Materials Physics and Mechanics 46 (2020) 168-181 Received: March 11, 2020

NUCLEAR GEOMETRY: SODIUM, MAGNESIUM, ALUMINUM

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Abstract. The nuclear geometry has been developed by analogy with the fullerene geometry. On the basis of this geometric approach, it was possible to design the structure of sodium, magnesium and aluminum isomers and their isotopes, which can be obtained by means of nuclear synthesis. The most stable nuclei can be classed into two groups: basic nuclei having equal number of protons and neutrons and isotopes having one or two more neutrons. The latter ensure their mechanical stability with respect to shear stresses, sending their electron to the coat of mail created by the basic nuclei.

Keywords: aluminum, graph representation, isomer, isotope, magnesium, nuclear electron, nuclear geometry, nuclear reaction, sodium

1. Introduction

Earlier, by analogy with fullerenes, the nuclear geometry has been designed The geometric models of nuclei, developed by analogy with the fullerene geometry, allow explain why the nuclei have a definite number of stable isotopes and isotopes having a large half-decay period. Contrary to the usual "arithmetic approach", when the nuclear reactions are written down simply as in chemistry, the geometric approach was used when the reactions are considered, if the reacting nuclei are compatible from the geometric standpoint.

In this contribution I expand the geometric approach which explains not only the generation of sodium, magnesium and aluminum but also that of their isotopes and isomers in the framework of one and the same unified modeling. It should be emphasized again that I use, instead of the vague notion "nuclear isomerism" [1], the clear notion accepted for molecules, i.e. I accept that space isomerism of nuclei is the phenomenon which consists in the existence of nuclei having an equal mass number but different positions of the nuclear constituents in the space.

2. Isomers of sodium and their isotopes

There is only one stable isotope of sodium, $_{11}Na^{23}$ (100 %), and an unstable isotope having a comparatively large half-decay period being equal to 2.602 y, $_{11}Na^{22}$ [2]. Previously it was suggested that nuclei can be separated into two main types: basic nuclei having equal number of protons and neutrons and isotopes having one or two more neutrons. First consider simpler basic nucleus, $_{11}Na^{22}$ that can be obtained by two ways:

through the use of reaction $d + {}_{10}Ne^{20} \rightarrow {}_{11}Na^{22}$, and

by means of two-stage reaction ${}^{12}C + {}^{4}He \rightarrow {}^{16}O$, ${}^{16}C + {}^{6}Li \rightarrow {}^{22}Na$.

Consider the first reaction more closely.

2.1. Joining a dimer to a (tetra-hexa)$_3$-penta$_6$ dodecahedron. The reaction is illustrated in Figs. 1, 2 and 3. Here a deuteron is incorporated into a basic nucleus of neon having three-fold symmetry. From Fig. 1 it follows that for neon only four protons from twenty take part

http://dx.doi.org/10.18149/MPM.4612020_16

really in the reaction. They are specially marked in the figure; the protons are pinked, the new proton-protons bonds are lilac, the old bonds, which were destroyed, are shown using red dot lines. Here tertions are omitted.

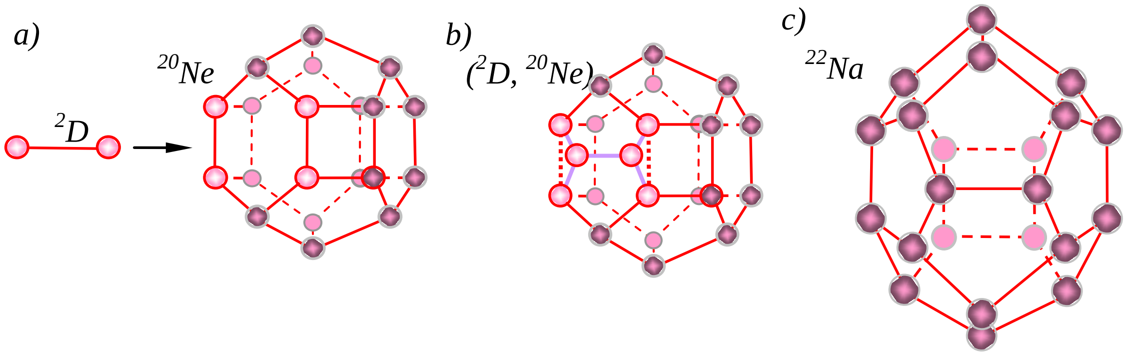

Fig. 1. Attachment of deuteron to neon: *a*) separate particles; *b*) intermediate compound; *c*) sodium ^{22}Na after relaxation

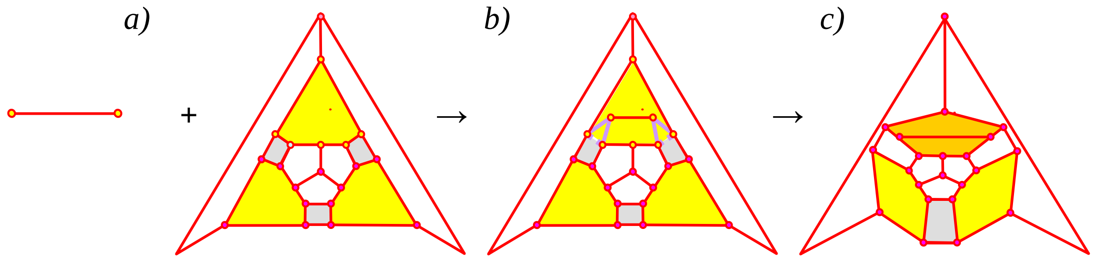

Fig. 2. Graph representation of the nuclear reaction $d + {}_{10}Ne^{20} \rightarrow {}_{11}Na^{22}$. Embedding the graph of deuteron into the graph of neon: *a*) separate graphs corresponding to a dimer (at the left) and to a tetra$_3$-penta$_6$-hexa$_3$ dodecahedron (at the right); *b*) embedding, *c*) graph of the tetra-penta$_{10}$-hexa$_2$ triacaidecahedron shown in Fig. 1c. All notations are the same as before

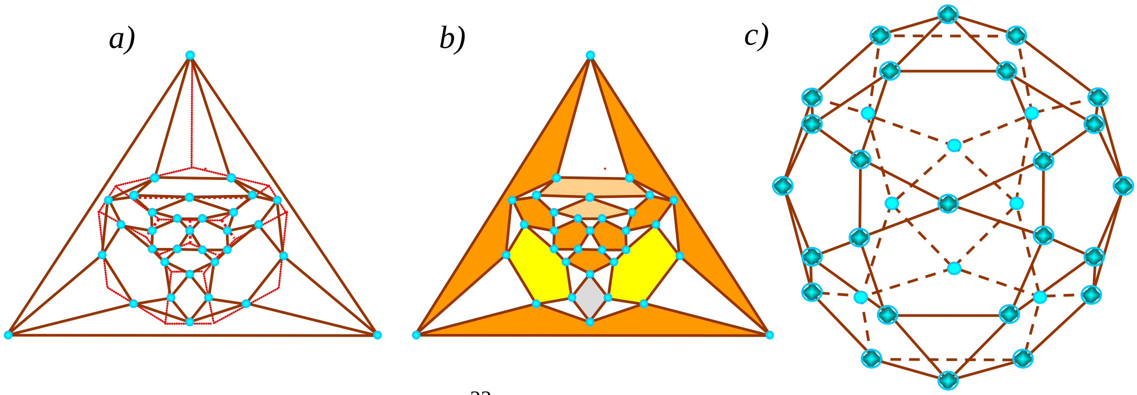

Fig. 3. Electronic structure of sodium ^{22}Na: *a*) graph of tertion net being constructed on the base of the proton-cell graph (red dot lines; *b*) separate graph of tertion net; *c*) tertion net

2.2. Two-stage reaction. Now consider the two-stage reaction which is written above as $^{12}C + {}^{4}He \rightarrow {}^{16}O$, $^{16}O + {}^{6}Li \rightarrow {}^{22}Na$. The first stage is illustrated in Fig. 4 and consists in joining a tetrahedron with a hexagonal prism. Earlier this reaction was already analyzed as the fist stage for obtaining neon having the shape of tetra$_3$-penta$_6$-hexa$_3$ dodecahedron. The second stage is shown in Fig. 5. It consists in joining a triangular prism to a cupola of three-fold symmetry which was formed at the fist stage. The graph representation of the two-stage reaction is presented in Fig. 6.

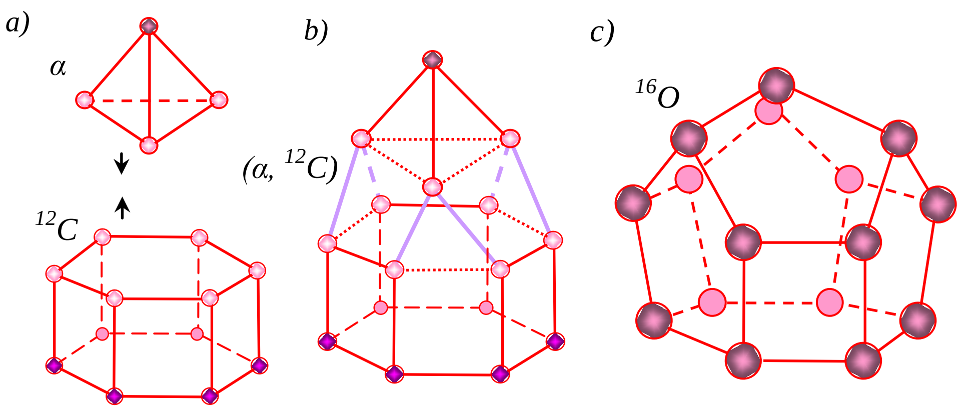

Fig. 4. Joining a tetrahedron (α-particle) to a hexagonal prism (^{12}C): *a*) separate tetrahedron and hexagonal prism; proton bonds (red lines), reacting protons (light pink spheres), neutral atoms (dark pink spheres); *b*) intermediate compound: old bonds to be destroyed (red dot lines), new bonds (lilac lines); *c*) cupola of three-fold symmetry (^{16}O)

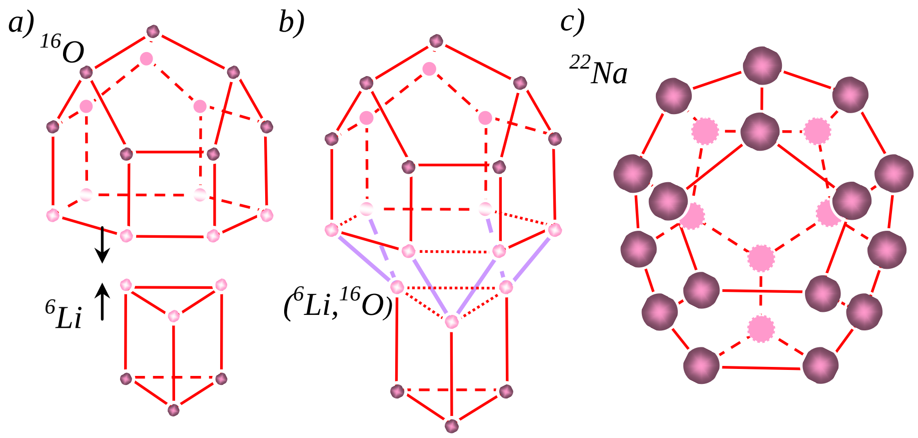

Fig. 5. Joining a triangular prism to a cupola: *a*) separate prism (^{6}Li) and cupola (^{16}O), *b*) intermediate compound, *c*) tri-penta$_9$-hexa$_3$ triacaidecahedron (^{22}Na)

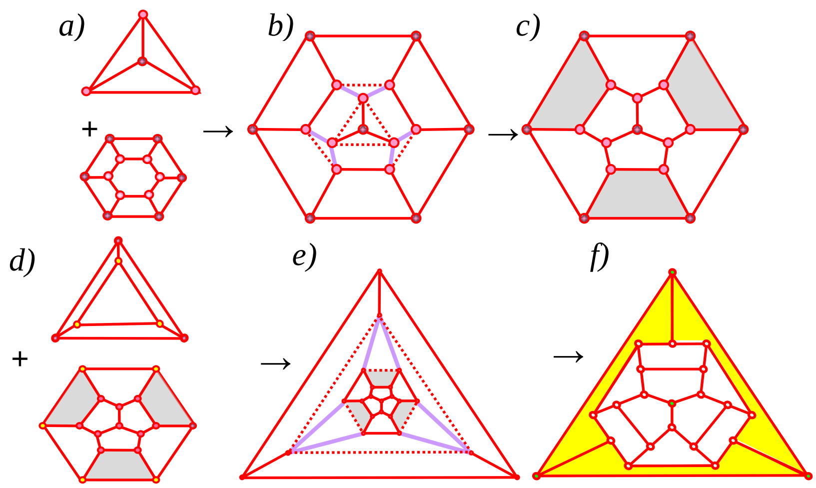

Fig. 6. Graph representation of the two-stage nuclear reaction $^{12}C + {}^{4}He \rightarrow {}^{16}O + {}^{6}Li \rightarrow {}^{22}Na$.
a) separate graphs corresponding to a tetrahedron (above) and to a hexagonal prism (down); b) embedding the graph of helium into the graph of carbon; c) graph of cupola oxygen; d) graphs corresponding to a triangular prism (above) and to a hexagonal cupola (below); e) embedding the graph of oxygen into the graph of lithium; f) graph of sodium

One additional remark is necessary. The final graph shown in Figure 6f reflects the three-fold symmetry of the sodium shape obtained. However we can consider this structure from another point of view. As noted above, the nuclei can be separated into two types: basic nuclei having equal number of protons and neutrons and isotopes having one or two more neutrons. Let's extend the classification. Among the basic nuclei we will recognize two subgroups: ideal (perfect) nuclei and imperfect ones. Similar to crystals, the perfect nuclei are highly symmetric. The imperfect nuclei have lost high symmetry and the loss is connected with structural defects. By analogy with crystals, one can consider the structure shown in Fig. 5f as a perfect classical dodecahedron by Plato which was spoiled by adding a defect in the form of an extra interstitial dimer. Following such approach developed for fullerenes [3] one defines such nuclei as having topological symmetry. The situation is illustrated in Fig. 7, where two graphs, corresponding with the shape shown in Fig. 5c, are presented. The graph at the left reflects the symmetry induced by an extra dimer, whereas the graph at the right emphasizes the topological symmetry.

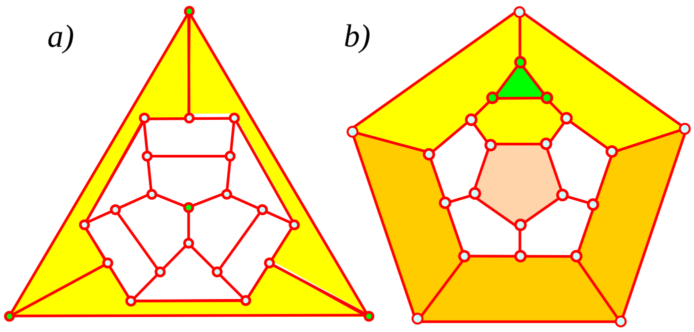

Fig. 7. Two graphs of one and the same structure shown in Fig. 5c, reflecting different sides of symmetry: *a*) ordinary symmetry; *b*) topological symmetry

To gain a more penetrating insight into the electronic structure of imperfect nuclei, it is better to use topological symmetry. The tertion net and its graph, which are presented in Fig. 8, are designed through the use of topological symmetry.

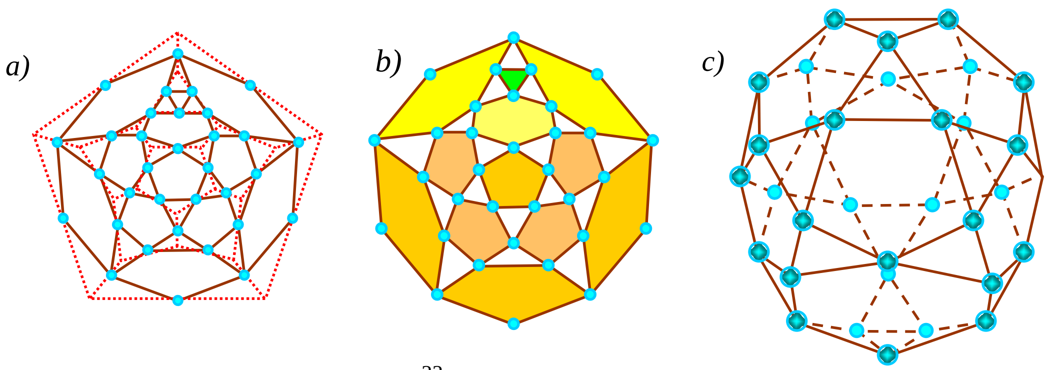

Fig. 8. Electronic structure of sodium ^{22}Na: *a*) graph of tertion net being constructed on the base of the proton-cell graph (red dot lines); *b*) separate graph of tertion net; *c*) tertion net

2.3. Stability due to neutron embedding. Let's replace in the previous reactions neon 20 having the shape of a (tetra-hexa)$_3$-penta$_6$ dodecahedron by neon 21 (the same dodecahedron but body centered). Then there arises the isotope of sodium (^{23}Na) shown in Fig. 9. If to replace in the previous reactions carbon 12 (hexagonal prism) by carbon 13 (body centered hexagonal prism), there appears the isotope of sodium 23 shown in Fig. 10. It should be noted that in the first case additional tertions have the charge of ½ *e*, in the second case of ⅓ *e*. The reasons are connected with the number of hexangular facets which incorporate the electrons of internal neutrons. So there leaves room for two isomers of sodium isotope, ^{23}Na.

Why the basic nuclei are unstable and the isotopes are stable? One can submit the following explanation. It is clear that any nucleus structure (proton cell) must be stable with respect to mechanical stresses which appear due to thermal vibrations of protons. It is assumed that the stability is insured by the coat of mail that dictates geometry of a proton cell. On the basis of the previous experience, one can envision that the coat of mail (tertion net), which ensures such stability, doesn't contain hexagons. The coat of mail of both basic isomers has the hexagons (Figs. 3 and 8). Embedding extra neutron eliminates the hexagons and does the coat of mail denser (Fig. 9 and 10). In its turn it leads to increasing stability and transforming the basic nuclei into the corresponding isotopes.

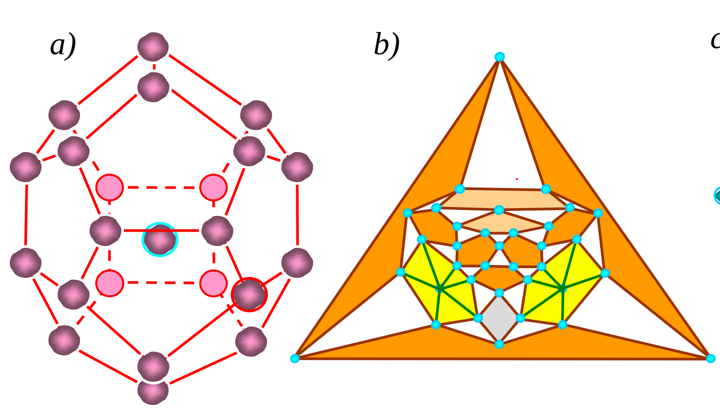

Fig. 9. Structure of isotope ^{23}Na: a) proton cell; b) graph of the tertion net; c) tertion net

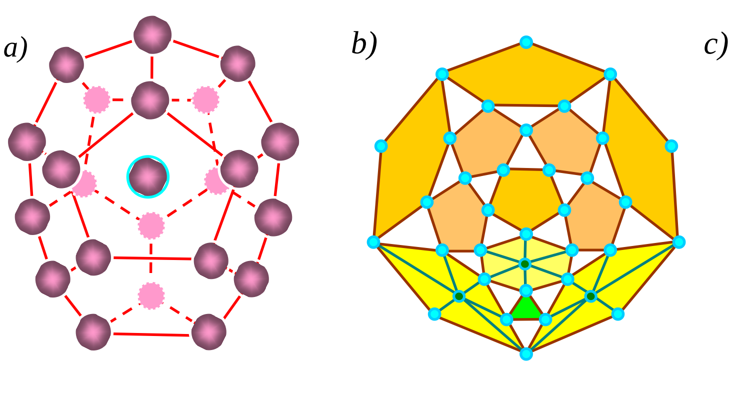

Fig. 10. Structure of another isotope ^{23}Na: a) proton cell; b) tertion-net graph; c) tertion net

3. Isomers of magnesium and their isotopes

There are three stable isotopes of magnesium: $_{12}$Mg^{24} (78.99 %), $_{12}$Mg^{25} (10.00 %) and $_{12}$Mg^{26} (11.01 %); beside there is an unstable isotope $_{12}$Mg^{28}, which has a reasonably small half-decay period being equal to 21.07 hours [2]. The crucial question is again how to obtain them and their space isomers in the framework of one and the same assumptions. Previously it was suggested that the nuclei can be separated into two main types: the basic nuclei having equal number of protons and neutrons and the isotopes having one or two more neutrons. A better understanding can be gained if to begin with the basic nucleus, $_{12}$Mg^{24} that can be obtained by various ways through the use of the most probable geometrically compatible reactions:

$^{12}C + {}^{12}C \rightarrow {}^{24}Mg, \quad {}^{8}Be + {}^{16}O + \rightarrow {}^{24}Mg.$

3.1. Joining two hexagonal prisms. The reaction is illustrated in Figs. 11, 12 and 13. Here both configurations have six-fold symmetry. From Fig. 11 it follows that for each carbon only half protons take part in the reaction. They are specially marked in the figure; the protons are pinked, the new proton-protons bonds are lilac, the old bonds, which were destroyed, are shown using dotted lines. Here tertions are also omitted.

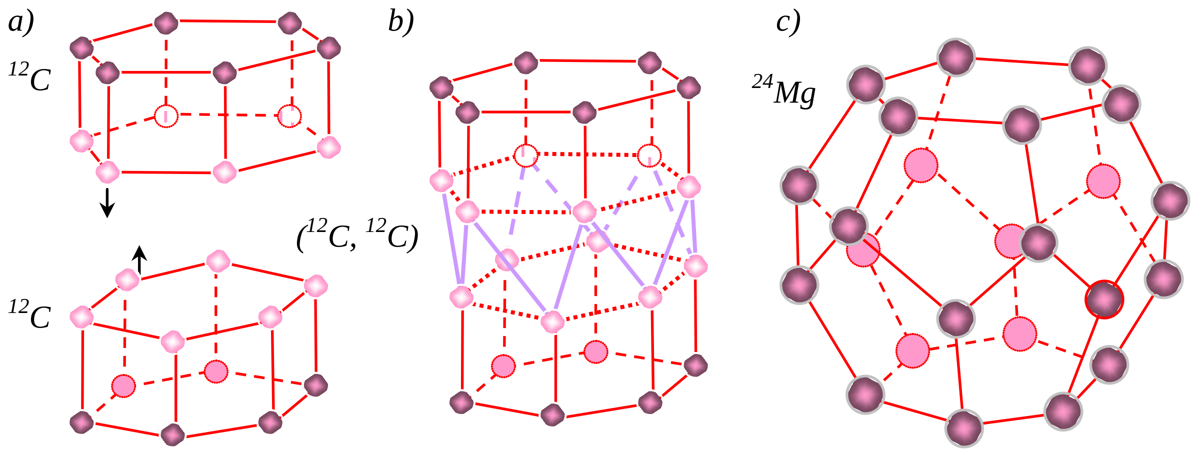

Fig. 11. Joining two hexagonal prisms: *a*) separate prisms (carbon); *b*) intermediate compound; *c*) regular penta$_{12}$-hexa$_{2}$ tettarecaidecahedron (^{24}Mg)

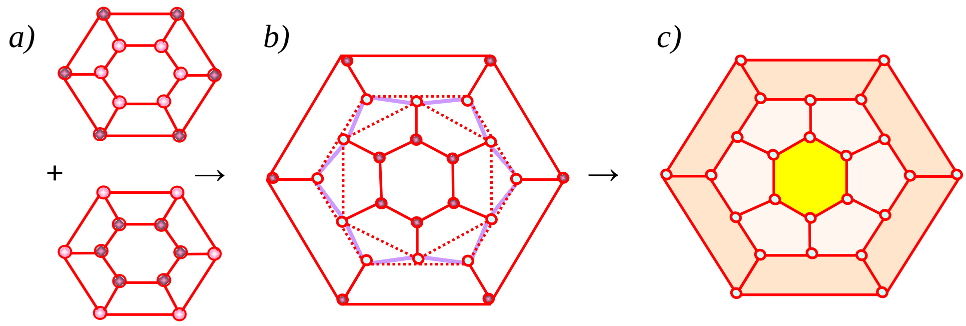

Fig. 12. Graph representation of the nuclear reaction ${}_6C^{12} + {}_6C^{12} \rightarrow {}_{12}Mg^{24}$; embedding a graph of carbon into another: *a*) separate graphs corresponding to carbon nuclei; *b*) embedding, *c*) graph of the penta$_{12}$-hexa$_{2}$ polyhedron (^{24}Mg)

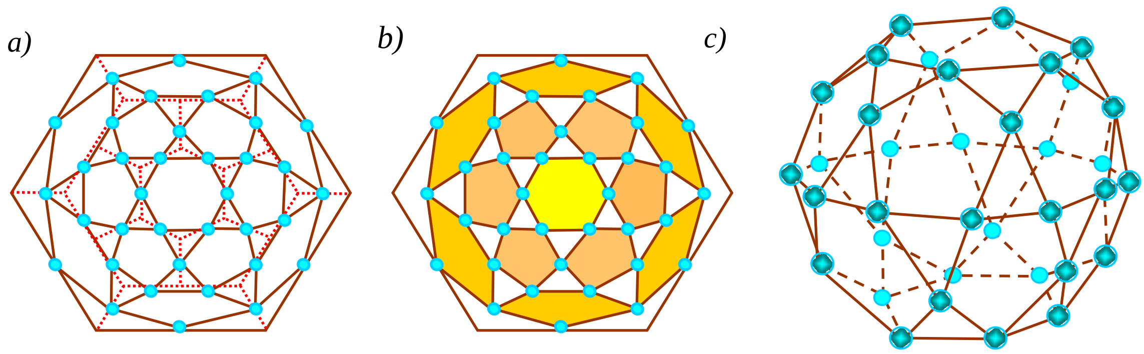

Fig. 13. Electronic structure of six-fold symmetry magnesium ^{24}Mg: *a*) graph of tertion net constructed on the base of the proton-cell graph (red dot lines; *b*) separate graph of tertion net; *c*) tertion net

3.2. Joining a cube to a square barrel. The reaction is shown in Figs. 14, 15 and 16. Here both configurations have four-fold symmetry. From Fig. 14 it follows that for beryllium half protons take part in the reaction but for oxygen only four protons from sixteen do it. They are specially marked in the figure; the protons are pinked, the new proton-protons bonds are lilac, the old bonds, which were destroyed, are shown using dotted lines. Here tertions are also omitted.

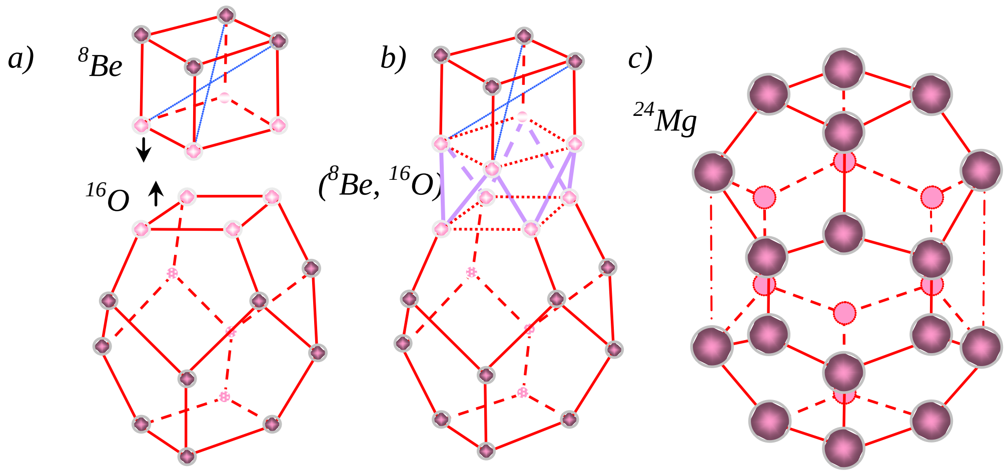

Fig. 14. Joining a cube to a square barrel: *a*) cube (beryllium) at the top, barrel (oxygen) at the bottom; *b*) intermediate compound; *c*) regular tetra$_2$-penta$_8$-hexa$_4$ polyhedron of four-fold symmetry (^{24}Mg)

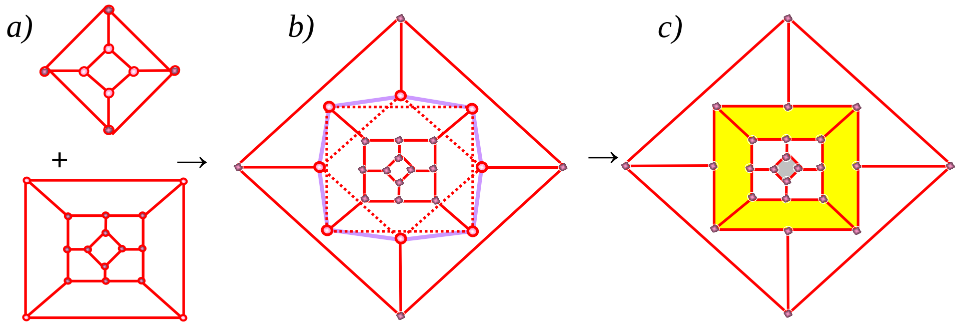

Fig. 15. Graph representation of the nuclear reaction ^8Be + ^{16}O + → ^{24}Mg; embedding the graph of oxygen into the graph beryllium: *a*) separate graphs corresponding to the nuclei; *b*) embedding, *c*) graph of the tetra$_2$-penta$_8$-hexa$_4$ polyhedron having four-fold symmetry (^{24}Mg)

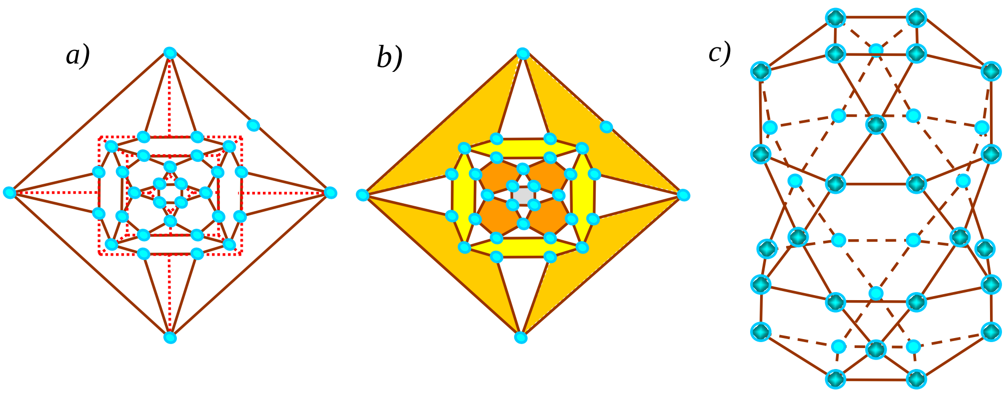

Fig. 16. Electronic structure of four-fold symmetry magnesium ^{24}Mg: *a*) graph of tertion net constructed on the basis of the proton-cell graph (red dot lines0;
b) separate graph of tertion net; *c*) tertion net

3.3. Joining two truncated tetrahedra I. The reaction is illustrated in Figs. 17, 18 and 19. As a result we have obtained a basic nucleus of three-fold symmetry having equal number of protons and neutrons.

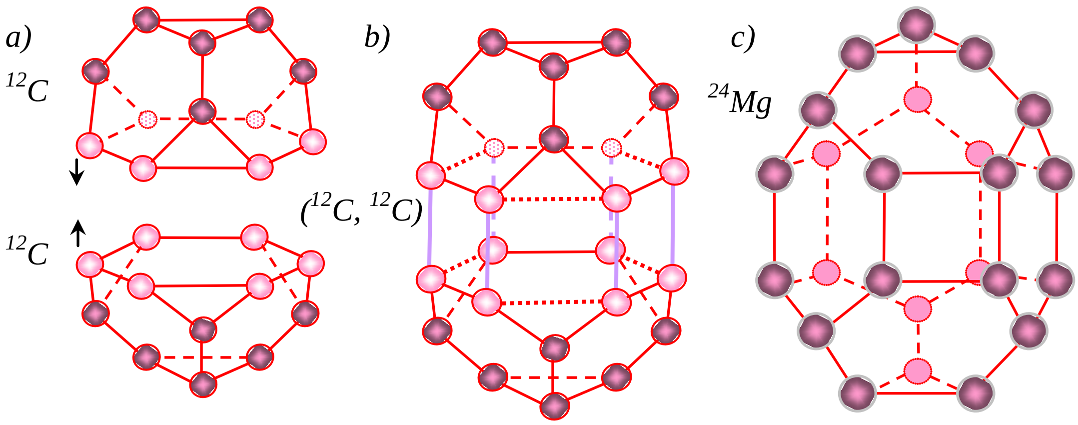

Fig. 17. Mirror-symmetry joining two truncated tetrahedra: *a*) separate tetrahedra (carbon); *b*) intermediate compound; *c*) regular tri$_2$-tetra$_3$-hexa$_9$ polyhedron of three-fold symmetry (^{24}Mg)

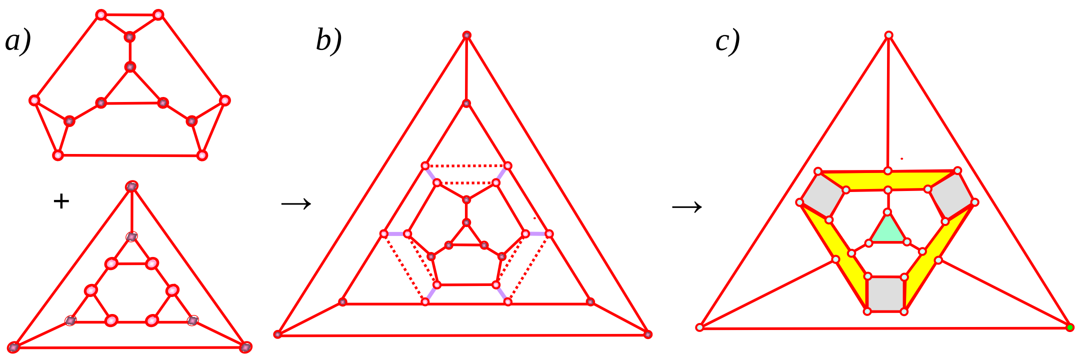

Fig. 18. Graph representation of the nuclear reaction ${}_6C^{12} + {}_6C^{12} \rightarrow {}_{12}Mg^{24}$; embedding a graph of carbon into another one: *a*) separate graphs corresponding to carbon nuclei; *b*) embedding, *c*) graph of the tri$_2$-tetra$_3$-hexa$_9$ polyhedron of three-fold symmetry (^{24}Mg)

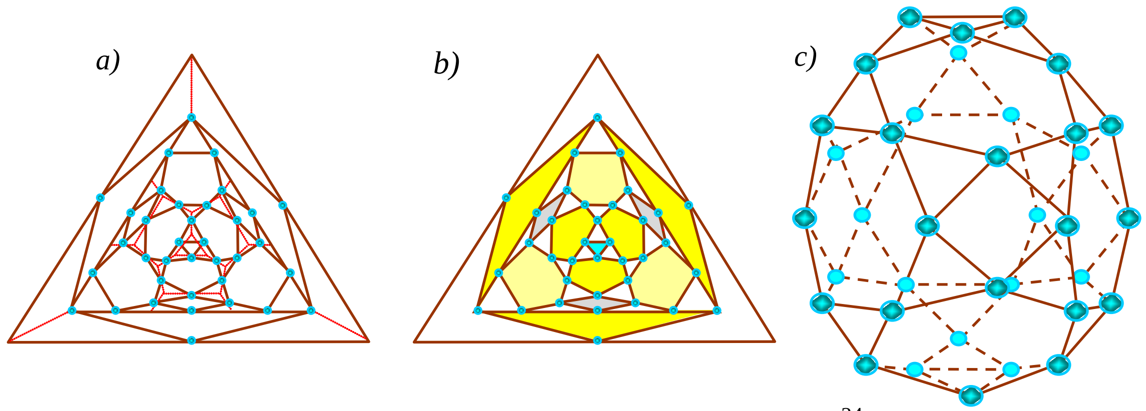

Fig. 19. Electronic structure of three-fold symmetry magnesium ^{24}Mg: *a*) graph of tertion net being constructed on the basis of the proton-cell graph (red dot lines); *b*) separate graph of tertion net; *c*) tertion net

Joining two truncated tetrahedra considered refers to mirror reflection. However there is also rotatory reflection joining. This type is illustrated by the example of fullerenes. By analogy with the fullerenes consider the corresponding reaction.

3.3. Joining two truncated tetrahedra II. The reaction is shown in Figs. 20, 21 and 22.

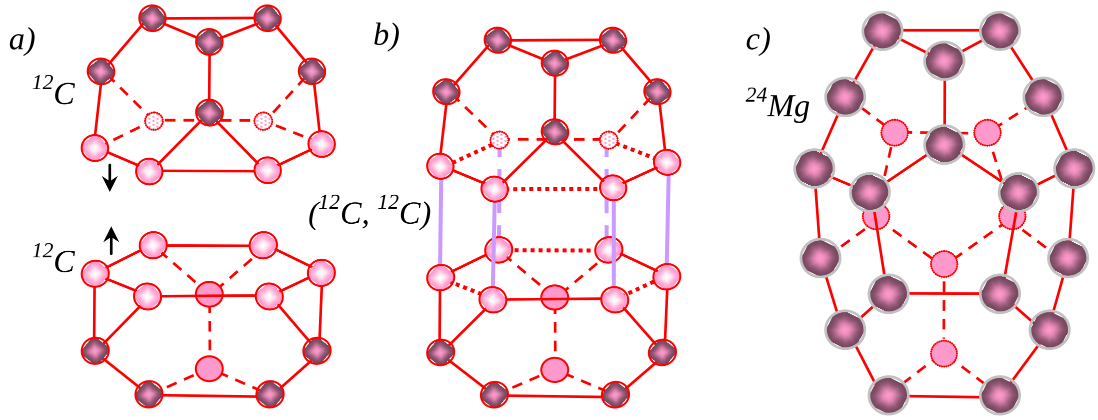

Fig. 20. Rotatory-reflection joining two truncated tetrahedra: *a*) separate tetrahedra (carbon); *b*) intermediate compound; *c*) tri$_2$-penta$_6$-hexa$_6$ polyhedron of three-fold symmetry (^{24}Mg)

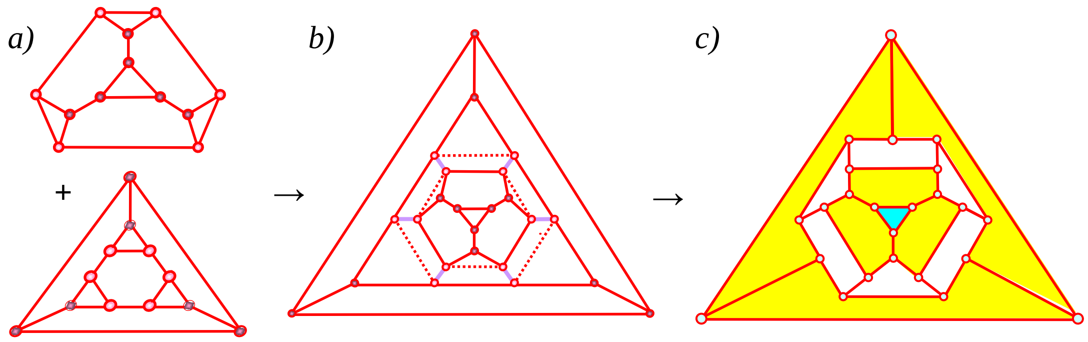

Fig. 21. Graph representation of the nuclear reaction $_6C^{12} + {}_6C^{12} \rightarrow {}_{12}Mg^{24}$; embedding a graph of carbon into another one: *a*) separate graphs corresponding to carbon nuclei; *b*) embedding, *c*) graph of the tri$_2$-tetra$_3$-hexa$_9$ polyhedron of three-fold symmetry (^{24}Mg)

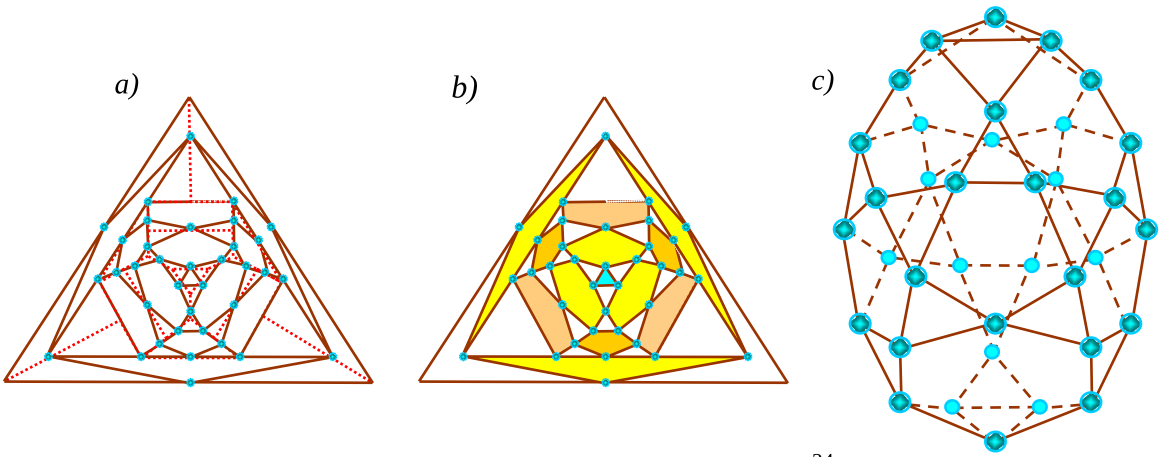

Fig. 22. Electronic structure of three-fold symmetry magnesium ^{24}Mg: *a*) graph of tertion net being constructed on the base of the proton-cell graph (red dot lines); *b*) separate graph of tertion net; *c*) tertion net

From the results obtained it follows that the basic nucleus magnesium 24 has few isomers having three, four and six-fold symmetry. Expanding the analogy, one can assume that the magnesium isotopes, having one, two or four internal neutrons, inherit also that symmetry.

3.4. One-neutron embedding isotopy. From the results obtained it may be concluded that magnesium isotopes ${}_{12}Mg^{25}$ having different symmetry can be formed by various ways. In the reactions described above

$$^{12}C + {}^{12}C \rightarrow {}^{24}Mg, \quad {}^{8}Be + {}^{16}O + \rightarrow {}^{24}Mg,$$

here only basic nuclei having equal number of protons and neutrons are taken into attention. Let us replace one carbon 12 (hexagonal prism) with carbon 13 (body centered hexagonal prism) and beryllium 8 (cubic cell) with beryllium 9 (body centered cube), i.e. think over the reactions

$$^{13}C + {}^{12}C \rightarrow {}^{25}Mg, \quad {}^{9}Be + {}^{16}O + \rightarrow {}^{25}Mg.$$

In the first case we obtain the isotope of magnesium 25 having six-fold symmetry. It is shown in Fig. 23. In the second case replacing leads to appearance of the isotope magnesium 25 having four-fold symmetry, which is illustrated in Fig. 24. It should be emphasized that here only the most probable reactions with isotopes are considered.

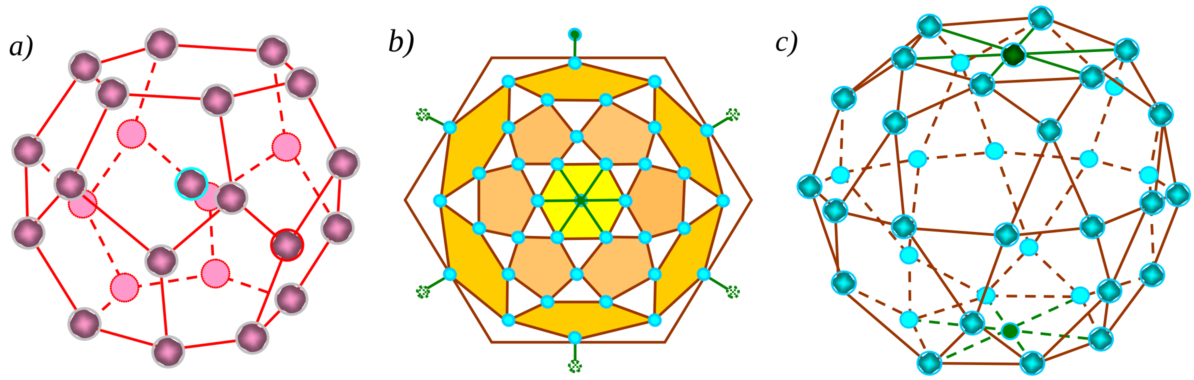

Fig. 23. Structure of six-fold symmetry isotope ^{25}Mg: a) proton cell; b) graph of the tertion net; c) tertion net

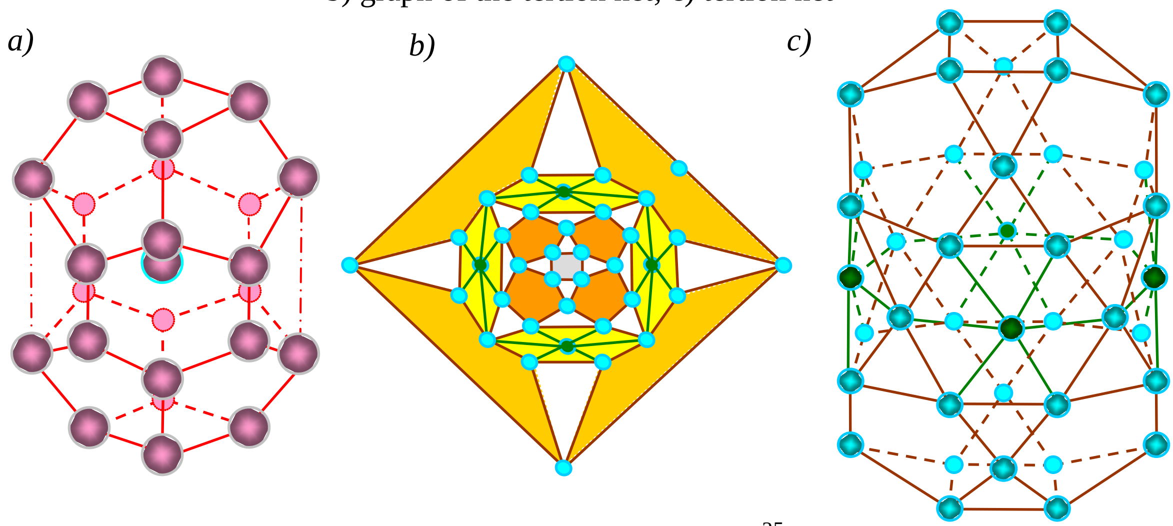

Fig. 24. Structure of four-fold symmetry isotope ^{25}Mg: a) proton cell; b) graph of the tertion net; c) tertion net

From the results obtained, it follows also that in the first case the neutron decays into a proton and two negatively charged particles, having charge ½ *e*. In the second case the charge is equal to ¼ *e*. It was shown previously that the difference can be attributed to the Stark effect, if to associate splitting with symmetry of a nuclear cell and the number of hexagonal faces.

3.5. Two-neutrons embedding isotopy. Magnesium isotopes ${}_{12}Mg^{26}$ having different symmetry can be obtained by different ways: through the use of geometrically compatible reactions ${}^{13}C + {}^{13}C \rightarrow {}^{26}Mg$, rotatory reflection (Fig. 25) and mirror reflection (Fig. 26), and rotatory reflection one ${}^{12}C + {}^{14}C \rightarrow {}^{26}Mg$ (Fig. 27).

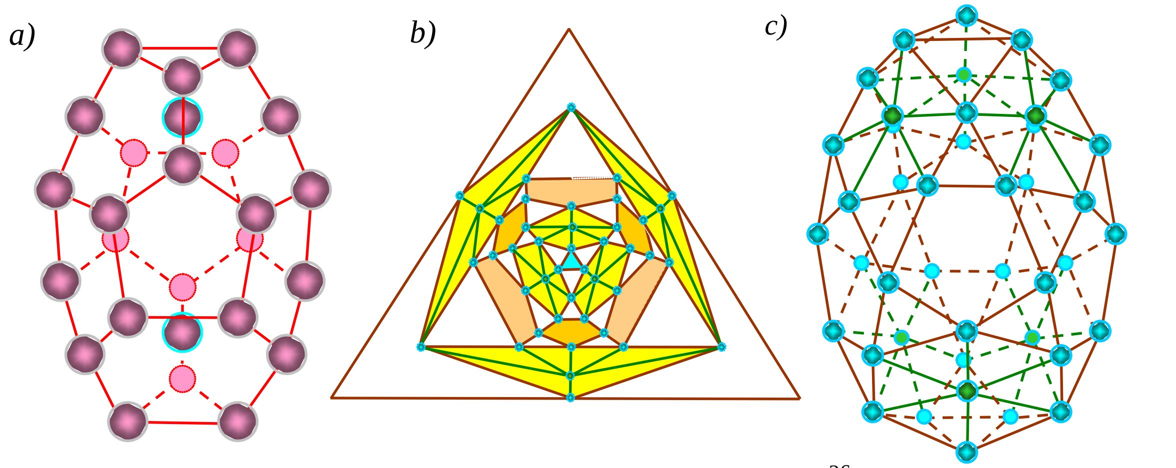

Fig. 25. Structure of three-fold mirror-symmetry isotope ^{26}Mg: a) proton cell; b) graph of the tertion net; c) tertion net

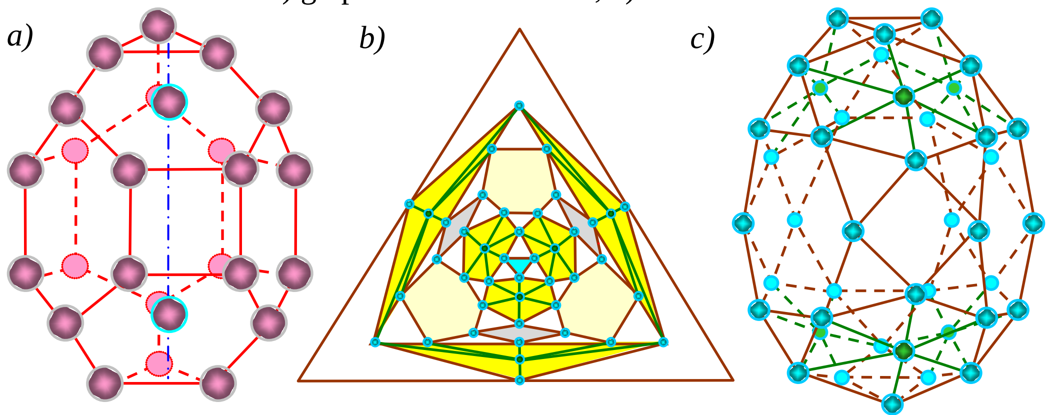

Fig. 26. Structure of isotope ^{26}Mg having three-fold rotatory reflection symmetry: a) proton cell; b) graph of the tertion net; c) tertion net

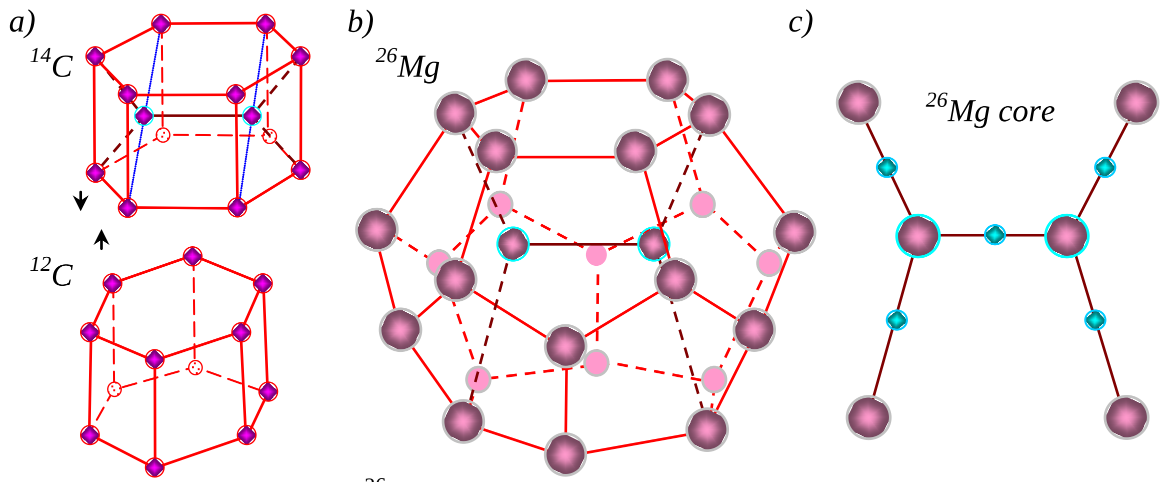

Fig. 27. Structure of isotope ^{26}Mg having six-fold symmetry: *a*) separate prisms; b) proton cell; c) core of the nucleus with the nearest cell protons and the binding tertions

3.6. Four-neutrons embedding isotopy. Magnesium isotopes $_{12}Mg^{28}$ can be obtained through the use of the geometrically compatible reaction $^{14}C + {}^{14}C \rightarrow {}^{28}Mg$. The reaction is shown in Fig. 27. Here one runs into a new phenomenon. Up to now, when there were two internal neutrons, they were decomposed into protons and tertions by the external surroundings, the protons being tightly connected with the electronic coat of mail and maybe only slightly connected, if are connected at all, one another. Now it is seen that the internal protons form the core, they being tightly connected one another and slightly connected with the external electronic coat of mail (Fig. 28). Four internal neutrons give 12 tertions; 6 of them form the coat of mail of the core, 4 refer to the bonds connecting the core and the external proton cell and 2 are incorporated into the external coat of mail. All tertions have the charge ⅓ *e*; they are specially marked in emerald in Fig. 28.

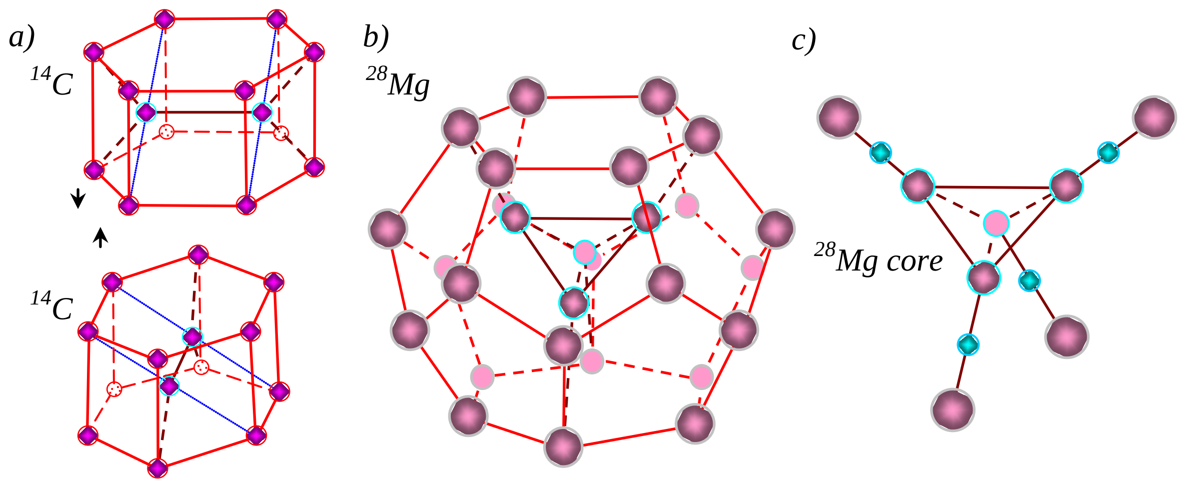

Fig. 27. Structure of isotope ^{28}Mg having six-fold symmetry: *a*) separate prisms (carbon 14); b) proton cell; c) core of the nucleus with the nearest cell protons and the binding tertions

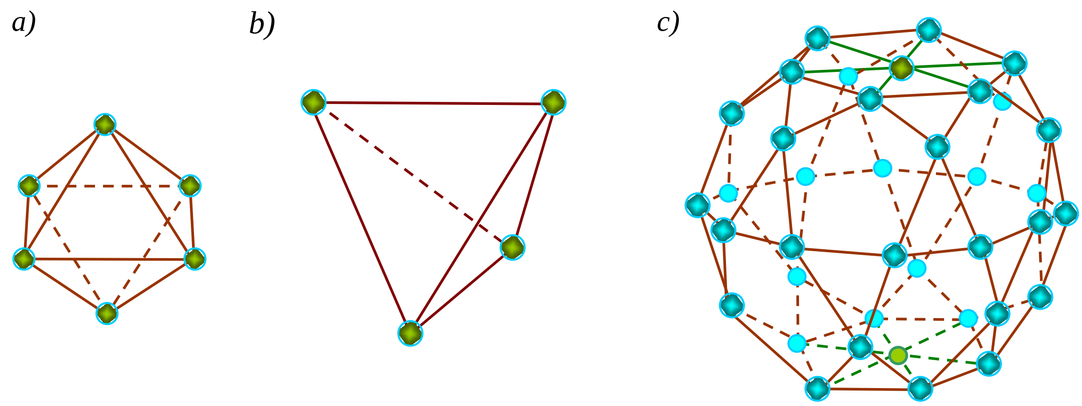

Fig. 28. Electronic structure of six-fold symmetry magnesium isotope ^{28}Mg: *a*) core tertion net (octahedron); *b*) binding tertion net (tetrahedron); *c*) tertion net of an external coat of mail

4. Isotopes of aluminum and their isomers

There is only one stable isotope of aluminum, $_{13}Al^{27}$ (100 %), and an ‘unstable’ isotope however having a very large half-decay period being equal to $7.2 \cdot 10^5$ y, $_{13}Al^{26}$ [2]. The latter is a basic nucleus having equal number of protons and neutrons. From the aforesaid, it follows that it is easier to consider at first a simpler basic nucleus. It can be obtained by several ways, but the simplest one is through the use of reaction $d + {}_{12}Mg^{24} \rightarrow {}_{13}Al^{26}$.

4.1. Incorporating a dimer into a penta$_{12}$-hexa$_2$ polyhedron. The reaction is illustrated in Figs. 29 and 30. Here a deuteron is incorporated into a basic nucleus of magnesium having six-fold symmetry. From Fig. 29 it follows that for magnesium only four protons from twenty four take part really in the reaction. They are specially marked in the figure; the protons are pinked, the new proton-protons bonds are lilac, the old bonds, which were destroyed, are not shown. Here tertions are omitted.

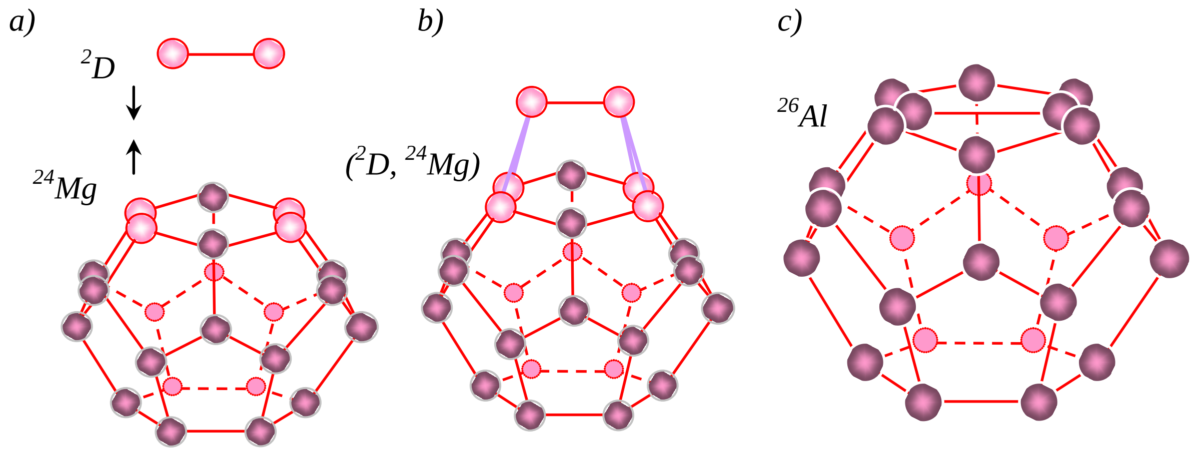

Fig. 29. Attachment of deuteron to magnesium: *a*) separate particles; *b*) intermediate compound; *c*) aluminum (^{26}Al) after relaxation

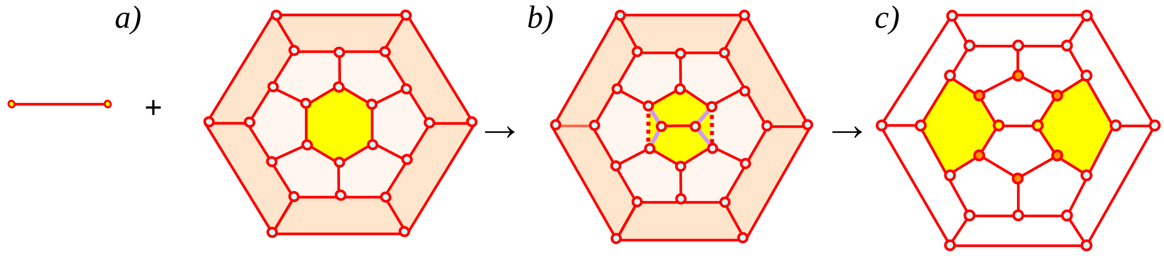

Fig. 30. Graph representation of the nuclear reaction $d + {}_{12}Mg^{24} \rightarrow {}_{13}Al^{26}$. Embedding the graph of deuteron into the graph of magnesium: *a*) separate graphs corresponding to a dimer (at the left) and to a penta$_{12}$-hexa$_2$ polyhedron (at the right); *b*) embedding, *c*) graph of the penta$_{12}$-hexa$_3$ polyhedron. All notations are the same as before

One can consider aluminum 26 obtained as a polyhedron having topological six-fold symmetry. At the same time it has ordinary three-fold symmetry as it shown in Fig. 31. The electronic structure corresponding to three-fold symmetry is illustrated in Fig. 32.

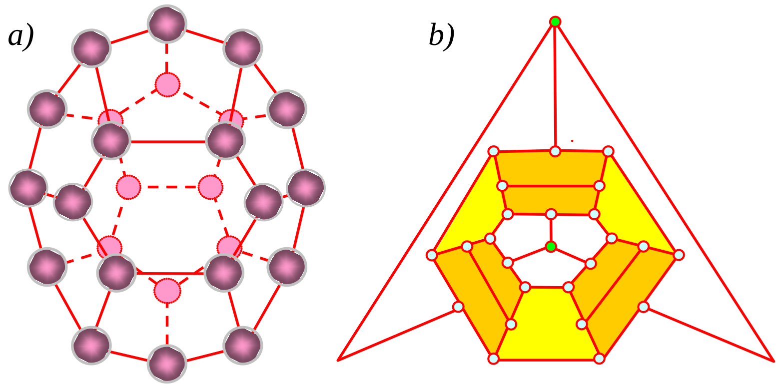

Fig. 31. Structure of aluminum 26 and its graph showing three fold symmetry

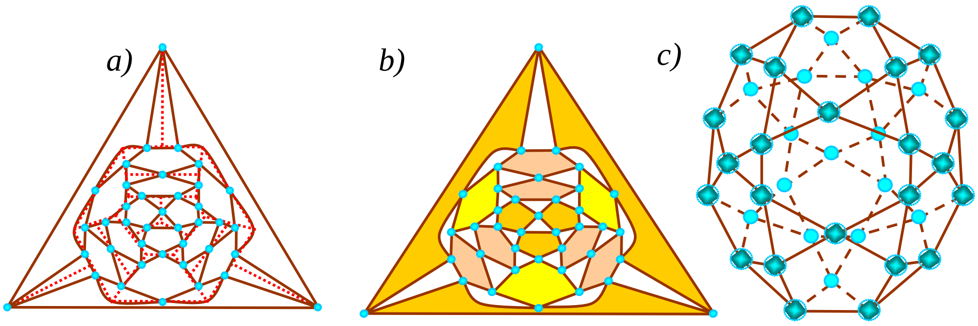

Fig. 32. Electronic structure of aluminum 26: *a*) graph of tertion net being constructed on the base of the proton-cell graph; *b*) separate graph of tertion net; *c*) tertion net

4.3. Neutron embedding. Let's replace in the previous reaction magnesium 24 having the shape of a penta$_{12}$-hexa$_2$-polyhedron by magnesium 25 (the same polyhedron but body centered). Then we obtain the isotope of aluminum (^{27}Al) shown in Fig. 33. It should be noted that in this case additional tertions have the charge of ⅓ *e*. The reasons are connected with the number of hexangular facets which incorporate the electrons of an internal neutron.

Why the basic nucleus and the isotope are stable? One can suggest the following explanation. It is clear that any nucleus structure (proton cell) must be stable with respect to mechanical stresses which appear due to thermal vibrations of protons. Assume that the stability is insured by the coat of mail that dictates geometry of a proton cell. On the basis of the previous experience, we came to conclusion that the coat of mail (tertion net), which ensures additional stability, doesn't contain hexagons.

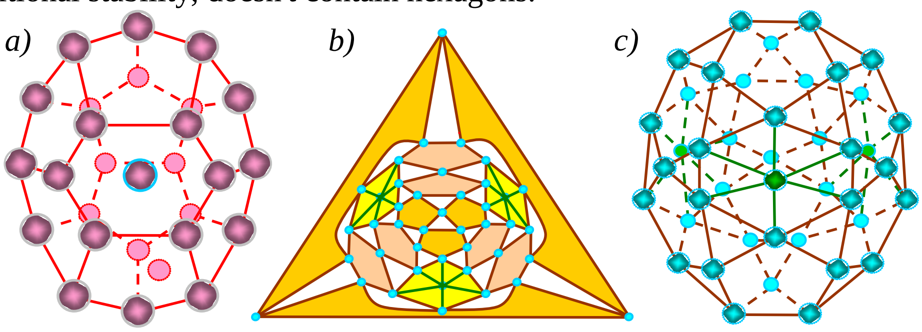

Fig. 33. Structure of isotope ^{27}Al: a) proton cell; b) graph of the tertion net; c) tertion net

5. Summary

By analogy with fullerenes, the nuclear geometry has been designed. For sodium, magnesium and aluminum, the protonic and electronic structures both for basic isomers and their isotopes were obtained. The most stable nuclei can be classed into two groups: basic nuclei having equal number of protons and neutrons and isotopes having one or two neutrons. The latter ensure their mechanical stability with respect to shear stresses, sending their electrons to the coat of mail created by the basic nuclei.

Acknowledgements. *No external funding was received for this study.*

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