$5.893 \cdot 10^{-7}$
		$\alpha_3^+$	$m_3^+$	$n_3^+$
		$2.301 \cdot 10^{-10}$	88.349	$3.711 \cdot 10^{-6}$
	$\nu_{km}^+(\sigma_i)$	$\lambda_{12}^+$	$\beta_{12}^+$	$\mu_{12}^+$

Sample test type	Technical parameter	1st element of nonlinear function	2nd element of nonlinear function	3rd element of nonlinear function
		0.158	$-3.106 \cdot 10^{-9}$	$2.192 \cdot 10^{-17}$
		$\lambda_{21}^+$	$\beta_{21}^+$	$\mu_{21}^+$
		0.103	$-1.79 \cdot 10^{-9}$	$9.106 \cdot 10^{-18}$
		$\lambda_{13}^+$	$\beta_{13}^+$	$\mu_{13}^+$
		0.203	$2.15 \cdot 10^{-9}$	$6.148 \cdot 10^{-17}$
		$\lambda_{23}^+$	$\beta_{23}^+$	$\mu_{23}^+$
		0.104	$0.87 \cdot 10^{-10}$	$6.741 \cdot 10^{-17}$
		$\lambda_{31}^+$	$\beta_{31}^+$	$\mu_{31}^+$
		0.146	$-0.146 \cdot 10^{-10}$	$6.971 \cdot 10^{-17}$
Uniaxial compression along the main axes of orthotropy	$E_k^-(\sigma_i), \text{ Pa}$	$\alpha_1^-$	$m_1^-$	$n_1^-$
		$9.988 \cdot 10^9$	-12.943	$6.71 \cdot 10^{-7}$
		$\alpha_2^-$	$m_2^-$	$n_2^-$
		$2.326 \cdot 10^{10}$	-436.81	$-6.077 \cdot 10^{-7}$
		$\alpha_3^-$	$m_3^-$	$n_3^-$
		$5.14 \cdot 10^9$	-129.15	$-78.31 \cdot 10^{-6}$
	$\nu_{km}^-(\sigma_i)$	$\lambda_{12}^-$	$\beta_{12}^-$	$\mu_{12}^-$
		0.118	$-1.457 \cdot 10^{-9}$	$2.136 \cdot 10^{-17}$
		$\lambda_{21}^-$	$\beta_{21}^-$	$\mu_{21}^-$
		0.06	$1.77 \cdot 10^{-9}$	$2.947 \cdot 10^{-17}$
		$\lambda_{13}^-$	$\beta_{13}^-$	$\mu_{13}^-$
		0.264	$-1.118 \cdot 10^{-9}$	$3.01 \cdot 10^{-17}$
		$\lambda_{23}^-$	$\beta_{23}^-$	$\mu_{23}^-$
		0.189	$2.156 \cdot 10^{-9}$	$2.104 \cdot 10^{-17}$
		$\lambda_{31}^-$	$\beta_{31}^-$	$\mu_{31}^-$
		0.134	$-0.457 \cdot 10^{-10}$	$5.819 \cdot 10^{-17}$
Offset in principal orthotropy planes	$G_{km}(\sigma_i), \text{ Pa}$	$g_{12}$	$p_{12}$	$q_{12}$
		$4.07 \cdot 10^9$	-1,6	$-8.38 \cdot 10^{-6}$
		$g_{23}$	$p_{23}$	$q_{23}$
		$1.723 \cdot 10^9$	16.899	$-1.1 \cdot 10^{-5}$
		$g_{31}$	$p_{31}$	$q_{31}$
		$2.43 \cdot 10^9$	-54.455	$-1.97 \cdot 10^{-5}$

For the posed problem, model assumptions, traditional for this class of problems, were introduced over the entire plate thickness [12,19,31] in the following formulation:

1) the normal to the median plane after deformation is rotated by an angle  $\psi_1$  relative to the axis  $x_1$  and by  $-\psi_2$  relative to the axis  $x_2$ ;

2) when determining the parameters of the stress state, the influence of normal stresses  $\sigma_3$  due to their smallness, is neglected.

Based on the above assumptions, for the displacements of the plate points we have:

$$u_1(x_1, x_2, x_3) = u_1(x_1, x_2) + x_3 \cdot \psi_2(x_1, x_2); \quad (16)$$

$$u_2(x_1, x_2, x_3) = u_1(x_1, x_2) + x_3 \cdot \psi_2(x_1, x_2); \quad (17)$$

$$u_3(x_1, x_2, x_3) = w(x_1, x_2); \quad (18)$$

where  $u_1, u_2, u_3$  – mid-surface displacement;  $\psi_1, \psi_2$  – angles of rotation of the plate sections relative to the axes;  $w$  – deflection.

In this case, the constitutive relations for a nonlinearly resistive orthotropic material (2-7), according to the adopted model, are represented as:

$$e_{11} = (A_{1111}(\sigma_i) + B_{1111}(\sigma_i) \cdot \alpha_{11}) \cdot \sigma_{11} + [A_{1122}(\sigma_i) + B_{1122}(\sigma_i) \cdot (\alpha_{11} + \alpha_{22})] \cdot \sigma_{22}; \quad (19)$$

$$e_{22} = [A_{1122}(\sigma_i) + B_{1122}(\sigma_i) \cdot (\alpha_{11} + \alpha_{22})] \cdot \sigma_{11} + (A_{2222}(\sigma_i) + B_{2222}(\sigma_i) \cdot \alpha_{22}) \cdot \sigma_{22}; \quad (20)$$

$$e_{33} = [A_{1133}(\sigma_i) + B_{1133}(\sigma_i) \cdot \alpha_{11}] \cdot \sigma_{11} + [A_{2233}(\sigma_i) + B_{2233}(\sigma_i) \cdot \alpha_{22}] \cdot \sigma_{22}; \quad (21)$$

$$2e_{12} = C_{1212}(\sigma_i) \cdot \tau_{12}; \quad (22)$$

$$2e_{13} = C_{1313}(\sigma_i) \cdot \tau_{13}; \quad (23)$$

$$2e_{23} = C_{2323}(\sigma_i) \cdot \tau_{23}; \quad (24)$$

where:

$$S = \sqrt{\sigma_{11}^2 + \sigma_{22}^2 + 2(\tau_{12}^2 + \tau_{23}^2 + \tau_{31}^2)}; \quad (25)$$

$$\sigma_i = \sqrt{\sigma_{11}^2 - \sigma_{11}\sigma_{22} + \sigma_{22}^2 + 3(\tau_{12}^2 + \tau_{23}^2 + \tau_{31}^2)}; \quad (26)$$

$$C_{1111} = A_{1111}(\sigma_i) + B_{1111}(\sigma_i) \cdot \alpha_{11}; \quad (27)$$

$$C_{1122} = A_{1122}(\sigma_i) + B_{1122}(\sigma_i) \cdot (\alpha_{11} + \alpha_{22}); \quad (28)$$

$$C_{1133} = A_{1133}(\sigma_i) + B_{1133}(\sigma_i) \cdot \alpha_{11}; \quad (29)$$

$$C_{2222} = A_{2222}(\sigma_i) + B_{2222}(\sigma_i) \cdot \alpha_{22}; \quad (30)$$

$$C_{2233} = A_{2233}(\sigma_i) + B_{2233}(\sigma_i) \cdot \alpha_{22}; \quad (31)$$

$$D_{1212} = C_{1212}(\sigma_i); \quad D_{2323} = C_{2323}(\sigma_i); \quad D_{3131} = C_{3131}(\sigma_i). \quad (32)$$

Expressing stresses through deformations, taking into account the simplifying equations (8-15, 19-24, 27-32), after simple mathematical manipulations, we come to the following dependencies:

$$\sigma_{11} = D_{1111}(u_{1,1} - x_3 \cdot \psi_{2,1}) + D_{1122}(u_{2,2} - x_3 \cdot \psi_{1,2}); \quad (33)$$

$$\sigma_{22} = D_{1122}(u_{1,1} - x_3 \cdot \psi_{2,1}) + D_{2222}(u_{2,2} - x_3 \cdot \psi_{1,2}); \quad (34)$$

$$\tau_{12} = \frac{u_{1,2} + u_{2,1} - x_3 \cdot (\psi_{1,1} + \psi_{2,2})}{D_{1212}}; \quad (35)$$

$$\tau_{23} = \frac{(\psi_2 + w_{,1})}{D_{2323}}; \quad (36)$$

$$\tau_{31} = \frac{(\psi_1 + w_{,2})}{D_{3131}}; \quad (37)$$

where:

$$D_{1111} = \frac{C_{2222}}{C_{1111} \cdot C_{2222} - C_{1122} \cdot C_{1122}}; \quad (38)$$

$$D_{1122} = \frac{C_{1122}}{C_{1111} \cdot C_{2222} - C_{1122} \cdot C_{1122}}; \quad (39)$$

$$D_{2222} = \frac{C_{1111}}{C_{1122} \cdot C_{1122} - C_{2222} \cdot C_{1111}}. \quad (40)$$

Deformations aren't explicitly included here, but they are easily calculated from the third equation of the system (19-24).

Taking into account that taking the new physical equations as a basis, we thus don't make changes in the dependences of the static-geometric nature, we represent the static conditions for a rectangular plate in a Cartesian coordinate system in the traditional form [12,19,31]:

$$N_{ij,j} = 0; \quad Q_{k,k} = -q; \quad M_{ij,j} - Q_i = 0. \quad (41)$$

Forces and moments are determined by integrating stresses (31-35) over the plate thickness:

$$N_{ij} = \int_{-h/2}^{h/2} \sigma_{ij} dx_3, \quad (i,j=1,2); \quad (42)$$

$$Q_k = \int_{-h/2}^{h/2} \tau_{k3} dx_3, \quad (k=1,2); \quad (43)$$

$$M_{ij} = \int_{-h/2}^{h/2} \sigma_{ij} \cdot x_3 dx_3, \quad (i,j=1,2). \quad (44)$$

Considering dependences (31-44) together, after some transformations, we obtain the resolving equations for bending of rectangular orthotropic plates of average thickness from a nonlinear material with different resistance:

$$\begin{aligned} & \psi_{2,11} \cdot B_{1111} + \psi_{2,1} \cdot B_{1111,1} + \psi_{1,21} \cdot B_{1122} + \psi_{1,2} \cdot B_{1122,1} + u_{2,21} \cdot B_{1122} + u_{2,2} \cdot B_{1122,1} + \\ & + u_{1,1} \cdot B_{1111,1} + u_{1,11} \cdot B_{1111} + 0,5 \cdot \psi_{1,12} \cdot B_{1212} + \\ & + 0,5 \cdot \psi_{1,1} \cdot B_{1212,2} + 0,5 \cdot \psi_{2,22} \cdot B_{1212} + 0,5 \cdot \psi_{2,2} \cdot B_{1212,2} + \\ & + 0,5 \cdot u_{1,12} \cdot D_{1212} + 0,5 \cdot u_{1,2} \cdot D_{1212,2} + 0,5 \cdot u_{2,12} \cdot D_{1212} + 0,5 \cdot u_{2,1} \cdot D_{1212,2} = 0; \end{aligned} \quad (45)$$

$$\begin{aligned} & \psi_{1,2} \cdot B_{2222,2} + \psi_{1,22} \cdot B_{2222} + \psi_{2,1} \cdot B_{1122,2} + \psi_{2,12} \cdot B_{1122} + u_{2,2} \cdot B_{2222,2} + u_{2,22} \cdot B_{2222} + \\ & + u_{1,12} \cdot B_{1122} + u_{1,1} \cdot B_{1122,2} + 0,5 \cdot \psi_{1,1} \cdot B_{1212,1} + \\ & + 0,5 \cdot \psi_{1,11} \cdot B_{1212} + 0,5 \cdot \psi_{2,2} \cdot B_{1212,1} + 0,5 \cdot \psi_{2,21} \cdot B_{1212} + \\ & + 0,5 \cdot u_{1,2} \cdot D_{1212,1} + 0,5 \cdot u_{1,21} \cdot D_{1212} + 0,5 \cdot u_{2,1} \cdot D_{1212,1} + 0,5 \cdot u_{2,11} \cdot D_{1212} = 0; \end{aligned} \quad (46)$$

$$\begin{aligned} & 0,5 \cdot \psi_2 \cdot D_{1313,1} + 0,5 \cdot \psi_{2,1} \cdot D_{1313} + 0,5 \cdot w_3 \cdot D_{1313,1} + 0,5 \cdot w_{,11} \cdot D_{1313} + \\ & + 0,5 \cdot w_1 \cdot D_{2323,2} + 0,5 \cdot w_{1,2} \cdot D_{2323} + 0,5 \cdot w_{,22} \cdot D_{2323} + 0,5 \cdot w_{,2} \cdot D_{2323,2} = -q; \end{aligned} \quad (47)$$

$$\begin{aligned} & \psi_{2,11} \cdot K_{1111} + \psi_{2,1} \cdot K_{1111,1} + \psi_{1,21} \cdot K_{1122} + \psi_{1,2} \cdot K_{1122,1} + u_{2,21} \cdot B_{1122} + \\ & + u_{2,2} \cdot B_{1122,1} + u_{1,11} \cdot B_{1111} + u_{1,1} \cdot B_{1111,1} + \\ & + 0,5 \cdot \psi_{1,1} \cdot K_{1212,2} + 0,5 \cdot \psi_{1,12} \cdot K_{1212} + 0,5 \cdot \psi_{2,22} \cdot K_{1212} + \\ & + 0,5 \cdot \psi_{2,2} \cdot K_{1212,2} + 0,5 \cdot u_{1,22} \cdot B_{1212} + 0,5 \cdot u_{1,2} \cdot B_{1212,2} + \\ & + 0,5 \cdot u_{2,1} \cdot B_{1212} + 0,5 \cdot u_{2,12} \cdot B_{1212,2} - 0,5 \cdot \psi_2 \cdot D_{1313} - 0,5 \cdot w_{,1} \cdot D_{1313} = 0; \end{aligned} \quad (48)$$

$$\psi_{1,22} \cdot K_{2222} + \psi_{1,2} \cdot K_{2222,2} + \psi_{2,12} \cdot K_{1122} + \psi_{2,1} \cdot K_{1122,2} + u_{2,22} \cdot B_{2222} +$$

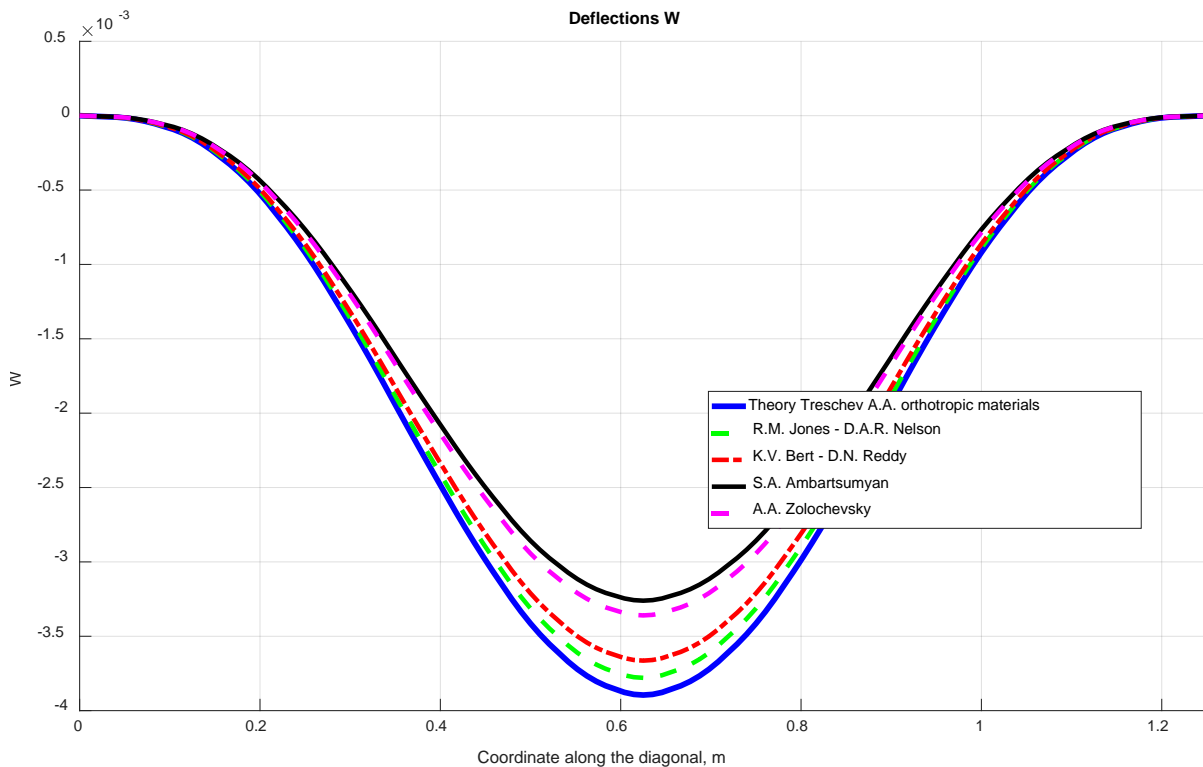
$$\begin{aligned}
& +u_{2,2} \cdot B_{2222,2} + u_{1,12} \cdot B_{1122} + u_{1,1} \cdot B_{1122,2} + \\
& +0,5 \cdot \psi_{1,11} \cdot K_{1212} + 0,5 \cdot \psi_{1,1} \cdot K_{1212,1} + 0,5 \cdot \psi_{2,21} \cdot K_{1212} + \\
& +0,5 \cdot \psi_{2,2} \cdot K_{1212,1} + 0,5 \cdot u_{1,21} \cdot B_{1212} + 0,5 \cdot u_{1,2} \cdot B_{1212,1} + \\
& +0,5 \cdot u_{2,11} \cdot B_{1212} + 0,5 \cdot u_{2,1} \cdot B_{1212,1} - 0,5 \cdot \psi_1 \cdot D_{2323} - 0,5 \cdot w_{,2} \cdot D_{2323} = 0.
\end{aligned} \tag{49}$$

To solve the obtained equations (45-49), we use the finite-difference method with the approximation of the second order of accuracy [33-34]. This procedure is implemented in the MATLAB environment.

### 3. Results and discussion

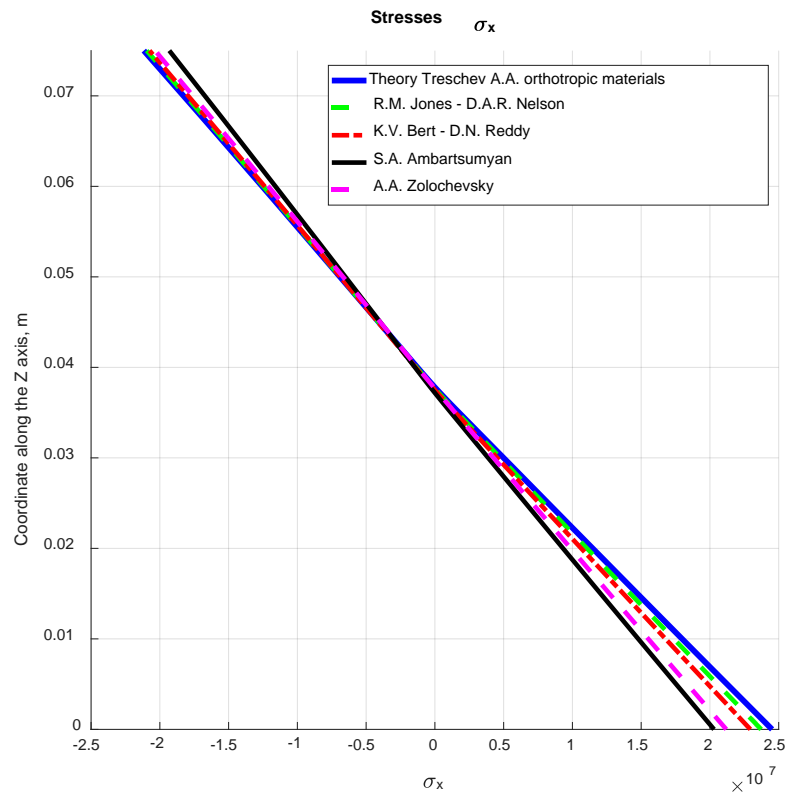
The main calculation results are given for a plate with dimensions of  $1.0 \times 0.75 \times 0.075$  (a×b×h) m, under the action of a uniformly distributed load  $q=1.45$  MPa. As a result of solving the problem posed for the transverse bending of a rigidly fixed (Figs. 2-19) and hingedly supported (Figs. 20-34) rectangular plate made of orthotropic nonlinear multi-resistive composite material AVCO Mod 3a [35], distributions of the main characteristics of the stress-strain state were obtained. Additionally, the effect of the load value on deflections and maximum moments in the plate was analyzed to demonstrate nonlinearity for various fixing options.

#### Rigidly clamped plate made of graphite AVCO Mod3a [35]

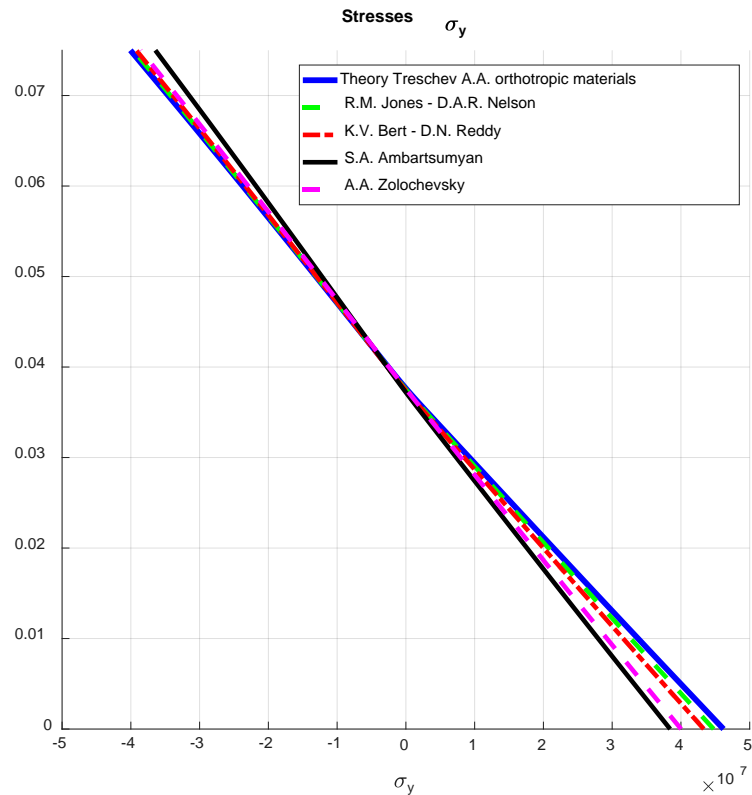


**Fig. 2.** Deflections W along the diagonal of the plate

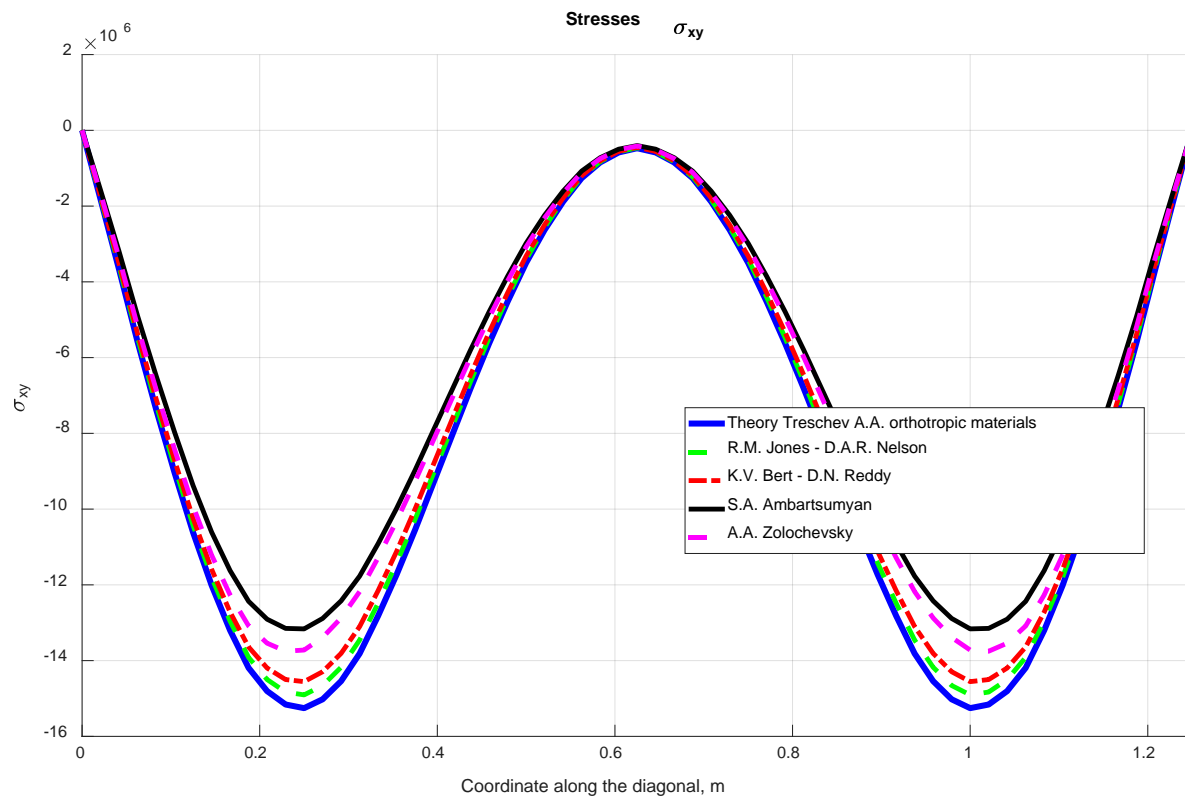




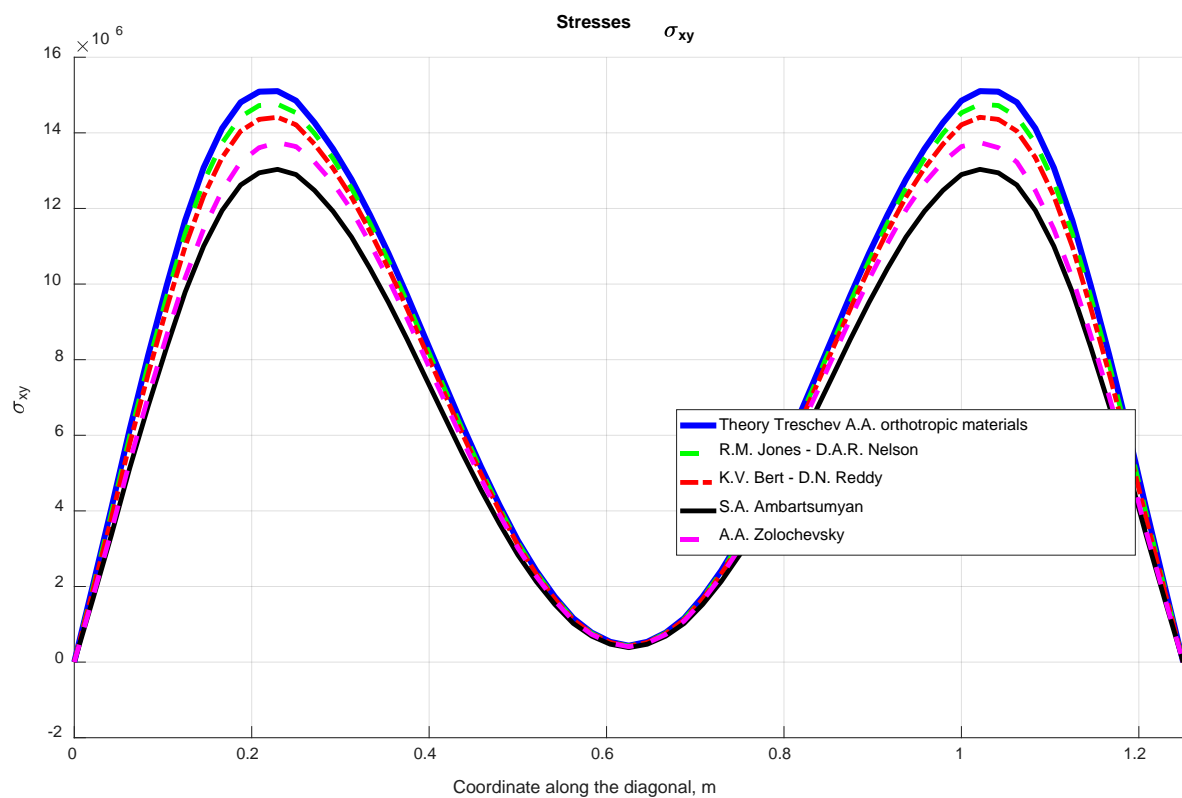
**Fig. 3.** Distribution of  $\sigma_x$  stresses over the plate thickness at point 0.5a 0.5b



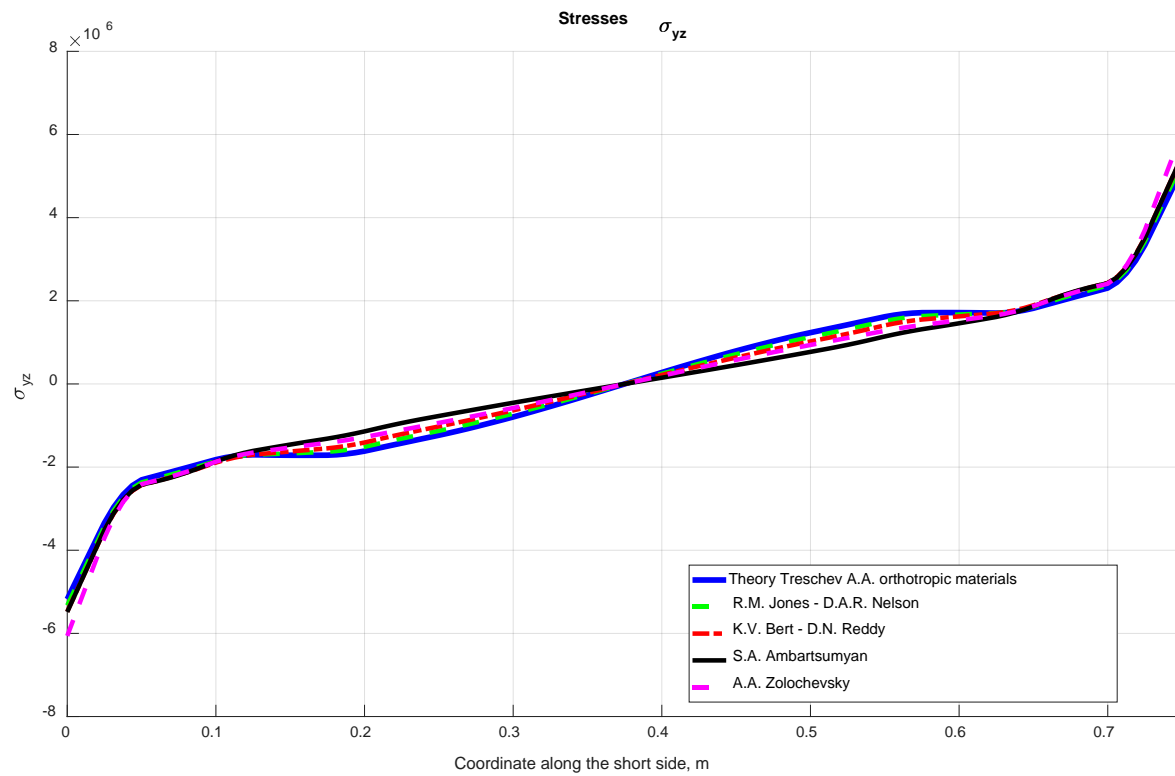
**Fig. 4.** Distribution of  $\sigma_y$  stresses over the plate thickness at point 0.5a 0.5b



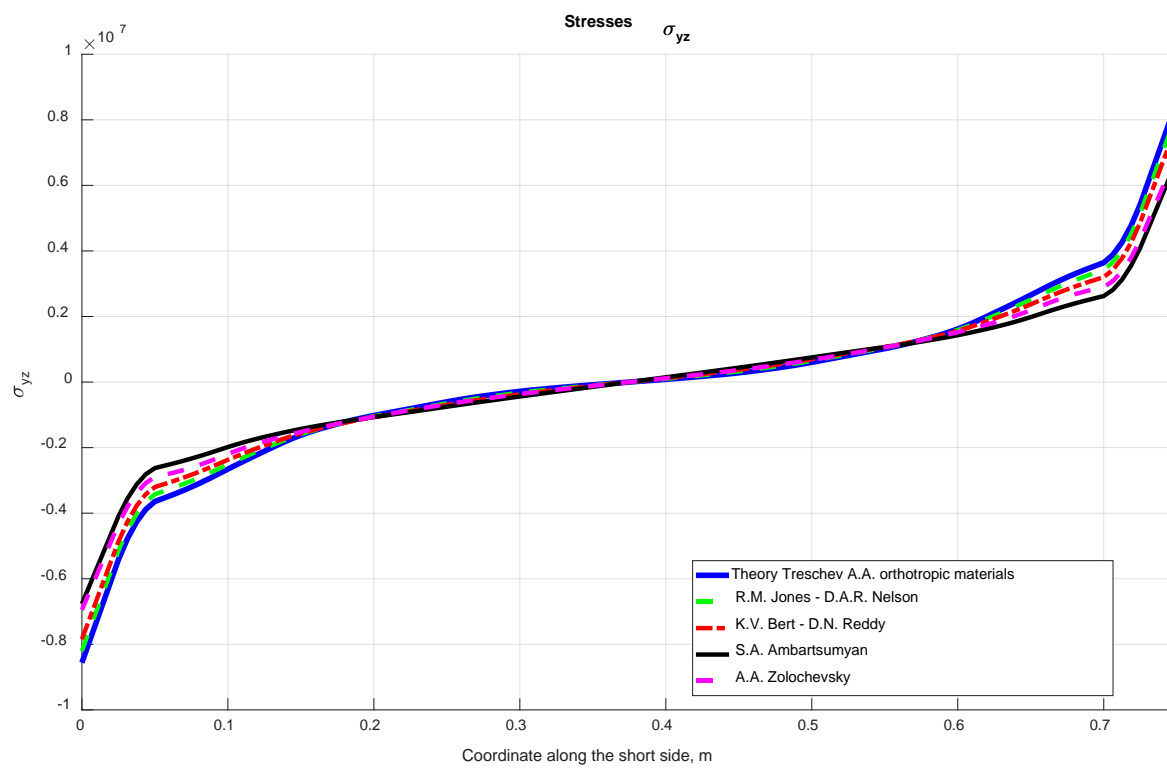
**Fig. 5.** Distribution of shear stresses  $\sigma_{xy}$  along the platinum diagonal in the lower section



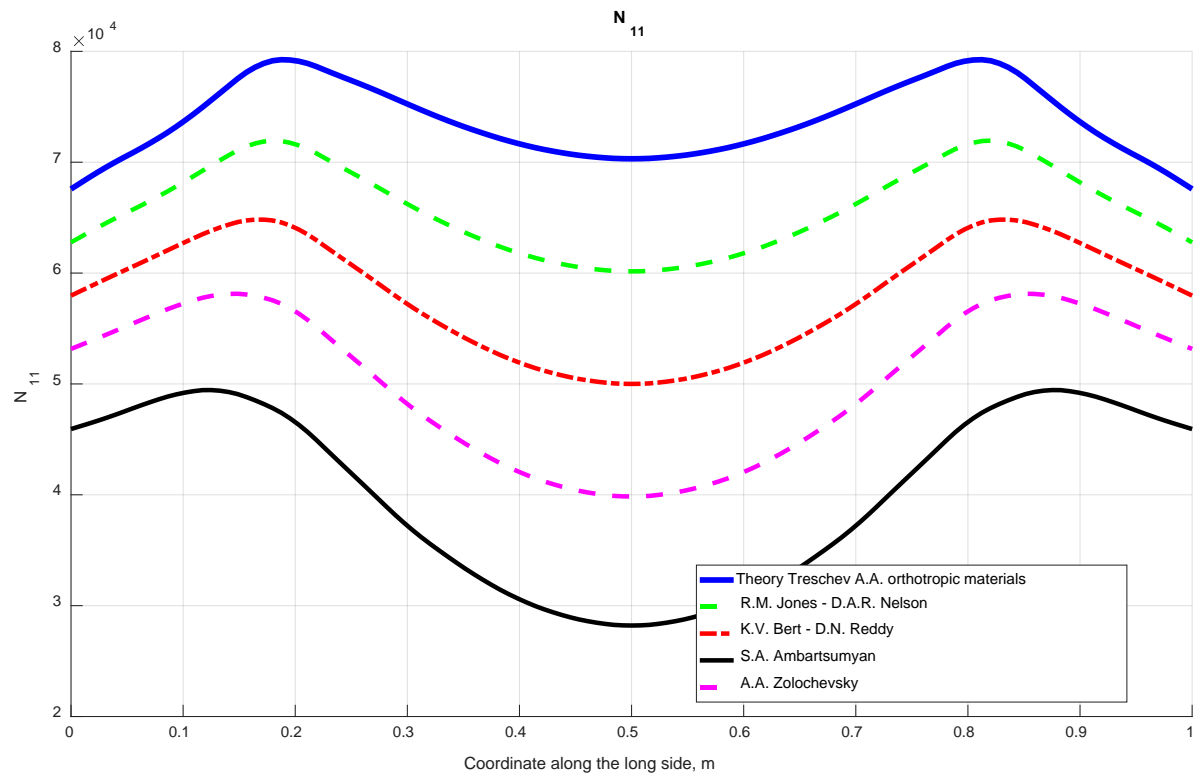
**Fig. 6.** Distribution of shear stresses  $\sigma_{xy}$  along the diagonal of platinum in the upper section



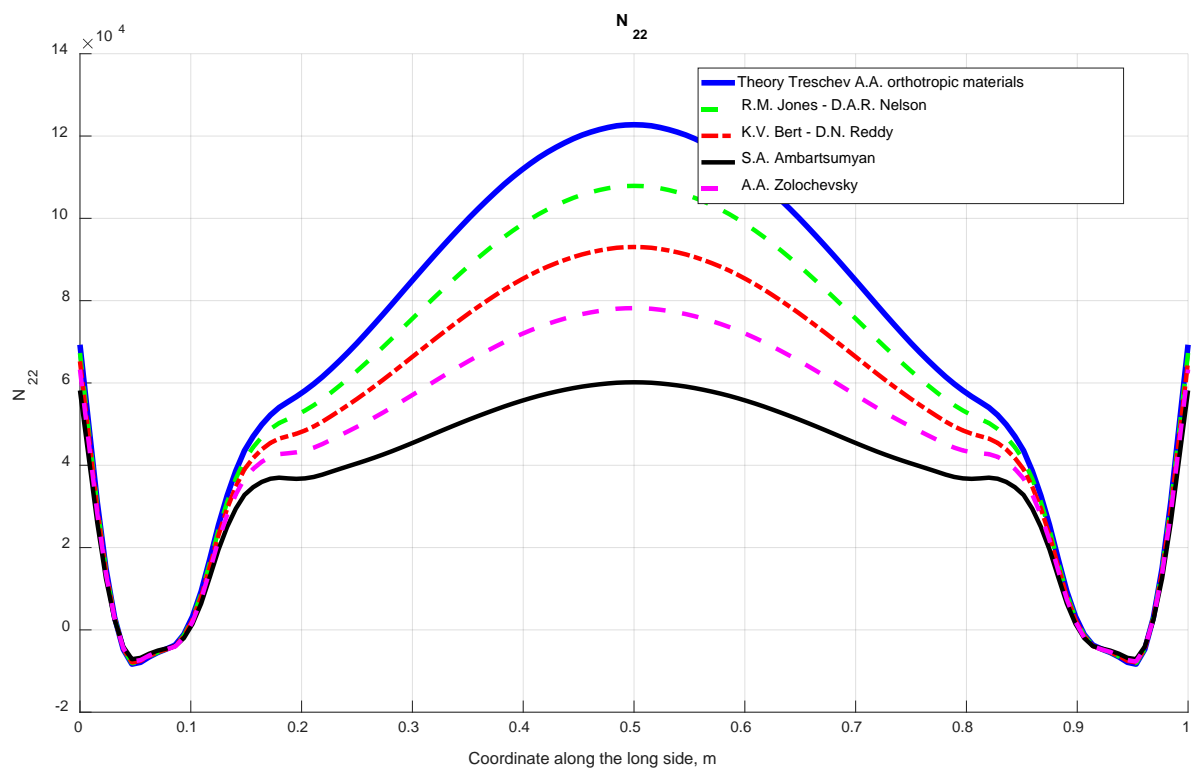
**Fig. 7.** Distribution of shear stresses  $\sigma_{yz}$  along the  $X_2$  axis in the lower section



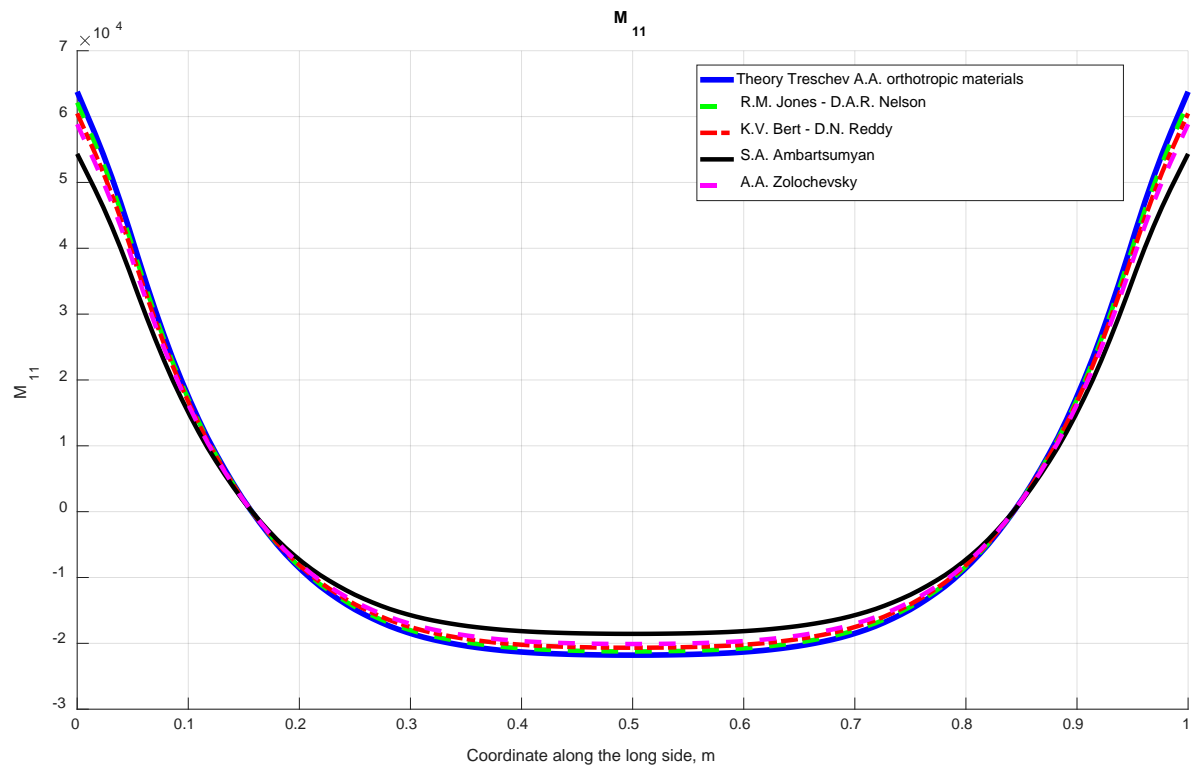
**Fig. 8.** Distribution of shear stresses  $\sigma_{yz}$  along the  $X_2$  axis in the upper section



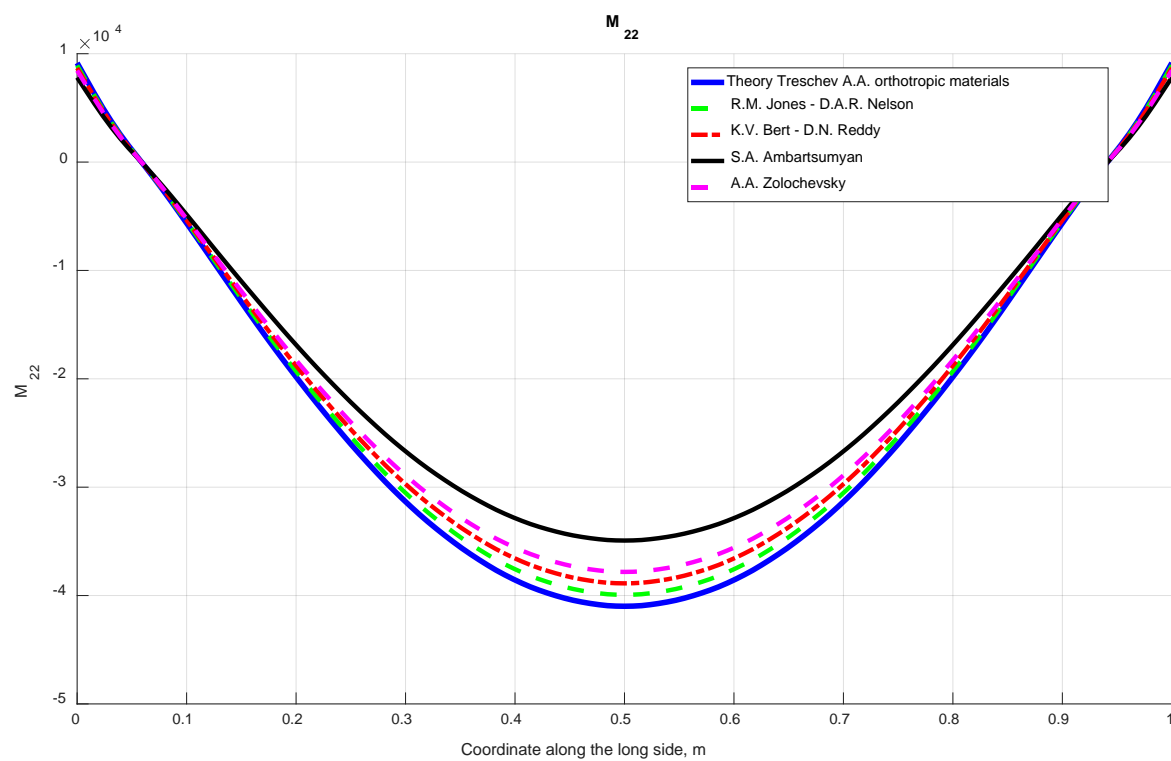
**Fig. 9.** Efforts  $N_{11}$  along the  $X_1$  axis



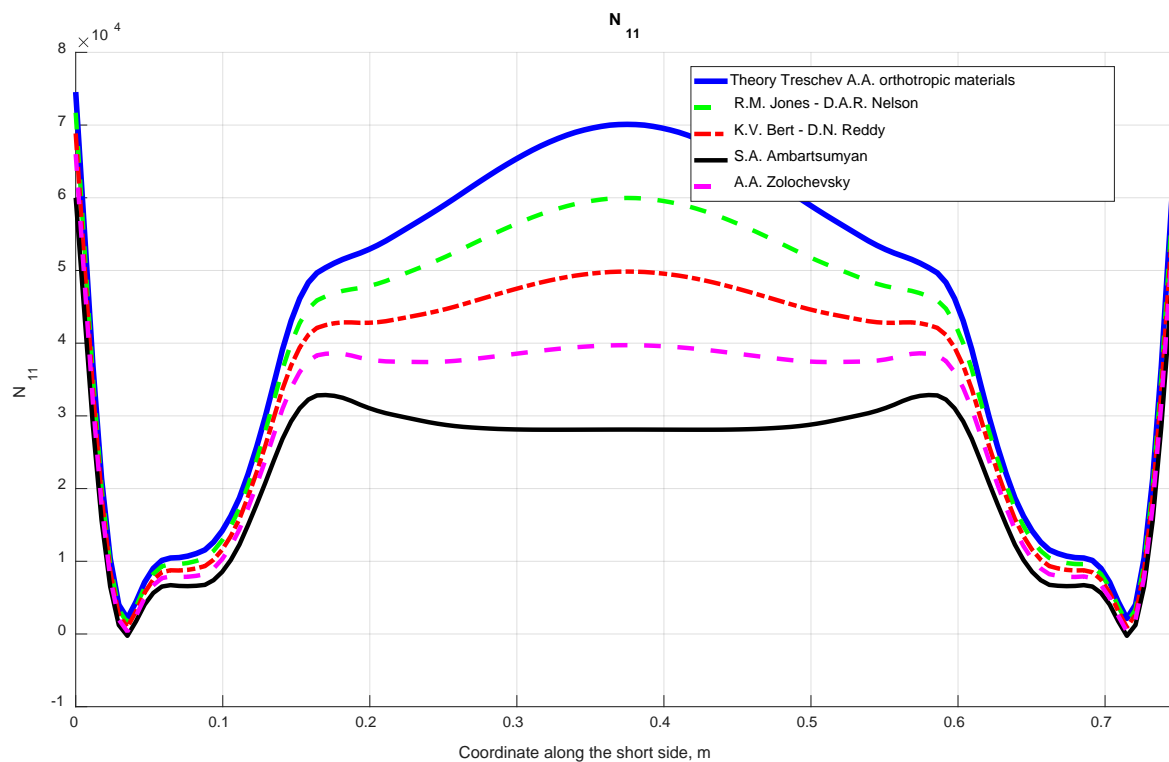
**Fig. 10.** Efforts  $N_{22}$  along the  $X_1$  axis



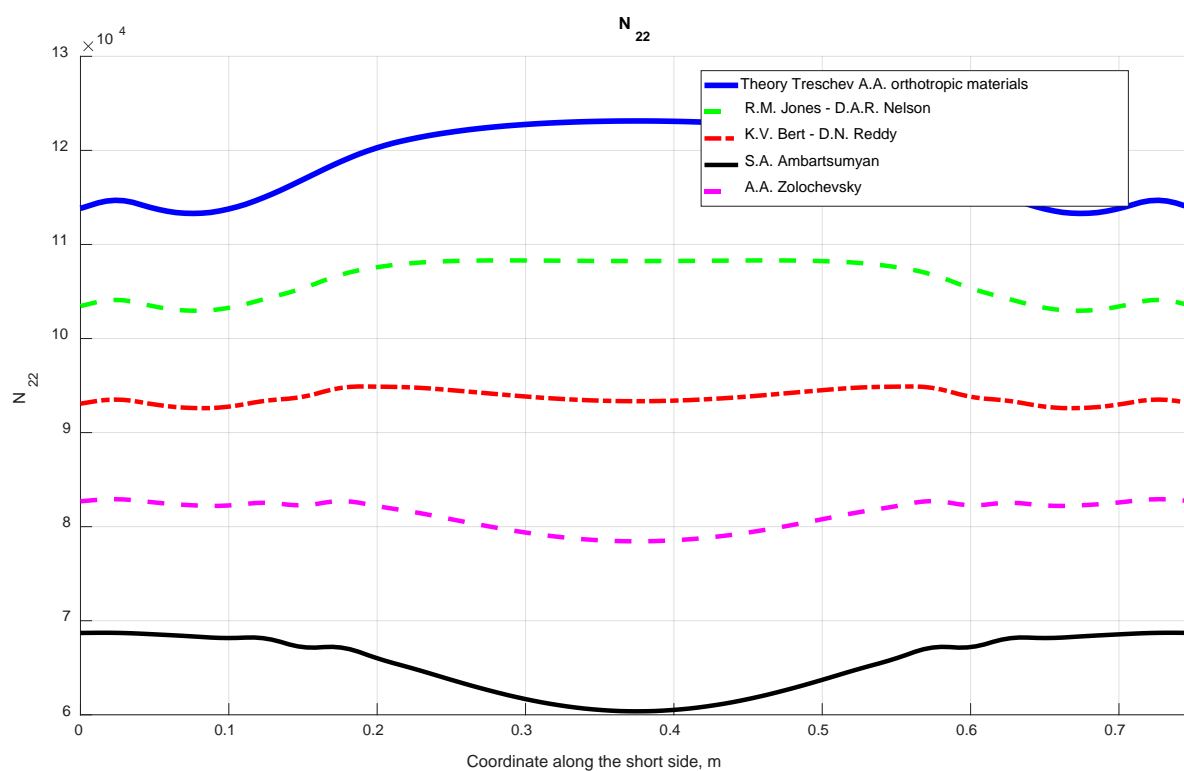
**Fig. 11.** Bending moment  $M_{11}$  along the  $X_1$  axis



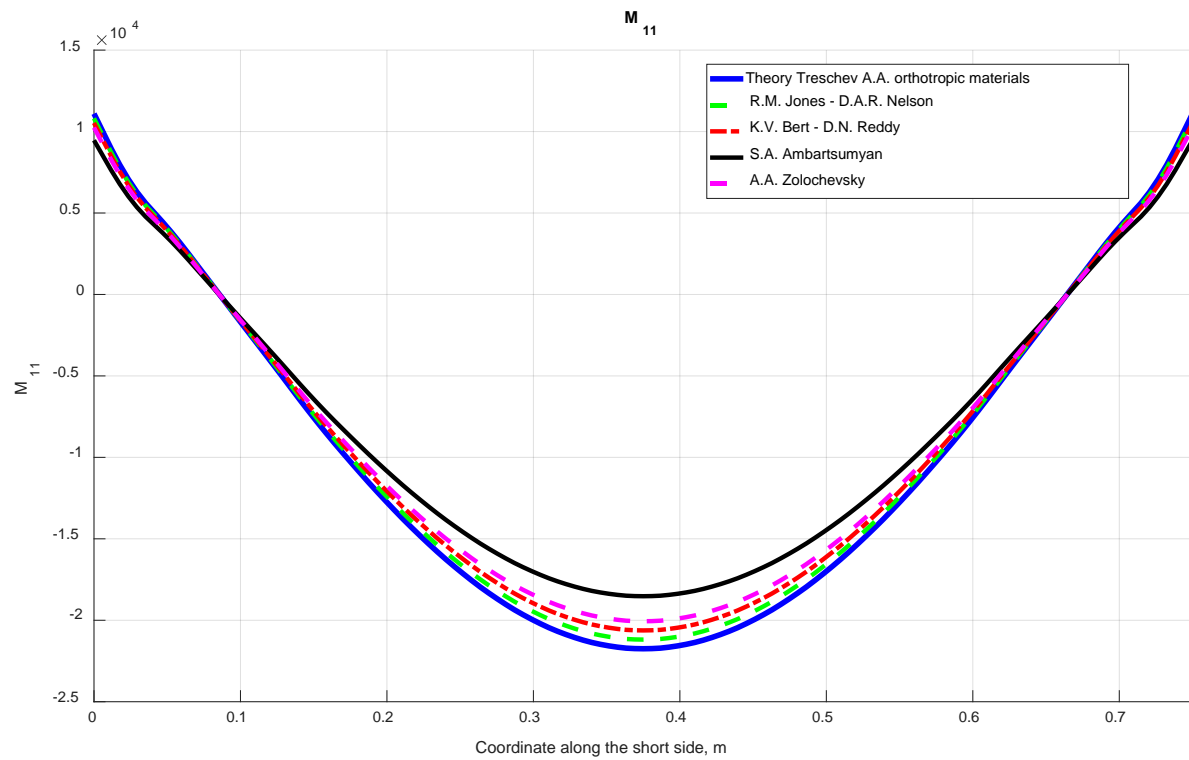
**Fig. 12.** Bending moment  $M_{22}$  along the  $X_1$  axis



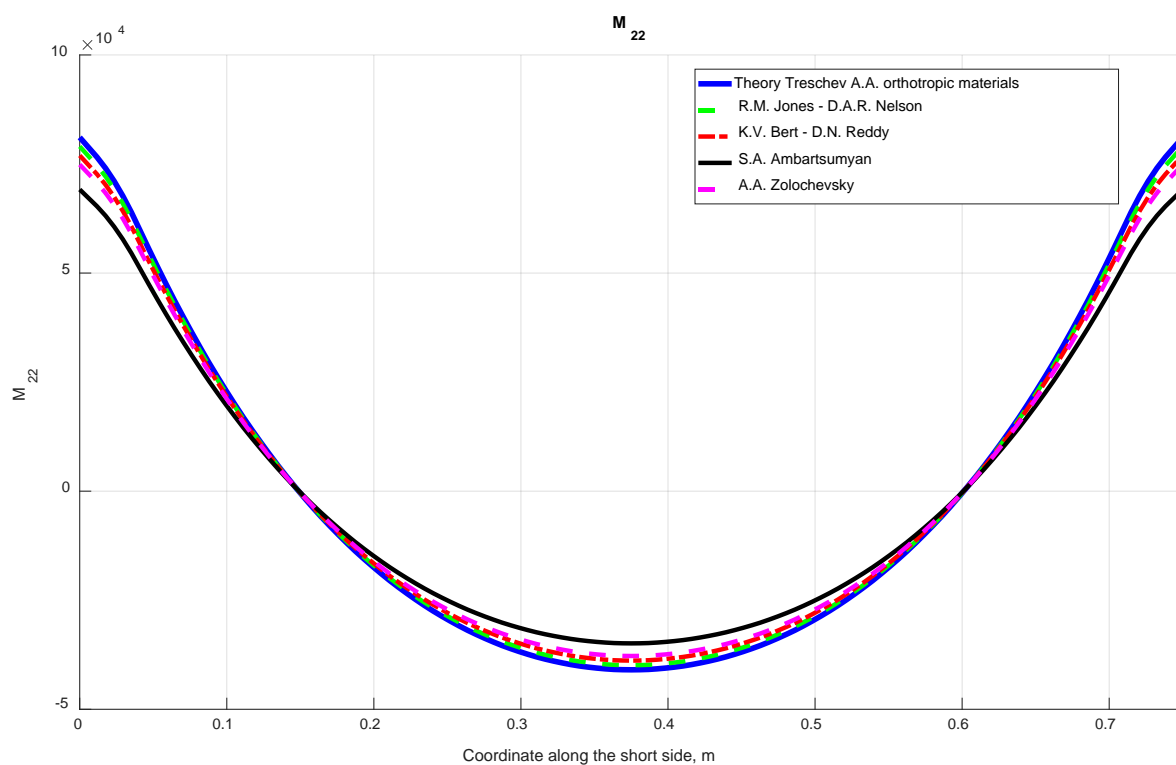
**Fig. 13.** Efforts  $N_{11}$  along the  $X_2$  axis



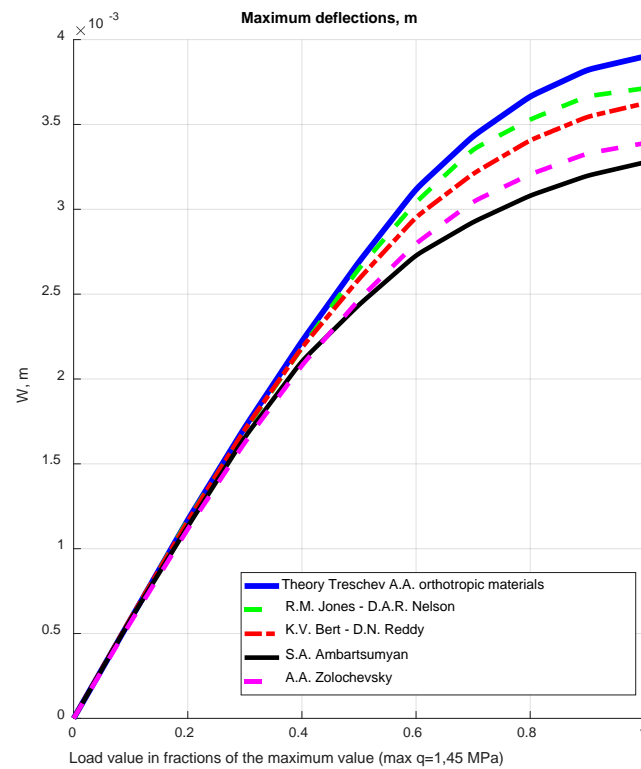
**Fig. 14.** Efforts  $N_{22}$  along the  $X_2$  axis



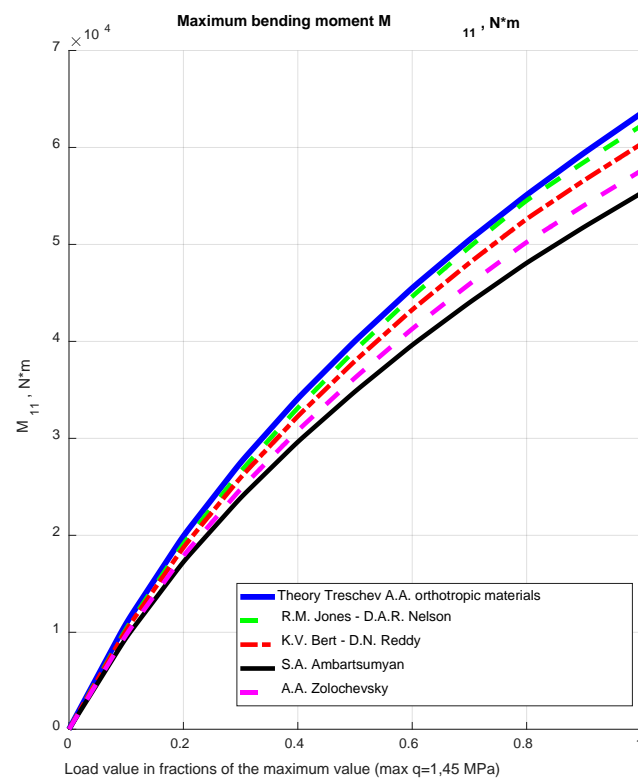
**Fig. 15.** Bending moment  $M_{11}$  along the  $X_2$  axis



**Fig. 16.** Bending moment  $M_{22}$  along the  $X_2$  axis

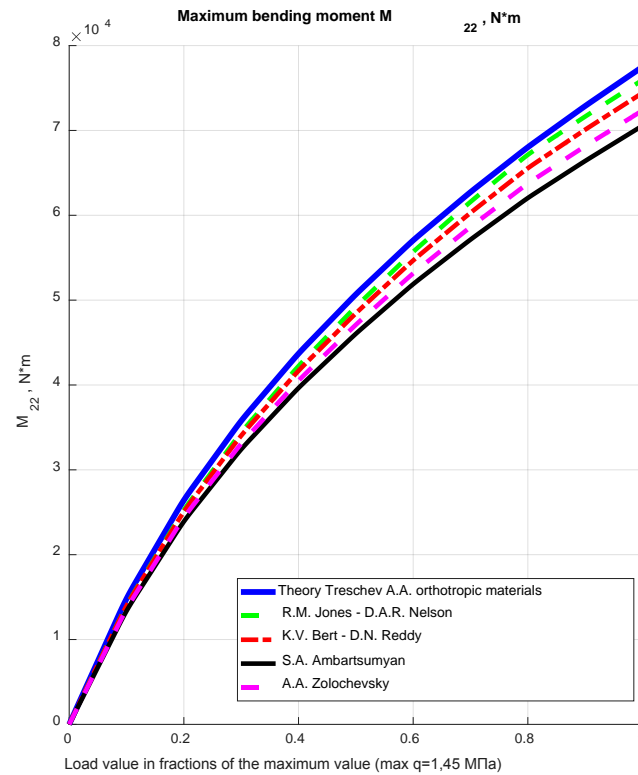


**Fig. 17.** Effect of load value on deflections



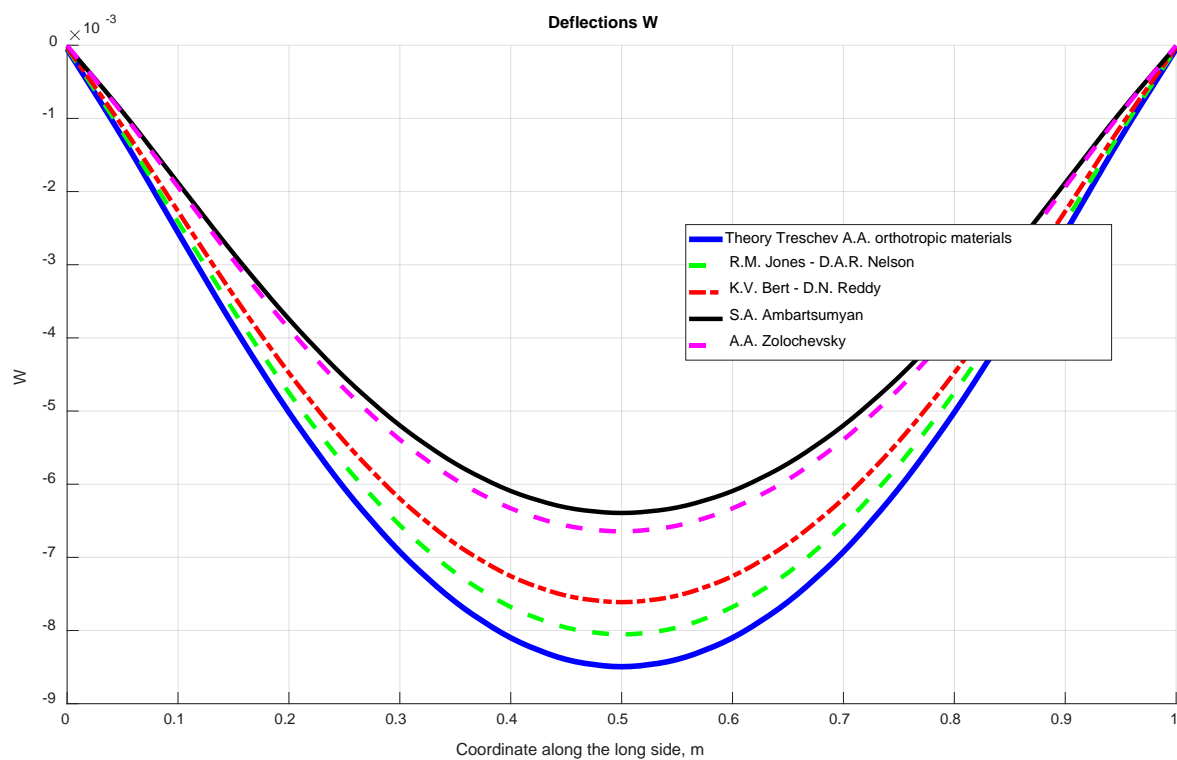
**Fig. 18.** Influence of the magnitude of the load on the maximum bending moment  $M_{11}$



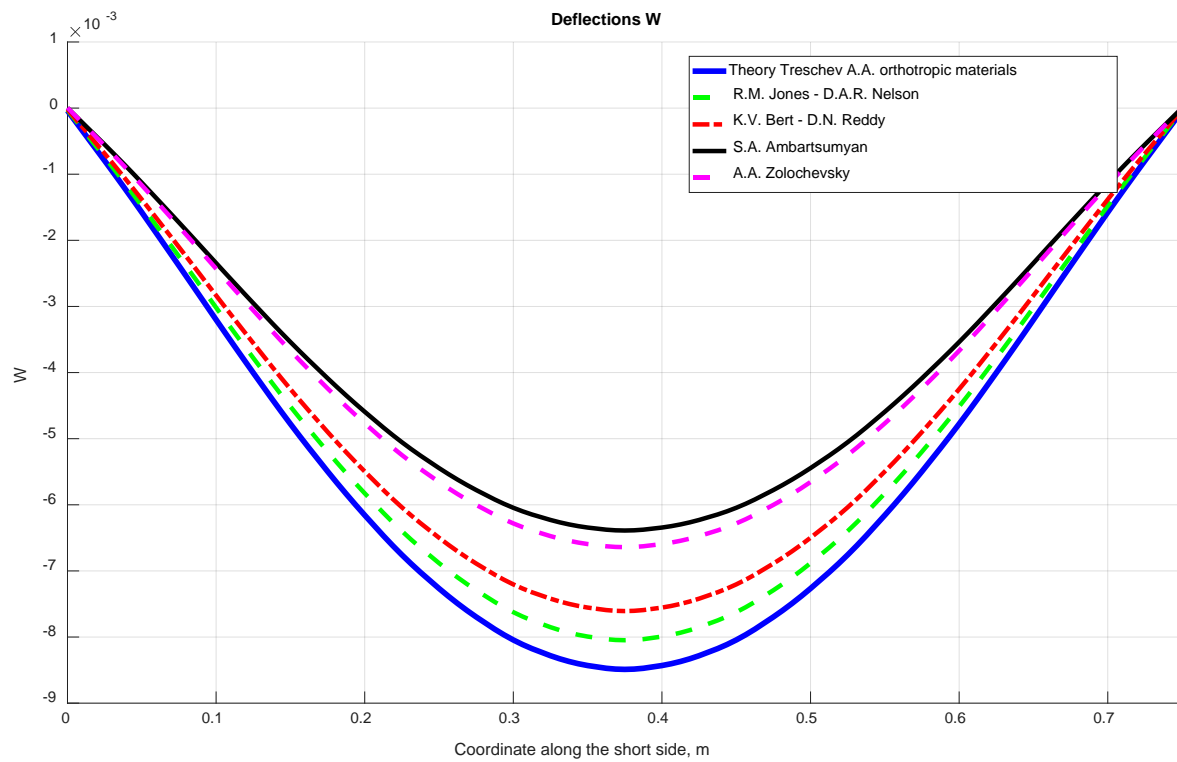


**Fig. 19.** Influence of the magnitude of the load on the maximum bending moment  $M_{22}$

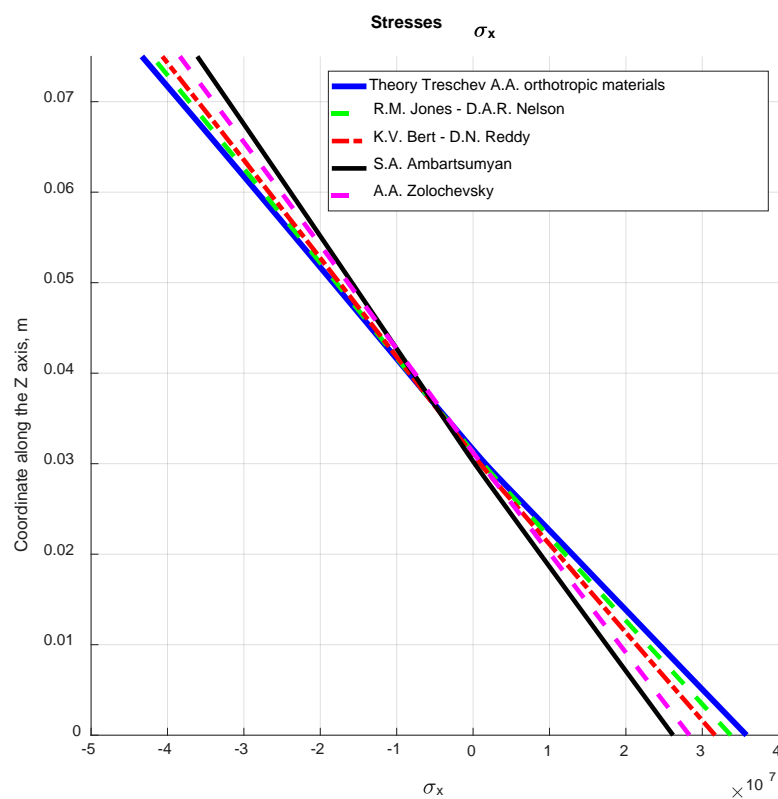
**Freely supported plate made of graphite AVCO Mod3a**



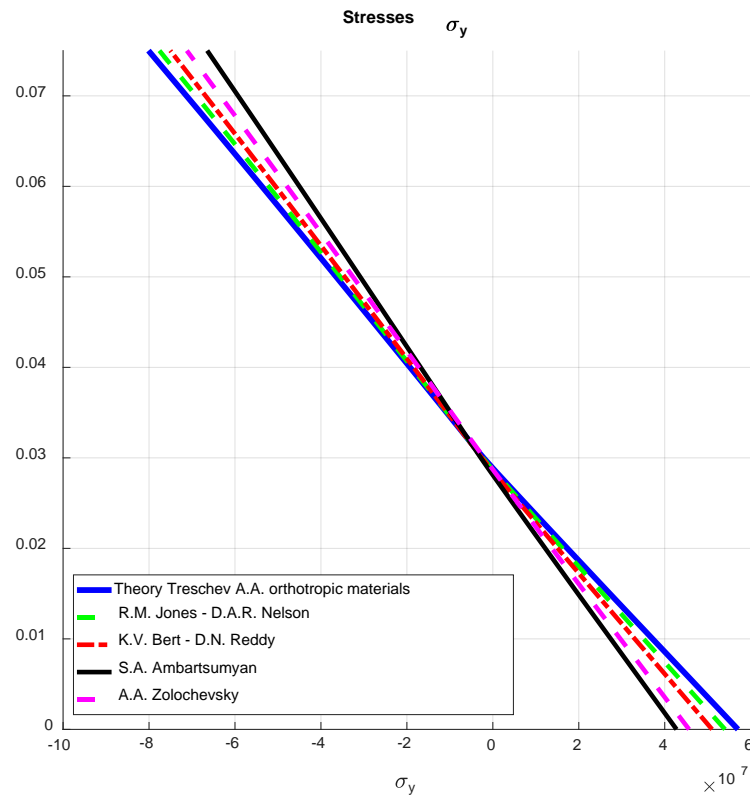
**Fig. 20.** Deflections  $W$  along the  $X_1$  axis



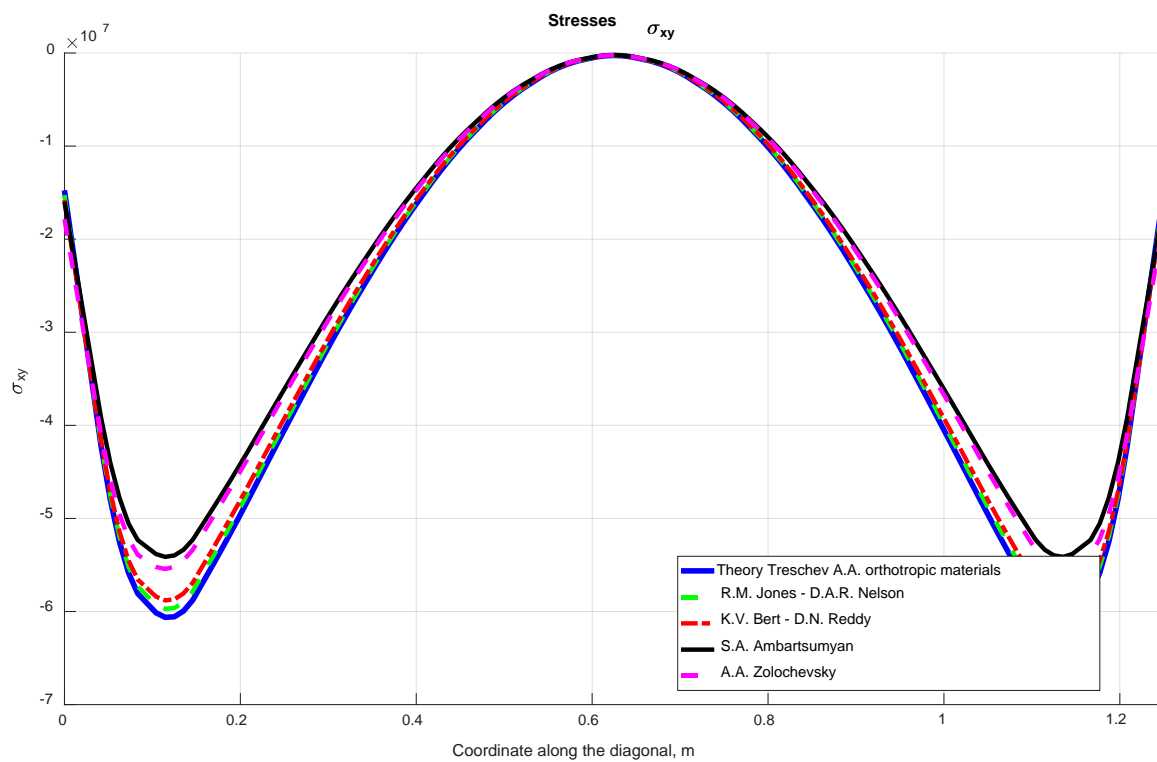
**Fig. 21.** Deflections W along the  $X_2$  axis



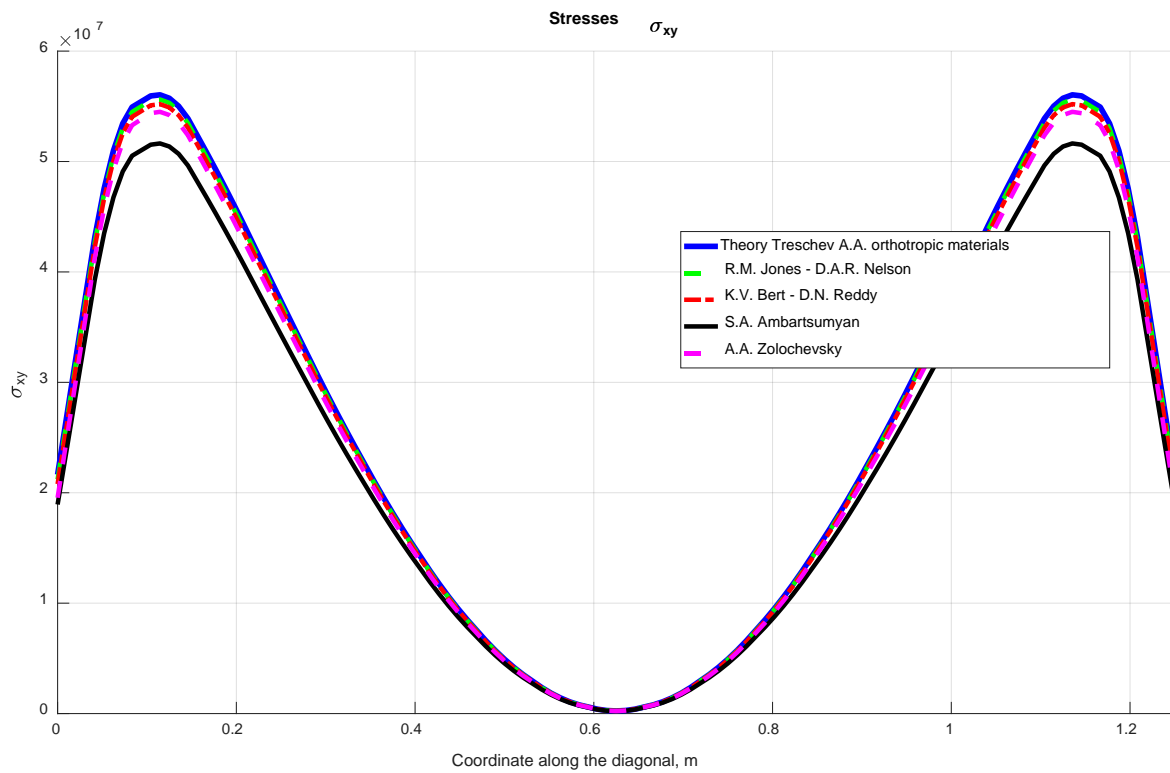
**Fig. 22.** Distribution of  $\sigma_x$  stresses over the plate thickness at point 0,5a 0,5b



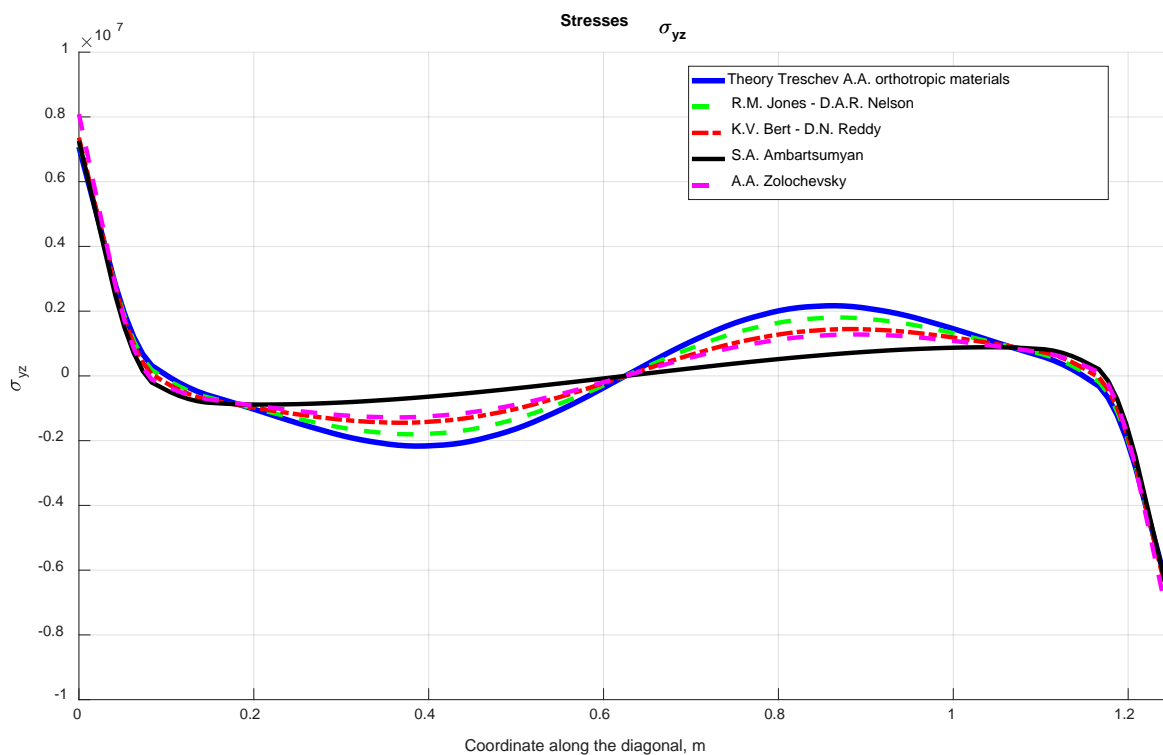
**Fig. 23.** Distribution of  $\sigma_y$  stresses over the plate thickness at point 0,5a 0,5b



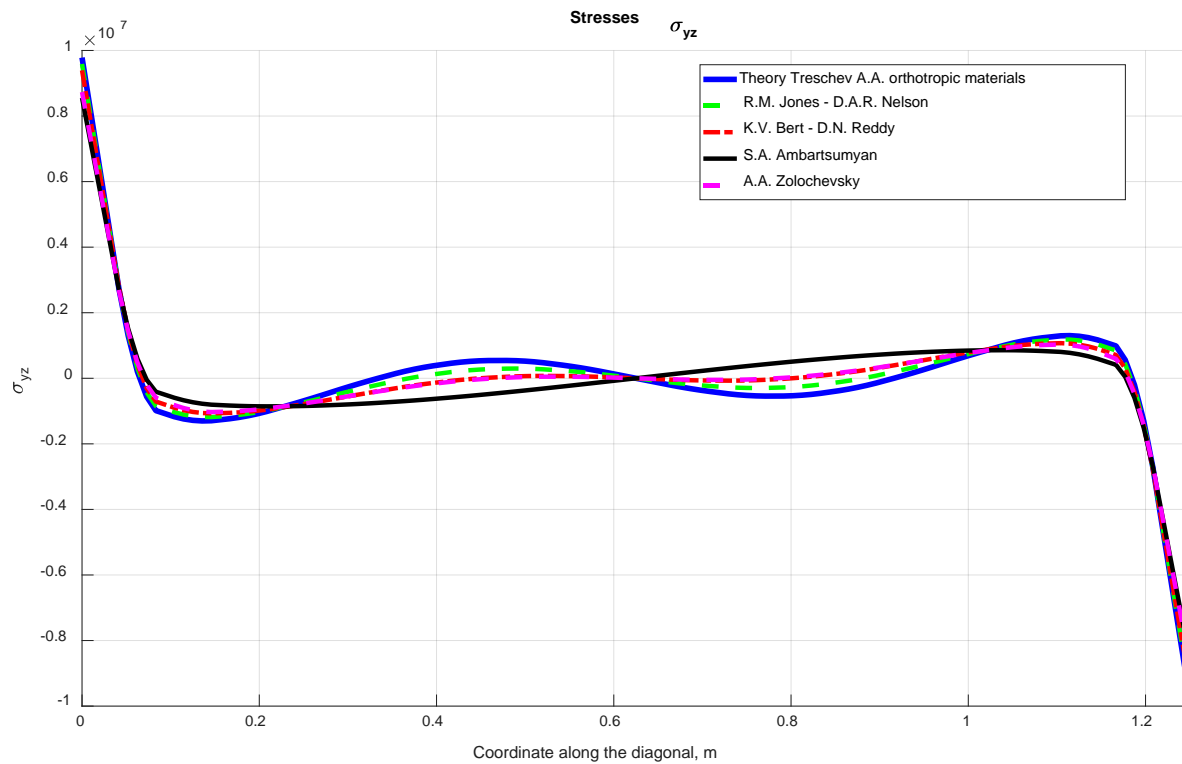
**Fig. 24.** Distribution of shear stresses  $\sigma_{xy}$  along the diagonal of platinum in the lower section



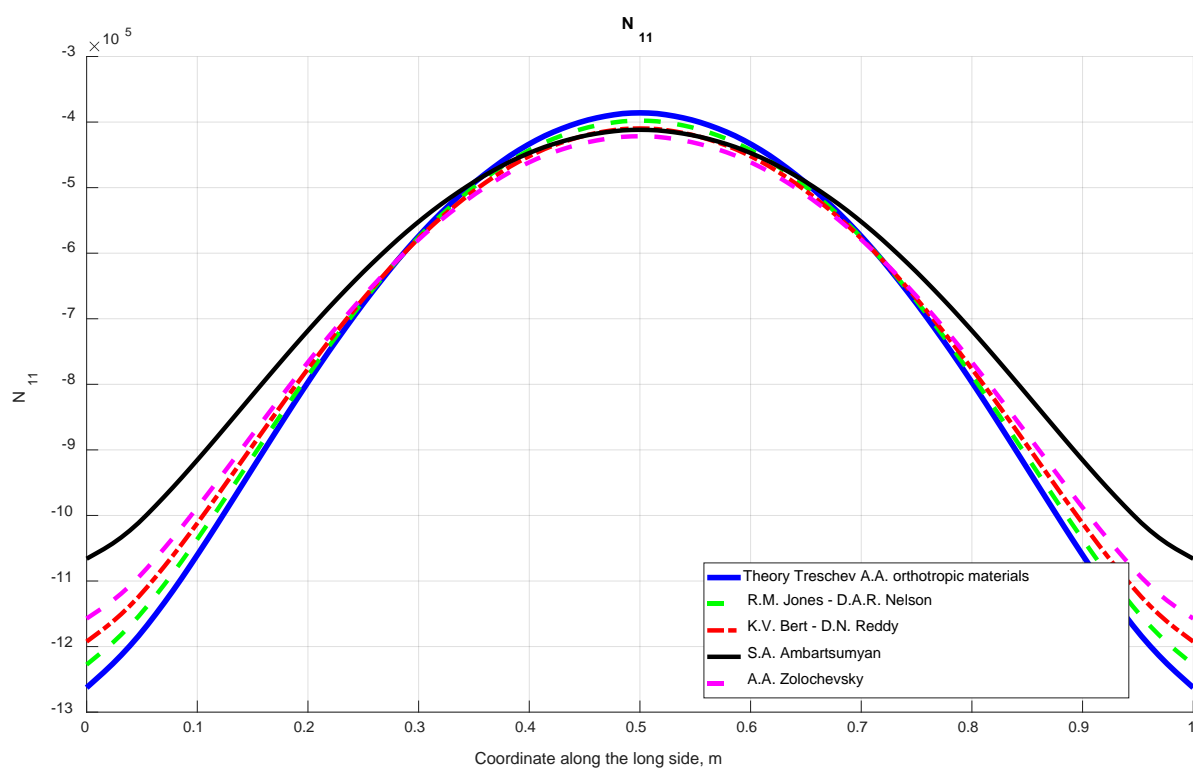
**Fig. 25.** Distribution of shear stresses  $\sigma_{xy}$  along the diagonal of platinum in the upper section



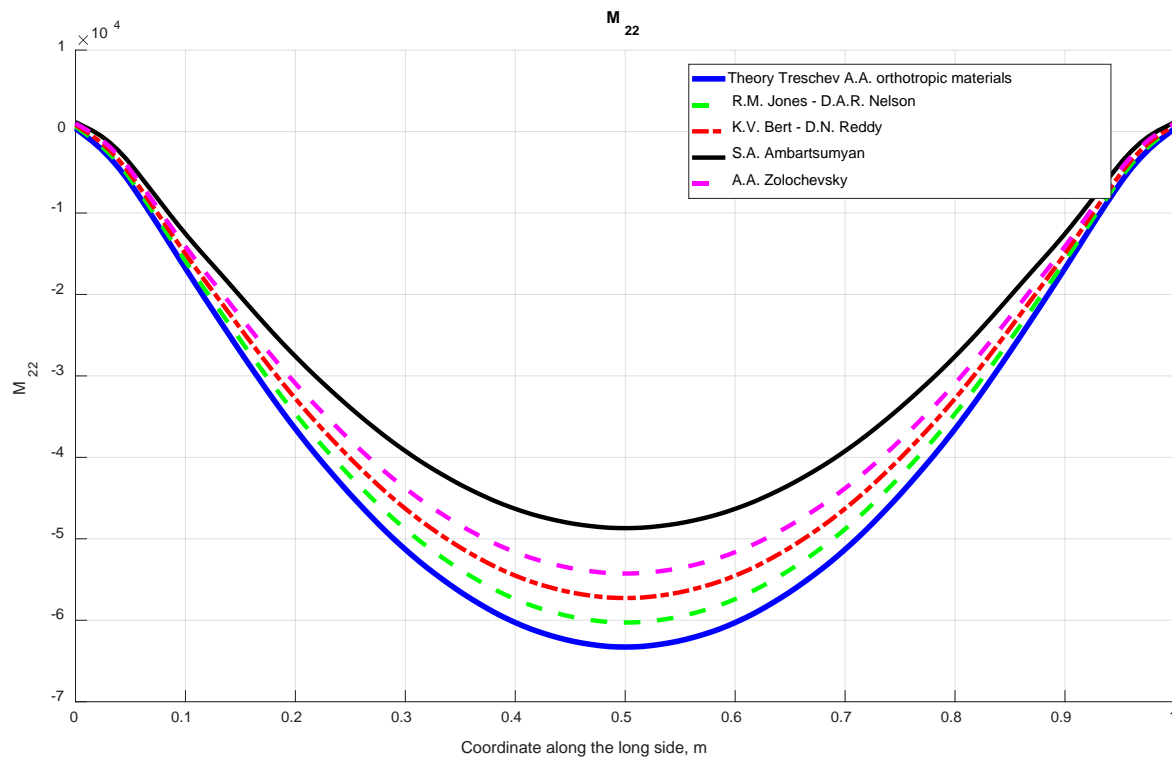
**Fig. 26.** Distribution of shear stresses  $\sigma_{yz}$  along the diagonal of the slab in the upper section



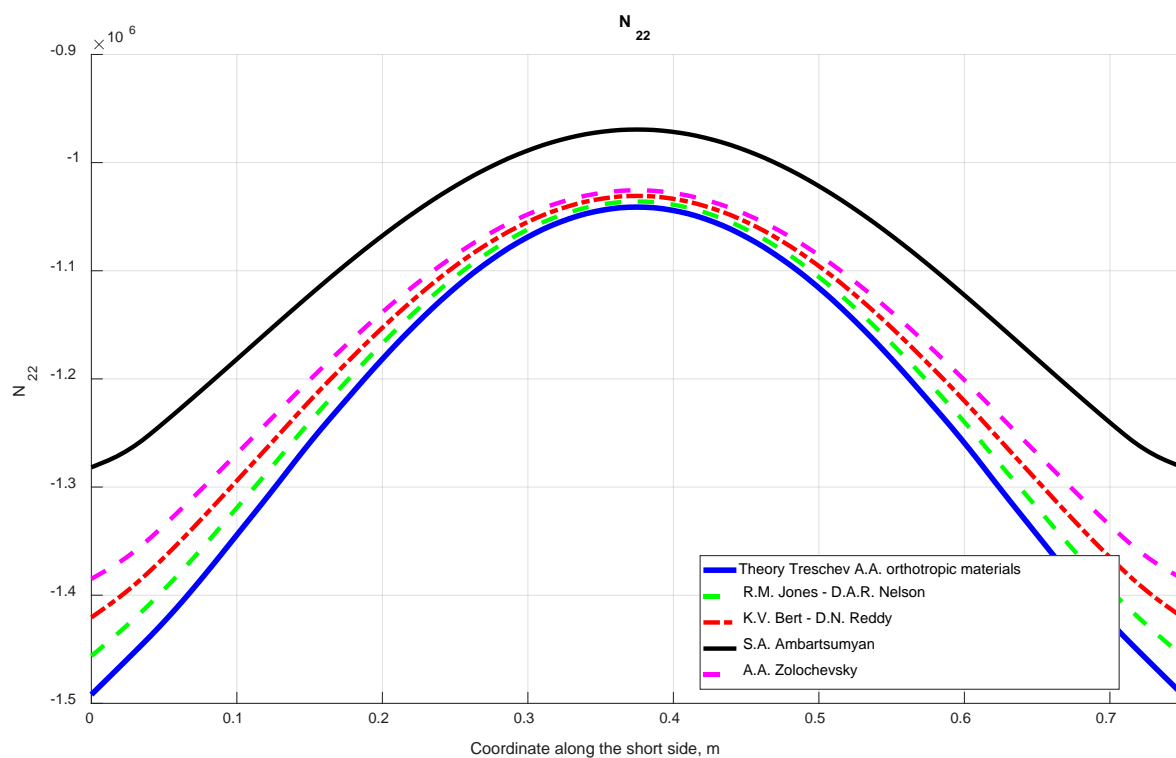
**Fig. 27.** Distribution of shear stresses  $\sigma_{yz}$  along the diagonal of the slab in the upper section



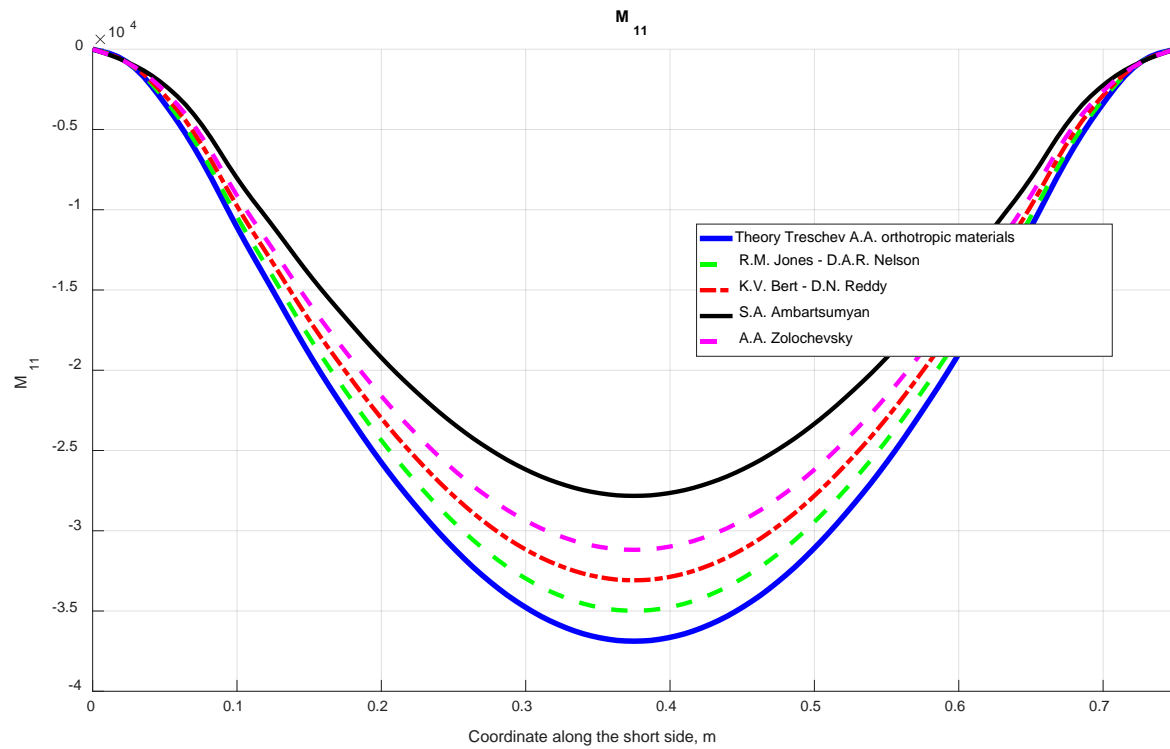
**Fig. 28.** Efforts  $N_{11}$  along the  $X_1$  axis



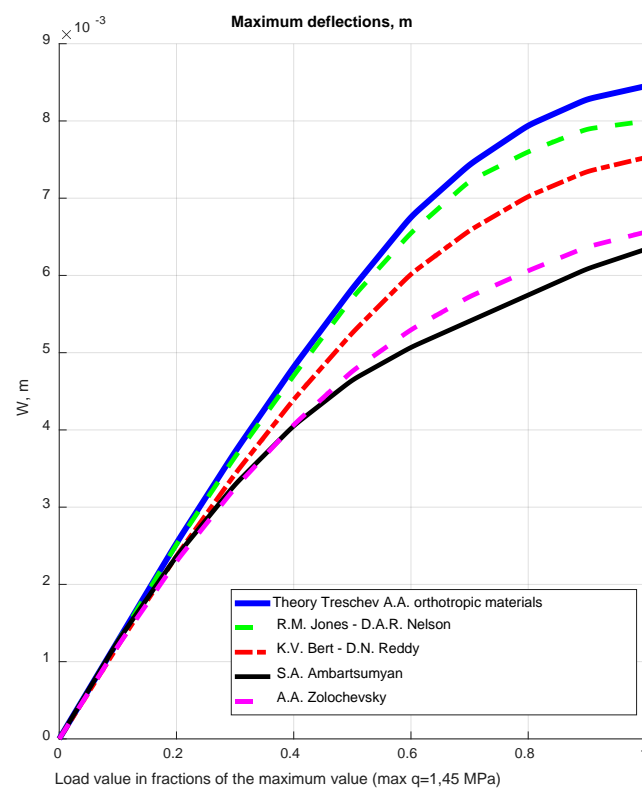
**Fig. 29.** Bending moment  $M_{22}$  along the  $X_1$  axis



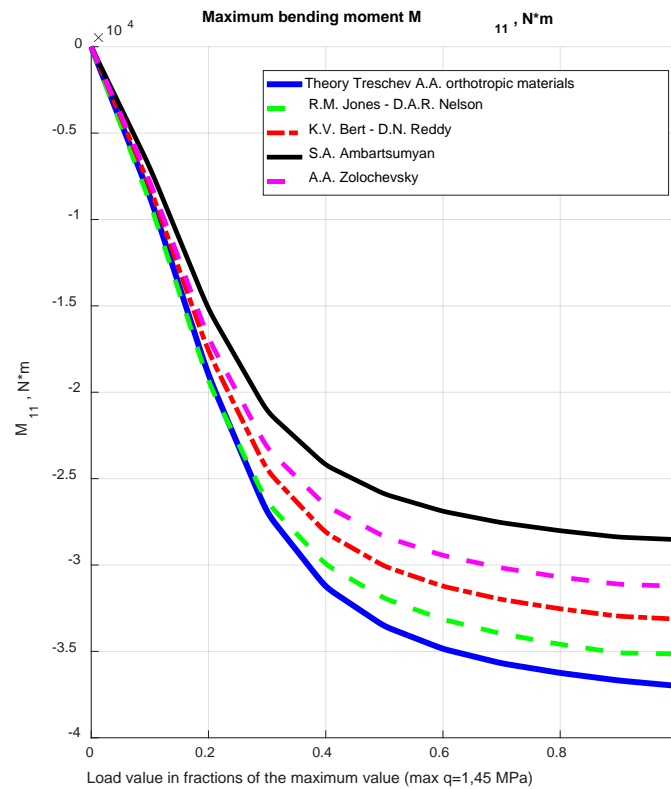
**Fig. 30.** Efforts  $N_{22}$  along the  $X_2$  axis



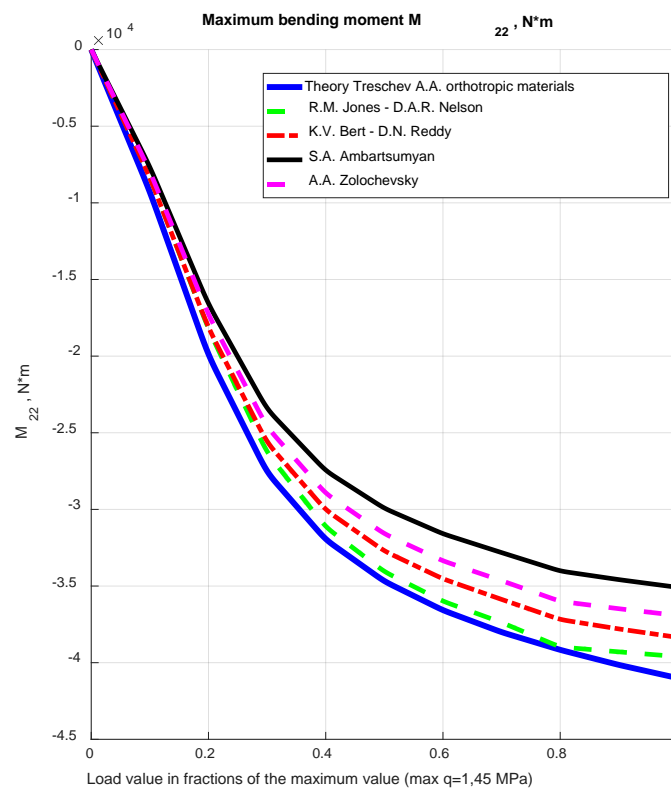
**Fig. 31.** Bending moment  $M_{11}$  along the  $X_2$  axis



**Fig. 32.** Effect of load value on deflections



**Fig. 33.** Influence of the magnitude of the load on the maximum bending moment  $M_{11}$



**Fig. 34.** Influence of the magnitude of the load on the maximum bending moment  $M_{22}$



#### 4. Summary

Analyzing the above graphical dependencies, it should be noted that the disregard for nonlinearity leads to a very significant error in the results. It should be said that when using materials in which the nonlinearity is more pronounced than that of the carbon fiber – carbon AVCO Mod 3a composite, and considering structures in large displacements and in a more complex stress-strain state, the stress-strain state will change even more significantly.

Based on the above, it can be concluded that this work is relevant and will serve as the first step towards a more accurate calculation of building structures within the framework of the proposed theory [32].

In a similar vein, the problem of axisymmetric transverse bending of an annular plate made of an orthotropic nonlinear material with different resistance was considered in [36]. During the deformation of the plate under consideration, there are no tangential stresses in the middle plane  $\sigma_{xy}$  and in the cross-section along the second coordinate  $\sigma_{yz}$ , and only tangential stresses in the cross-section along the radial coordinate  $\sigma_{rz}$  are taken into account. The presence of tangential stresses  $\sigma_{xy}$  and  $\sigma_{yz}$  (Figs. 5-8 and Figs. 24-27) significantly complicate the picture of the stress-strain state of rectangular plates due to the redistribution of stresses. Taking these stresses into account gives a more complete picture of the process of plate deformation during transverse bending, expanding the understanding of the picture of the stress-strain state of bent plates (especially the moment of the onset of the limiting state).

#### 5. Conclusions

The main results and conclusions of the work are as follows:

1. A number of theories of deformation of nonlinear anisotropic materials with different resistances are considered, it is concluded that the constitutive relations used by the authors describe the process of deformation of plates made of these materials most fully.
2. The governing equations for the bending of rectangular plates of average thickness made of an orthotropic physically nonlinear material sensitive to the type of stress state are obtained in a geometrically linear formulation.
3. For a comparative analysis of the calculations of the proposed variant of deformation, the calculation of a rectangular plate of average thickness was carried out. The analysis of the results is demonstrated by the example of a rectangular plate with characteristic dimensions of  $1.0 \times 0.5 \times 0.075$  (a**x**b) m.
4. An algorithm for solving the resulting resolving equations based on the method of variable parameters of elasticity and finite-difference approximation of the second order of accuracy has been developed. Developed special software which implements the algorithm using MATLAB computing system to calculate the stress-strain state of the plates.
5. The results of the calculation of the plates showed that taking into account the phenomenon of nonlinear differential resistance allows obtaining more precise results, in comparison with the "linear theory of elasticity" and theories: S.A. Ambartsumyan, A.A. Zolochovsky, R.M. Jones – D.A.R. Nelson, C.W. Bert – J.N. Reddy up to 26.8% for maximum displacements, up to 38% for maximum stresses, and in some cases the difference for force factors can reach 60%;
6. The analysis of the results obtained allows us to conclude that it is necessary to take into account the phenomenon of nonlinear resistance of the material when carrying out strength calculations, due to the fact that this phenomenon has a significant effect on the qualitative and quantitative characteristics of the stress-strain state of structures (in particular, for a rectangular plate of average thickness).

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## References

- [1] Ambartsumyan SA. *Theory of anisotropic plates: strength, stability, vibrations*. Moscow: Nauka; 1967. (In Russian)
- [2] Ambartsumyan SA. Basic equations and ratios of the multi-modular theory of elasticity of an anisotropic body. *Izv. Academy of Sciences of the USSR. MTT*. 1969;3: 51-61.
- [3] Ambartsumyan SA. *Multi-modular theory of elasticity*. Moscow: Nauka; 1982. (In Russian)
- [4] Ambartsumyan SA, Khachatryan AA. The basic equations of the theory of elasticity for materials of different resistance to tension and compression. *Inzh. journals MTT*. 1966;2: 44-53.
- [5] Ambartsumyan SA, Khachatryan AA. Towards a multimodular theory of elasticity. *Ing. journals MTT*. 1966;6: 64-67.
- [6] Jones RM. A Nonsystemmetric Compliance Matrix Approach to Nonlinear Multimodulus Orthotropic Materials. *AIAA Journal*. 1977;15(10): 1436-1443.
- [7] Jones RM, Nelson DAR. Material for nonlinear Deformation. *AIAA Journal*. 1976;14(6): 709-716.
- [8] Jones RM. Modeling Nonlinear Deformation of the Carbon-Carbon Composite Materials. *AIAA Journal*. 1980;18(8): 995-1001.
- [9] Jones RM. Stress-Strain Relations for Materials with Moduli in Tension and Compression. *AIAA Journal*. 1977;15(1): 16-25.
- [10] Bert CW. Models for fibrous composites with different properties in tension and compression. *J. Eng. Matls. and Tech., Trans. ASME*. 1977;99HD(4): 344-349.
- [11] Bert CW. Micromechanics of the different elastic behavior of filamentary composites in tension and compression. In: Bert CW. (ed.) *Mechanics of Bimodulus Materials*. 1979. p.17-28.
- [12] Bert CW, Gordaninejad F. Multi-modular Materials. *International Journal for Numerical Methods in Engineering*. 1984;20: 479-503.
- [13] Bert CW, Reddy JN, Chao WC. Bending of Thick Rectangular Plates Laminated of Bimodulus Composite Materials. *AIAA Journal*. 1981;19(10): 1342-1349.
- [14] Zolochovsky AA. To tensor coupling in the theories of elasticity and plasticity of anisotropic composite materials that are differently opposed to tension and compression. *Mechanics of Composite Materials*. 1985;1: 53-58.
- [15] Zolochovsky AA. Determining equations and some problems of the multimodular theory of elasticity of anisotropic materials. *PMTF*. 1985;4: 131-138.
- [16] Zolochovsky AA. On the theory of plasticity of materials differently resisting tension and compression. *Izv. universities. Engineering*; 1986;6: 13-16.
- [17] Zolochovsky AA. On the relations of the theory of elasticity of anisotropic multi-modular materials. *Dynamics and durability of machines*. 1981;34: 3-8.
- [18] Zolochovsky AA. Ratios of the multimodular theory of elasticity of anisotropic materials based on three mixed invariants. *Dynamics and Strength of Machines*. 1987;46: 85-89.
- [19] Zolochovsky AA, Morachkovsky OK. The directions of development of models and methods for calculating the nonlinear deformation of bodies and elements of engineering structures. *Dynamics and strength of machines*. 1989;50: 3-9.
- [20] Zolochovsky AA, Sklepus SN. On the theory of plasticity with three invariants of the stress state. *Izv. universities. Engineering*. 1987;5: 7-10.
- [21] Zolochovsky AA. Concerning consideration of the multiresistance in the theory of creep of isotropic and anisotropic materials. *PMTF*. 1982;4: 140-144.
- [22] Lomakin EV. Multi-modularity of composite materials. *Mechanics of composite materials*. 1981;1: 23-29.

- [23] Lomakin EV. Relations of the theory of elasticity for an anisotropic body, the deformation characteristics of which depend on the type of stress state. *Izv. Academy of Sciences of the USSR. MTT*. 1983;3: 63-69.
- [24] Berezin IS, Zhidkov NP. *Computing methods vol. 1*. Moscow: State publishing house physical. lit-ry; 1959. (In Russian)
- [25] Treschev AA. *The theory of deformation and strength of materials sensitive to the type of stress state. Defining relations*. Moscow: RAASN; 2008. (In Russian)
- [26] Matchenko NM, Treschev AA. *Theory of deformation of materials with different resistance. Determining relations*. Tula: TSU; 2000. (In Russian)
- [27] Treschev AA. To bending of thick rectangular orthotropic slabs made of nonlinear materials sensitive to the type of stress state. In: *Proceedings of the 7th International Conference «Actual Problems of Construction and Building Industry»*. Tula: TulSU; 2006. (In Russian)
- [28] Treschev AA. *The theory of deformation and strength of materials with initial and induced sensitivity to the type of stress state. Defining relations: monograph*. Moscow: RAASN; 2016. (In Russian)
- [29] Treschev AA. Description of deformation of nonlinear anisotropic materials. *Architectural and construction materials science at the turn of the century: materials of reports of the International conference*. Belgorod: BelGTASM; 2002. (In Russian)
- [30] Treschev AA. Description of nonlinear deformation of anisotropic materials *Problems and achievements of construction materials science*. Belgorod: BSTU; 2005. p. 233-234. (In Russian)
- [31] Treschev AA. *Anisotropic plates and shells of materials with different resistance: monograph*. Moscow: RAACS; 2007. (In Russian)
- [32] Treschev AA, Romashin DA. Defining relations for nonlinear anisotropic materials sensitive to the type of stress state. *Bulletin of Nizhny Novgorod University. N.I. Lobachevsky*. 2011;4(4): 1740-1742.
- [33] Birger IA, Mavlyutov RR. *Resistance materials: Tutorial*. Moscow: Science. Ch. ed. Phys. Mat. lit.; 1986. (In Russian)
- [34] Pisarenko GS, Mozharovsky NS. *Equations and boundary value problems of the theory of plasticity and creep. Reference book*. Kiev: Science; 1981. (In Russian)
- [35] Jones RM, Nelson DAR. Theoretical-experimental correlation of material models for nonlinear deformation of graphite. *AIAA Journal*. 1976;14(10): 1427-1435.
- [36] Zhurin EA, Treschev AA. Bending of ring plates, performed from an orthotropic nonlinear differently resistant material. *International Journal for Computational Civil and Structural Engineering*. 2020;16(1): 130-146.