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THERMOMECHANICAL INTERACTIONS DUE TO TIME HARMONIC SOURCES IN A TRANSVERSELY ISOTROPIC MAGNETO THERMOELASTIC ROTATING SOLIDS IN LORD-SHULMAN MODEL

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Abstract. The present research deals with the mathematical modelling of two dimensional transversely isotropic magneto thermoelastic initially stressed solid due to time-harmonic source with generalized Lord-Shulman (LS) theory of thermoelasticity. The Fourier transform has been used to find the solution to the problem. The expressions for the displacement components, stress components, and temperature distribution are obtained in the transformed domain. The effect of time-harmonic source is depicted graphically on the resulting quantities.

Keywords: transversely isotropic Magneto thermoelastic, mechanical and thermal stresses, time-harmonic source

Nomenclature

δ_{ij}	Kronecker delta,
D_{ijkl}	Elastic parameters,
β_{ij}	Thermal elastic coupling tensor,
T	Absolute temperature,
T_0	Reference temperature,
φ	conductive temperature,
t_{ij}	Stress tensors,
e_{ij}	Strain tensors,
P_{mi}	Pre stress tensor,
u_i	Components of displacement,
ρ	Medium density,
C_E	Specific heat,
α_{ij}	Linear thermal expansion coefficient,
K_{ij}	Thermal conductivity,
ω	Frequency,
τ_0	Relaxation Time,
$\mathbf{\Omega}$	Angular Velocity of the Solid,
F_i	Components of Lorentz force,
$\boldsymbol{H_0}$	Magnetic field intensity vector,
$\boldsymbol{J}$	Current Density Vector,
$\boldsymbol{u}$	Displacement Vector,
μ_0	Magnetic permeability,
ε_0	Electric permeability,
$\delta(t)$	Dirac's delta function,
$\boldsymbol{h}$	induced magnetic field vector,
$\boldsymbol{E}$	induced electric field vector,

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1. Introduction

A lot of research and attention has been given to deformation and heat flow in a continuum using thermoelasticity theories during the past few years. When sudden heat/external force is applied in a solid body, it transmits time-harmonic wave by thermal expansion. The change at some point in the medium is beneficial to detect the deformed field near mining shocks, seismic and volcanic sources, thermal power plants, high-energy particle accelerators, and many emerging technologies. The study of the time-harmonic source is one of the broad and dynamic areas of continuum dynamics. Therefore, an unbounded rotating elastic medium with angular velocity, with rotation and relaxation time, and without energy dissipation in generalized thermoelasticity has been studied in this research.

Marin [1] had proved the Cesaro means of strain and kinetic energies of dipolar bodies with finite energy. Marin [2] investigated and solved the initial-boundary value problem without recourse either to an energy conservation law or to any boundedness assumptions on the thermoelastic coefficients in thermoelastic bodies with voids. Ailawalia et al. [3] had studied a rotating generalized thermoelastic medium in the presence of two temperatures beneath hydrostatic stress and gravity with different kinds of sources using integral transforms. Singh and Yadav [4] solved the transversely isotropic rotating magnetothermoelastic medium equations by cubic velocity equation of three plane waves without anisotropy, rotation, and thermal and magnetic effects. Banik and Kanoria (2012) studied the thermoelastic interaction in an isotropic infinite elastic body with a spherical cavity for the TPL(Three-Phase-Lag) heat equation with two-temperature generalized thermoelasticity theory and has shown variations between two models: the two-temperature GN theory in presence of energy dissipation and two-temperature TPL model and has shown the effects of ramping parameters and two-temperature.

Mahmoud [5] had considered the impact of rotation, relaxation times, magnetic field, gravity field, and initial stress on Rayleigh waves and attenuation coefficient in an elastic half-space of granular medium and obtained the analytical solution of Rayleigh waves velocity by using Lame’s potential techniques. Abd-alla and Alshaikh [6] had discussed the influence of magnetic field and rotation on plane waves in transversely isotropic thermoelastic medium under the GL theory in presence of two relaxation times to show the presence of three quasi plane waves in the medium. Marin et al. [7] has modelled a micro stretch thermoelastic body with two temperatures and eliminated divergences among the classical elasticity and research.

Sharma et al. [8] investigated the 2-D deception in transversely isotropic homogeneous thermoelastic solids in presence of two temperatures in GN-II theory with an inclined load (linear combination of normal load and tangential load). Kumar et al. [9] investigated the impact of Hall current in a transversely isotropic magnetothermoelastic in the presence and absence of energy dissipation due to the normal force. Kumar et al. [10] studied the conflicts caused by thermomechanical sources in a transversely isotropic rotating homogeneous thermoelastic medium with a magnetic effect as well as two temperatures and applied to the thermoelasticity Green–Naghdi theories with and without energy dissipation using thermomechanical sources. Lata et al. [11] studied two temperature and rotation aspect for GN-II and GN-III theory of thermoelasticity in a homogeneous transversely isotropic magnetothermoelastic medium for the case of the plane wave propagation and reflection. Ezzat et al. [12] proposed a mathematical model of electro-thermoelasticity for heat conduction with memory-dependent derivative. Kumar et al. [13] analyzed the Rayleigh waves in a transversely isotropic homogeneous magnetothermoelastic medium in presence of two temperatures, with Hall current and rotation.

Marin et.al. [14] studied the GN-thermoelastic theory for a dipolar body using mixed initial BVP and proved a result of Hölder's-type stability. Lata [15] studied the impact of energy dissipation on plane waves in sandwiched layered thermoelastic medium of uniform thickness, with two temperatures, rotation, and Hall current in the context of GN Type-II and Type-III theory of thermoelasticity. Ezzat and El-Bary [16] had applied the magneto-thermoelasticity model to a one-dimensional thermal shock problem of functionally graded half-space based on memory-dependent derivative. Kumar et al. [17] investigated the deformations in a homogeneous transversely isotropic magneto-Visco thermoelastic medium under GN type I and II theories in presence of rotation and two temperatures with thermomechanical sources. Despite this several researchers working on a different theory of thermoelasticity as Marin [18], Marin [19], Atwa [20], Marin [21], Marin and Baleanu [22], Bijarnia and Singh [23], Ezzat et al. [24], Ezzat et al. [25], Ezzat et al. [26], Ezzat and El-Bary [27], Ezzat and El-Bary [28], Ezzat et al. [29], Chauthale et al**.** [30] and Shahani and Torki [31].

In spite of these, not much work has been carried out in thermomechanical interactions in transversely isotropic magneto thermoelastic solid with rotation and relaxation time and without energy dissipation due to time-harmonic source in generalized LS theories of thermoelasticity. Keeping these considerations in mind, analytic expressions for the displacement components, stress components, and temperature distribution in two-dimensional homogeneous, transversely isotropic magneto-thermoelastic solids without energy dissipation, with rotation and various frequencies of the time-harmonic source.

2. Basic equations

Following Zakaria [32], the simplified Maxwell's linear equation of electrodynamics for a slowly moving and conducting elastic solid are

$$curl\,\boldsymbol{h} = \boldsymbol{J} + \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}, \tag{1}$$

$$curl\,\boldsymbol{E} = -\,\mu_0 \frac{\partial \boldsymbol{h}}{\partial t}, \tag{2}$$

$$\boldsymbol{E} = -\,\mu_0 \left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{H_0}\right), \tag{3}$$

$$div\,\boldsymbol{h} = 0. \tag{4}$$

Maxwell stress components are given by

$$T_{ij} = \mu_0 (H_i h_j + H_j h_i - H_k h_k \delta_{ij}). \tag{5}$$

For an anisotropic pre-stressed thermoelastic medium, the constitutive equation is given by

$$t_{ij} = D_{ijkl} e_{kl} + e_{jm} P_{mi} - \beta_{ij} T, \tag{6}$$

and the equation of motion for a uniformly rotating pre-stressed medium in the rotating frame of reference with angular velocity $\boldsymbol{\Omega}$ in the presence of external force is given by

$$t_{ij,j} + F_i = \rho\{\ddot{\boldsymbol{u}}_i + (\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{u}))_i + (2\boldsymbol{\Omega} \times \dot{\boldsymbol{u}})_i\}, \tag{7}$$

where $\boldsymbol{\Omega} = \Omega \boldsymbol{n}$, $\boldsymbol{n}$ is a unit vector representing the direction of the axis of rotation. The term $\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{u})$ is the additional centripetal acceleration due to the time-varying motion only, and the term $2\boldsymbol{\Omega} \times \dot{\boldsymbol{u}}$ is the Coriolis acceleration which occurs for a moving frame of reference only. The total stress in the half-space is composed of Hooke's mechanical stress and Maxwell's stress. The Lorentz force F_i is

$$F_i = \mu_0 (\boldsymbol{J} \times \boldsymbol{H_0}) . \tag{8}$$

The Lord-Shulman model of heat conduction equation in the presence of an external source of heat Q is given by

$$K_{ij} T_{,ij} + \rho\big(Q + \tau_0 \dot{Q}\big) = \beta_{ij} T_0\big(\dot{e}_{ij} + \tau_0 \ddot{e}_{ij}\big) + \rho C_E\big(\dot{T} + \tau_0 \ddot{T}\big), \tag{9}$$

where

$$\beta_{ij} = C_{ijkl} \alpha_{ij}, \tag{10}$$

$$e_{ij} = \frac{1}{2}\big(u_{i,j} + u_{j,i}\big), \quad i,j = 1,2,3, \tag{11}$$

$\beta_{ij} = \beta_i\delta_{ij}$, $K_{ij} = K_i\delta_{ij}, C_{ijkl} = D_{ijkl} + \delta_{jl}P_{ki}$, i is not summed.

Here $D_{ijkl}(D_{ijkl} = D_{klij} = D_{jikl} = D_{ijlk})$ are elastic parameters.

3. Formulation and solution of the problem

We consider a pre-stressed 2-D homogeneous transversely isotropic magneto thermoelastic medium, permeated by an initial magnetic field $\boldsymbol{H_0} = (0, H_0, 0)$ acting along y-axis. This initial magnetic field produces an induced magnetic field $\boldsymbol{h} = (0, h, 0)$ and induced electric field $\boldsymbol{E} = (E_1, 0, E_3)$. The rectangular Cartesian co-ordinate system (x, y, z) having origin on the surface $(z = 0)$ with z-axis pointing vertically into the medium is introduced. The surface of the half-space is subjected to a thermomechanical force acting at $z = 0$. In addition, we consider that

$$\boldsymbol{\Omega} = (0, \Omega, 0). \tag{12}$$

Following Lata et al**.** [10] from the generalized Ohm's law we have

$$J_2 = 0\,. \tag{13}$$

And the density components J_1 and J_3 are given as

$$J_1 = -\varepsilon_0\mu_0 H_0 \frac{\partial^2 w}{\partial t^2}, \tag{14}$$

$$J_3 = \varepsilon_0\mu_0 H_0 \frac{\partial^2 u}{\partial t^2}. \tag{15}$$

In addition, the equations of displacement vector $\boldsymbol{u} = (u, v, w)$ and temperature change T for 2-dimensional motion of transversely isotropic thermoelastic solid are:

$$u = u(x, z, t), v = 0, w = w(x, z, t) \text{ and } T = T(x, z, t). \tag{16}$$

Following Slaughter [33] using the appropriate transformation on equations (7) and (9) with the aid of equation (16), yields

$$C_{11}\frac{\partial^2 u}{\partial x^2} + C_{13}\frac{\partial^2 w}{\partial x\partial z} + C_{44}\left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x\partial z}\right) - \beta_1\frac{\partial T}{\partial x} - \mu_0 J_3 H_0 = \rho\left(\frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega\frac{\partial w}{\partial t}\right), \tag{17}$$

$$(C_{13} + C_{44})\frac{\partial^2 u}{\partial x\partial z} + C_{44}\frac{\partial^2 w}{\partial x^2} + C_{33}\frac{\partial^2 w}{\partial z^2} - \beta_3\frac{\partial T}{\partial z} - \mu_0 J_1 H_0 = \rho\left(\frac{\partial^2 w}{\partial t^2} - \Omega^2 w - 2\Omega\frac{\partial u}{\partial t}\right), \tag{18}$$

$$K_1\frac{\partial^2 T}{\partial x^2} + K_3\frac{\partial^2 T}{\partial z^2} + \rho\left(Q + \tau_0\dot{Q}\right) = \rho C_E\left(\dot{T} + \tau_0\ddot{T}\right) + T_0\frac{\partial}{\partial t}\left\{\beta_1\left(1 + \tau_0\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial x} + \beta_3\left(1 + \tau_0\frac{\partial}{\partial t}\right)\frac{\partial w}{\partial z}\right\}, \tag{19}$$

and

$$t_{11} = C_{11}e_{11} + C_{13}e_{13} - \beta_1 T, \tag{20}$$

$$t_{33} = C_{13}e_{11} + C_{33}e_{33} - \beta_3 T, \tag{21}$$

$$t_{13} = 2C_{44}e_{13}, \tag{22}$$

where $C_{11} = D_{11} + P_{11}, C_{13} = D_{13}, C_{33} = D_{33} + P_{33}, C_{44} = D_{44} + P_{11}$.

$\beta_1 = (C_{11} + C_{12})\alpha_1 + C_{13}\alpha_3,$

$\beta_3 = 2C_{13}\alpha_1 + C_{33}\alpha_3,$

where α_1, α_3 are linear thermal expansion coefficients.

We consider that the medium is initially at rest. Therefore, the preliminary and symmetry conditions are given by

$u(x, z, 0) = 0 = \dot{u}(x, z, 0),$

$w(x, z, 0) = 0 = \dot{w}(x, z, 0),$

$T(x, z, 0) = 0 = \dot{T}(x, z, 0)$ for $z \geq 0, -\infty < x < \infty,$

$u(x, z, t) = w(x, z, t) = T(x, z, t) = 0 \; for \; t > 0$ when $z \to \infty$.

Assuming the time-harmonic behaviour as

$$(u, w, T, Q)(x, z, t) = (u, w, T, Q)(x, z)e^{i\omega t}, \tag{23}$$

where ω is the angular frequency.

To simplify the solution, mention below dimensionless quantities are used:

$$x' = \frac{x}{L}, \quad u' = \frac{\rho c_1^2}{L\beta_1 T_0}u, \quad t' = \frac{c_1}{L}t, \quad w' = \frac{\rho c_1^2}{L\beta_1 T_0}w, \quad T' = \frac{T}{T_0}, t'_{11} = \frac{t_{11}}{\beta_1 T_0},$$
$$t'_{33} = \frac{t_{33}}{\beta_1 T_0}, t'_{31} = \frac{t_{31}}{\beta_1 T_0}, \quad z' = \frac{z}{L}, h' = \frac{h}{H_0}, \quad \Omega' = \frac{L}{C_1}\Omega . \tag{24}$$

Making use of (23) and then using dimensionless variables of eq. (24) in Eqs. (17)-(19), after suppressing the primes, yield

$$\frac{\partial^2 u}{\partial x^2} + \delta_4\frac{\partial^2 w}{\partial x\partial z} + \delta_2\left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x\partial z}\right) - \frac{\partial T}{\partial x} = \left(\frac{\varepsilon_0\mu_0^2 H_0^2}{\rho} + 1\right)(-\omega^2 u) - \Omega^2 u + 2\Omega i\omega w, \tag{25}$$

$$\delta_1\frac{\partial^2 u}{\partial x\partial z} + \delta_2\frac{\partial^2 w}{\partial x^2} + \delta_3\frac{\partial^2 w}{\partial z^2} - \frac{\beta_3}{\beta_1}\frac{\partial T}{\partial z} = \left(\frac{\varepsilon_0\mu_0^2 H_0^2}{\rho} + 1\right)(-\omega^2 w) - \Omega^2 w + 2\Omega i\omega u, \tag{26}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{K_3}{K_1}\frac{\partial^2 T}{\partial z^2} + \rho\left(1 + \tau_0\frac{c_1}{L}i\omega\right)Q = \delta_5\frac{\partial}{\partial t}\left(1 + \tau_0\frac{c_1}{L}i\omega\right)T +$$
$$+\delta_6 i\omega\left(1 + \tau_0\frac{c_1}{L}i\omega\right)\left[\beta_1\frac{\partial u}{\partial x} + \beta_3\frac{\partial w}{\partial z}\right], \tag{27}$$

where $\delta_1 = \frac{C_{13}+C_{44}}{C_{11}}, \delta_2 = \frac{C_{44}}{C_{11}}, \delta_3 = \frac{C_{33}}{C_{11}}, \delta_4 = \frac{C_{13}}{C_{11}}, \ \delta_5 = \frac{\rho C_E C_1 L}{K_1}, \delta_6 = -\frac{T_0\beta_1 L}{\rho C_1 K_1}$.

Apply Fourier transforms defined by

$$\hat{f}(\xi, z, \omega) = \int_{-\infty}^{\infty} f(x, z, \omega)e^{i\xi x}\,dx. \tag{28}$$

On Eqs. (25)–(27), we obtain a system of equations

$$[-\xi^2 + \delta_2 D^2 + \delta_7\omega^2 + \Omega^2]\hat{u}(\xi, z, \omega) + [\delta_4 Di\xi + \delta_2 Di\xi - 2\Omega i\omega]\widehat{w}(\xi, z, \omega) +$$
$$+(-\mathrm{i}\xi)\hat{T}(\xi, z, \omega) = 0, \tag{29}$$

$$[\delta_1 Di\xi + 2\Omega i\omega]\hat{u}(\xi, z, \omega) + [-\delta_2\xi^2 + \delta_3 D^2 + \delta_7\omega^2 + \Omega^2]\widehat{w}(\xi, z, \omega) -$$
$$-\frac{\beta_3}{\beta_1}D\hat{T}(\xi, z, \omega) = 0, \tag{30}$$

$$[-\delta_6\omega\delta_8\beta_1\xi]\hat{u}(\xi, z, \omega) + [\delta_6 i\omega\delta_8\beta_3 D]\widehat{w}(\xi, z, \omega) +$$
$$+\left[\xi^2 - \frac{K_3}{K_1}D^2 + \delta_5\delta_8 i\omega\right]\hat{T}(\xi, z, \omega) = \rho\delta_8\hat{Q}(\xi, z, \omega), \tag{31}$$

where $\delta_7 = \frac{\varepsilon_0\mu_0^2 H_0^2}{\rho} + 1, \ \delta_8 = 1 + \tau_0\frac{C_1}{L}i\omega$.

Here we are considering that there is no external supply of heat source i.e. by taking $\hat{Q}(\xi, z, s) = 0$, the non-trivial solution exists if the determinant of the coefficient matrix of $[\hat{u}, \widehat{w}, \hat{T}]$ of (29)-(31) vanishes and the characteristic equation is

$$(AD^6 + BD^4 + CD^2 + E)(\hat{u}, \widehat{w}, \hat{T}) = 0, \tag{32}$$

where

$$\mathrm{A} = \delta_2\delta_3\vartheta_7 - \vartheta_5\delta_2\frac{\beta_3}{\beta_1},$$

$$\mathrm{B} = \delta_3\vartheta_1\vartheta_7 + \delta_2\delta_3\vartheta_6 + \delta_2\vartheta_7\vartheta_3 - \vartheta_5\vartheta_9\delta_2 - \vartheta_8\delta_1 i\xi\vartheta_7,$$

$$\mathrm{C} = \delta_3\vartheta_1\vartheta_6 + \vartheta_1\vartheta_3\vartheta_7 - \vartheta_1\vartheta_5\vartheta_9 + \delta_2\vartheta_6\vartheta_3 + \vartheta_4\vartheta_8\vartheta_9 - \vartheta_8\delta_1 i\xi\vartheta_6 - 4\Omega^2\omega^2\vartheta_7 + \vartheta_2\delta_1 i\xi\vartheta_5 - \vartheta_2\vartheta_4\delta_3,$$

$$E = \vartheta_3\vartheta_1\vartheta_6 - 4\Omega^2\omega^2\vartheta_6 - \vartheta_2\vartheta_4\vartheta_3,$$

$$\vartheta_1 = -\xi^2 + \delta_7\omega^2 + \Omega^2,$$

$$\vartheta_2 = -i\xi,$$

$$\vartheta_3 = -\delta_2\xi^2 + \delta_7\omega^2 + \Omega^2,$$

$$\vartheta_4 = -\delta_6\delta_8\omega\beta_1\xi,$$

$\vartheta_5 = \delta_6\delta_8 i\omega\beta_3,$

$\vartheta_6 = \xi^2 + \delta_5\delta_8 i\omega,$

$\vartheta_7 = -\frac{K_3}{K_1},$

$\vartheta_8 = \delta_1 i\xi,$

$\vartheta_9 = -\frac{\beta_3}{\beta_1}.$

The roots of the Eq. (32) are $\pm\lambda_j$, $(j = 1,2,3)$, are obtained by using the radiation condition $\tilde{u}, \tilde{w}, \tilde{T} \to 0\ as\ z \to \infty$ and can be written as

$$\hat{u}(\xi, z, \omega) = \sum_{j=1}^{3} A_j e^{-\lambda_j z}, \tag{33}$$

$$\hat{w}(\xi, z, \omega) = \sum_{j=1}^{3} d_j A_j e^{-\lambda_j z}, \tag{34}$$

$$\hat{T}(\xi, z, \omega) = \sum_{j=1}^{3} l_j A_j e^{-\lambda_j z}, \tag{35}$$

where $A_j(\xi,\omega), j = 1,2,3$ being undetermined constants and d_j and l_j are given by

$$d_j = \frac{\delta_2\zeta_7\lambda_j^4 + (\vartheta_7\vartheta_1 + \delta_2\vartheta_6)\lambda_j^2 + \vartheta_1\vartheta_6 - \vartheta_4\vartheta_2}{(\delta_3\vartheta_7)\lambda_j^4 + (\delta_3\vartheta_6 + \vartheta_3\vartheta_7 - \vartheta_5\vartheta_9)\lambda_j^2 + \vartheta_3\vartheta_6},$$

$$l_j = \frac{\delta_2\delta_3\lambda_j^4 + (\delta_2\zeta_3 + \vartheta_1\delta_3 - \delta_1\vartheta_8 i\xi)\lambda_j^2 - 4\Omega^2\omega^2 + \vartheta_3\vartheta_1}{(\delta_3\vartheta_7)\lambda_j^4 + (\delta_3\vartheta_6 + \vartheta_3\vartheta_7 - \vartheta_5\vartheta_9)\lambda_j^2 + \vartheta_3\vartheta_6}.$$

4. Boundary conditions

Impulsive line tractions on the surface (z = 0). We consider that the mechanical force is applied on the surface $(z = 0)$ i.e.

$$t_{33} = -F_1\psi_1(x)e^{i\omega t}, \tag{36}$$

$$t_{31} = -F_2\psi_2(x)e^{i\omega t}, \tag{37}$$

$$\frac{\partial T}{\partial z} + hT = 0\,, \tag{38}$$

where F_1, F_2 is the magnitude of the force applied on the boundary, $\psi_1(x)$ and $\psi_2(x)$ specify the vertical and horizontal traction distribution functions respectively along the x-axis, h is heat transfer coefficient and $h \to 0$ equation (38) corresponds to the insulated boundary and $h \to \infty$, equation (38) corresponds to the isothermal boundary.

Applying the Fourier transform defined by (28) on the boundary conditions (36)-(38), and using the value of t_{33} and t_{31} from equations (21)-(22) and using values of $\hat{u}$, $\hat{w}$ and $\hat{T}$ from equations (33)-(35) in the transformed equations (36)-(38), we find the solution of the problem by using Cramer's rule to obtain the values of A_j and again using these values of A_j,j=1,2,3 in equations (33)-(35) and equations (21)-(22) we obtain the components of displacement, temperature, and stress, in the transformed domain as

$$\hat{u} = \frac{F_1\hat{\psi}_1(\xi)}{\Gamma}\left[\sum_{j=1}^{3}\Gamma_{1j}e^{-\lambda_j z}\right]e^{i\omega t} + \frac{F_2\hat{\psi}_2(\xi)}{\Gamma}\left[\sum_{j=1}^{3}\Gamma_{2j}e^{-\lambda_j z}\right]e^{i\omega t}, \tag{39}$$

$$\hat{w} = \frac{F_1\hat{\psi}_1(\xi)}{\Gamma}\left[\sum_{j=1}^{3}d_j\Gamma_{1j}e^{-\lambda_j z}\right]e^{i\omega t} + \frac{F_2\hat{\psi}_2(\xi)}{\Gamma}\left[\sum_{j=1}^{3}d_j\Gamma_{2j}e^{-\lambda_j z}\right]e^{i\omega t}, \tag{40}$$

$$\hat{T} = \frac{F_1\hat{\psi}_1(\xi)}{\Gamma}\left[\sum_{j=1}^{3}l_j\Gamma_{1j}e^{-\lambda_j z}\right]e^{i\omega t} + \frac{F_2\hat{\psi}_2(\xi)}{\Gamma}\left[\sum_{j=1}^{3}l_j\Gamma_{2j}e^{-\lambda_j z}\right]e^{i\omega t}, \tag{41}$$

$$\widehat{t_{11}} = \frac{F_1\hat{\psi}_1(\xi)}{\Gamma}\left[\sum_{j=1}^{3} S_j\Gamma_{1j}e^{-\lambda_j z}\right]e^{i\omega t} + \frac{F_2\hat{\psi}_2(\xi)}{\Gamma}\left[\sum_{j=1}^{3} S_j\Gamma_{2j}e^{-\lambda_j z}\right]e^{i\omega t}, \tag{42}$$

$$\widehat{t_{13}} = \frac{F_1\hat{\psi}_1(\xi)}{\Gamma}\left[\sum_{j=1}^{3} N_j\Gamma_{1j}e^{-\lambda_j z}\right]e^{i\omega t} + \frac{F_2\hat{\psi}_2(\xi)}{\Gamma}\left[\sum_{j=1}^{3} N_j\Gamma_{2j}e^{-\lambda_j z}\right]e^{i\omega t}, \tag{43}$$

$$\widehat{t_{33}} = \frac{F_1\hat{\psi}_1(\xi)}{\Gamma}\left[\sum_{j=1}^{3} M_j\Gamma_{1j}e^{-\lambda_j z}\right]e^{i\omega t} + \frac{F_2\hat{\psi}_2(\xi)}{\Gamma}\left[\sum_{j=1}^{3} M_j\Gamma_{2j}e^{-\lambda_j z}\right]e^{i\omega t}, \tag{44}$$

where

$\Gamma_{11} = -N_2R_3 + R_2N_3,$

$\Gamma_{12} = N_1R_3 - R_1N_3,$

$\Gamma_{13} = -N_1R_2 + R_1N_2,$

$\Gamma_{21} = M_2R_3 - R_2M_3,$

$\Gamma_{22} = -M_1R_3 + R_1M_3,$

$\Gamma_{23} = M_1R_2 - R_1M_2,$

$\Gamma = -M_1\Gamma_{11} - M_2\Gamma_{12} - M_3\Gamma_{13},$

$N_j = -\delta_2\lambda_j + i\xi d_j,$

$M_j = i\xi - \delta_3 d_j\lambda_j - \frac{\beta_3}{\beta_1}l_j,$

$R_j = (-\lambda_j + h)l_j,$

$S_j = -i\xi - \delta_4 d_j\lambda_j - l_j.$

Concentrated normal force. We obtain the solution with concentrated normal force on the surface $(z = 0)$ by taking

$$\psi_1(x) = \delta(x), \psi_2(x) = \delta(x). \tag{45}$$

Applying Fourier transform defined by Eq. (28) on Eq. (45), we obtain

$$\hat{\psi}_1(\xi) = 1, \hat{\psi}_2(\xi) = 1. \tag{46}$$

Using eq. (46) in (39)-(44), the components of displacement, stress, and temperature change are obtained.

Uniformly distributed force. We obtain the solution with uniformly distributed force applied on the surface $(z = 0)$for the case of a uniform strip load of non-dimensional width 2m applied at the origin of coordinate system x = z = 0 by taking

$$\psi_1(x), \psi_2(x) = \begin{cases} 1 \text{ if } |\mathrm{x}| \le \mathrm{m} \\ 0 \text{ if } |\mathrm{x}| > m \end{cases} \tag{47}$$

The Fourier transforms of $\psi_1(x)$ and $\psi_2(x)$ with respect to the pair (x, ξ) in the dimensionless form after suppressing the primes becomes

$$\hat{\psi}_1(\xi) = \hat{\psi}_2(\xi) = \left\{\frac{2\sin(\xi m)}{\xi}\right\},\ \xi \neq 0. \tag{48}$$

Using (48) in (39)-(44), the components of displacement, stress, and temperature can be obtained.

Linearly distributed force. We obtain the solution with linearly distributed force applied on the surface $(z = 0)$having 2 m as the width of the strip load by taking

$$\{\psi_1(x), \psi_2(x)\} = \begin{cases} 1 - \frac{|x|}{m} \text{ if } |\mathrm{x}| \le \mathrm{m} \\ 0 \text{ if } |\mathrm{x}| > m \end{cases} \tag{49}$$

by using () and applying the transform defined by (20) on (41), we get

$$\hat{\psi}_1(\xi) = \hat{\psi}_2(\xi) = \left\{\frac{2\{1 - \cos(\xi m))}{\xi^2 m}\right\},\ \xi \neq 0 \tag{50}$$

by using () and applying the transform defined by (20) on (41), we get

$$\hat{\psi}_1(\xi) = \hat{\psi}_2(\xi) = \left\{\frac{2\{1-\cos(\xi m)\}}{\xi^2 m}\right\},\ \xi \neq 0. \tag{50}$$

Using (50) in (39)-(44), the components of displacement, stress, and temperature are obtained.

Thermoelastic interactions due to the thermal source. Thermal source is applied at surface $(z = 0)$, so the boundary conditions become

$$t_{33} = 0, \tag{51}$$

$$t_{31} = 0, \tag{52}$$

$$\frac{\partial T}{\partial z} + hT = F_3 \psi_1(x) e^{i\omega t}, \tag{53}$$

where, $\psi_1(x)$ specifies the source distribution function along z-axis, F_3 is the constant temperature applied on the boundary surface. If $h = 0$, Eq. (53) corresponds to temperature gradient boundary whereas h → ∞, Eq. (53) corresponds to the temperature input boundary. Applying the Fourier transform defined by (28) on the boundary conditions (51)-(53), and using the value of t_{33} and t_{31} from Eqs. (21)-(22) and using values of $\hat{u}$, $\hat{w}$ and $\hat{T}$ from Eqs. (33)-(35) in the transformed Eqs. (51)-(53), we find the solution of the problem by using Cramer's rule to obtain the values of A_j and again using these values of A_j,j=1,2,3 in Eqs. (33)-(35) and Eqs. (21)-(22) we obtain the components of displacement, temperature, and stress, in the transformed domain as

$$\hat{u} = \frac{\widehat{F_3\psi_1(\xi)}}{\Gamma}\left[\sum_{j=1}^{3} \Gamma_{2j} e^{-\lambda_j z}\right] e^{i\omega t}, \tag{54}$$

$$\hat{w} = \frac{F_3\hat{\psi}_1(\xi)}{\Gamma}\left[\sum_{j=1}^{3} d_i \Gamma_{2j} e^{-\lambda_j z}\right] e^{i\omega t}, \tag{55}$$

$$\hat{T} = \frac{F_3\hat{\psi}_1(\xi)}{\Gamma}\left[\sum_{j=1}^{3} l_j \Gamma_{2j} e^{-\lambda_j z}\right] e^{i\omega t}, \tag{56}$$

$$\widehat{t_{11}} = \frac{F_3\hat{\psi}_1(\xi)}{\Gamma}\left[\sum_{j=1}^{3} S_j \Gamma_{2j} e^{-\lambda_j z}\right] e^{i\omega t}, \tag{57}$$

$$\widehat{t_{13}} = \frac{F_3\hat{\psi}_1(\xi)}{\Gamma}\left[\sum_{j=1}^{3} N_j \Gamma_{2j} e^{-\lambda_j z}\right] e^{i\omega t}, \tag{58}$$

$$\widehat{t_{33}} = \frac{F_3\hat{\psi}_1(\xi)}{\Gamma}\left[\sum_{j=1}^{3} M_j \Gamma_{2j} e^{-\lambda_j z}\right] e^{i\omega t}, \tag{59}$$

where

$\Gamma_{11} = -N_2R_3 + R_2N_3,$
$\Gamma_{12} = N_1R_3 - R_1N_3,$
$\Gamma_{13} = -N_1R_2 + R_1N_2,$
$\Gamma_{21} = M_2R_3 - R_2M_3,$
$\Gamma_{22} = -M_1R_3 + R_1M_3,$
$\Gamma_{23} = M_1R_2 - R_1M_2,$
$\Gamma = -M_1\Gamma_{11} - M_2\Gamma_{12} - M_3\Gamma_{13},$
$N_j = -\delta_2\lambda_j + i\xi d_j$; j=1,2,3,
$M_j = i\xi - \delta_3 d_j \lambda_j - \frac{\beta_3}{\beta_1} l_j$; j=1,2,3,
$R_j = \left(-\lambda_j + h\right) l_j$; j=1,2,3,

$S_j = -i\xi - \delta_4 d_j \lambda_j - l_j$.

Concentrated thermal source. We obtained the solution with concentrated normal *thermal source* on the surface $(z = 0)$by taking

$$\psi_1(x) = \delta(x). \tag{60}$$

Applying Fourier transform defined by Eq. (28) on Eq. (60), we obtain

$$\hat{\psi}_1(\xi) = 1. \tag{61}$$

Using (53) in (54)-(59), the components of displacement, stress, and temperature are obtained.

Uniformly distributed thermal source. We obtained the solution with uniformly distributed *thermal source* applied to the surface $(z = 0)$for the case of a uniform strip load of non-dimensional width 2m applied at the origin of coordinate system x = z = 0 by taking

$$\psi_1(x) = \begin{cases} 1 \text{ if } |x| \le m \\ 0 \text{ if } |x| > m \end{cases} \tag{62}$$

The Fourier transforms of $\psi_1(x)$ with respect to the pair (x, ξ) in the dimensionless form after suppressing the primes becomes

$$\hat{\psi}_1(\xi) = \left\{\frac{2\sin(\xi m)}{\xi}\right\},\ \xi \neq 0. \tag{63}$$

Using (55) in (54)-(59), the components of displacement, stress, and temperature are obtained.

Linearly distributed thermal source. We obtained the solution with linearly distributed force applied on the surface $(z = 0)$having 2 m as the width of the strip load by taking

$$\{\psi_1(x),\} = \begin{cases} 1 - \dfrac{|x|}{m} \text{ if } |x| \le m \\ \quad 0 \text{ if } |x| > m \end{cases} \tag{64}$$

by using (24) and applying the transform defined by (28) on (56), we get

$$\hat{\psi}_1(\xi) == \left\{\frac{2\{1-\cos(\xi m))}{\xi^2 m}\right\},\ \xi \neq 0. \tag{65}$$

Using (64) in (54)-(59), the components of displacement, stress, and temperature are obtained.

5. Inversion of the transformation

For obtaining the expressions of displacement component *u*, normal displacement component *w*, temperature change T, stress components t_{11}, t_{13} and t_{33} given by Eqs. (39)-(44) and (54)-(59) in the physical domain, we invert the expressions using the formula

$$\tilde{f}(x,z,\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\xi x}\,\hat{f}(\xi,z,\omega)d\xi = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\cos(\xi x) f_e - i\sin(\xi x) f_o|\, d\xi, \tag{66}$$

where f_o is odd and f_e is the even parts of $\hat{f}(\xi,z,s)$ respectively, which is solved numerically by using the software MATLAB

6. Results and discussion

To demonstrate the theoretical results and effect of angular frequency, and relaxation time, the physical data for cobalt material, which is transversely isotropic, is taken from Dhaliwal & Singh [34] is given as $D_{11} = 3.07 \times 10^{11} Nm^{-2}$, $D_{33} = 3.581 \times 10^{11} Nm^{-2}$, $D_{13} = 1.027 \times 10^{10} Nm^{-2}$, $D_{44} = 1.510 \times 10^{11} Nm^{-2}$, $\beta_1 = 7.04 \times 10^{6} Nm^{-2} deg^{-1}$, $\beta_3 = 6.90 \times 10^{6} Nm^{-2} deg^{-1}$, $\rho = 8.836 \times 10^{3} Kgm^{-3}$, $C_E = 4.27 \times 10^{2} jKg^{-1} deg^{-1}$, $K_1 = 0.690 \times 10^{2} Wm^{-1} Kdeg^{-1}$, $K_3 = 0.690 \times 10^{2} Wm^{-1} K^{-1}$, $T_0 = 298$ K, $H_0 = 1 Jm^{-1} nb^{-1}$, $\varepsilon_0 = 8.838 \times 10^{-12} Fm^{-1}$, L = 1, Ω =0.5, $F_1 = 1, F_2 = 1$, $F_3 = 1$, $P = P_{11} = P_{33} = 0.1$.

Using the above values, the displacement component *u*, normal displacement component *w*, temperature change T, stress components t_{11}, t_{13} and t_{33} in Eqs. (39)-(44) and (54)-(59) are further obtained in the physical domain by using the software MATLAB and have been plotted graphically for transversely isotropic magneto-thermoelastic medium to study the effect of frequency of time-harmonic sources.

Case 1: Mechanical force and concentrated load with time-harmonic source frequency and rotation

Figure 1 to Figure 6 show the variations of the displacement components (*u* and *w)*, temperature T and stress components (t_{11}, t_{13} and t_{33}) for a transversely isotropic magneto-thermoelastic medium with mechanical force and concentrated load and with combined effects of relaxation time, rotation, time-harmonic source frequency in generalized thermoelasticity without energy dissipation respectively. The displacement component *u* illustrates the same pattern for $\omega = 0.25, 0.5$ and 0.75 but having different magnitudes and shows the opposite pattern for $= 1.0$. The displacement component *w* illustrates the same pattern with different magnitude temperature T sharply decreases for the initial range of x for all the values of for ω. Stress components (t_{11}, t_{13} and t_{33}) in Fig. 4 to Fig. 6 vary (increase or decrease) during the initial range of distance near the loading surface of the time-harmonic source and follow a small oscillatory pattern for the rest of the range of distance.

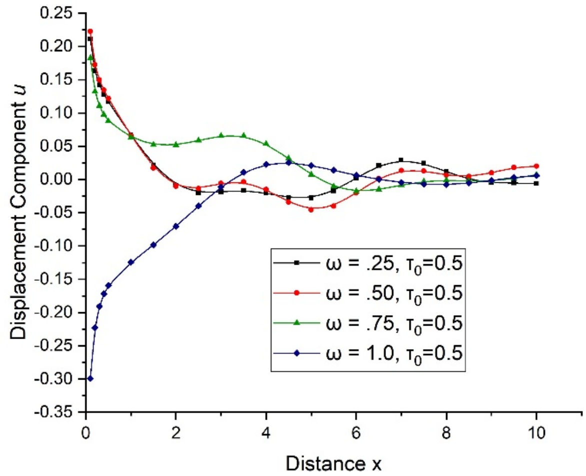

Fig. 1. Variations of displacement component u with distance x

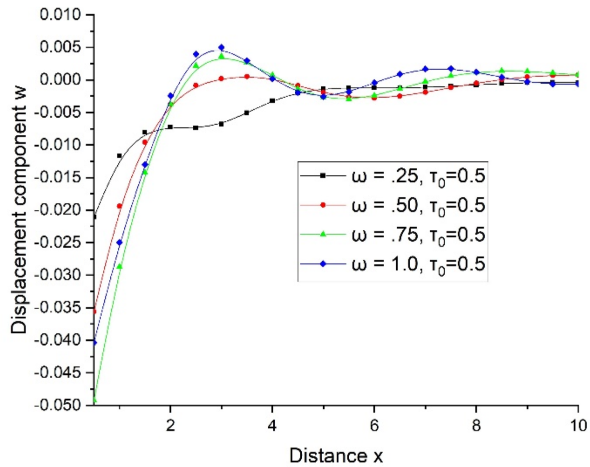

Fig. 2. Variations of displacement component w with distance x

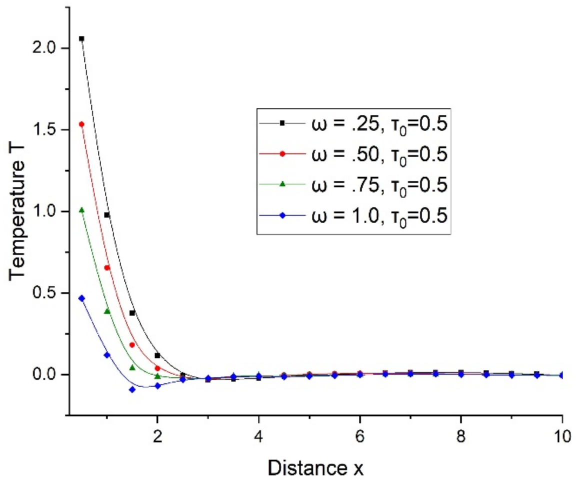

Fig. 3. Variations of temperature T with distance x

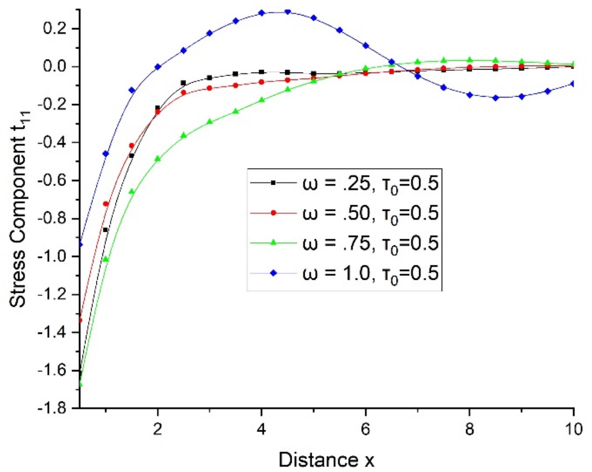

Fig. 4. Variations of stress component t_{11} with distance x

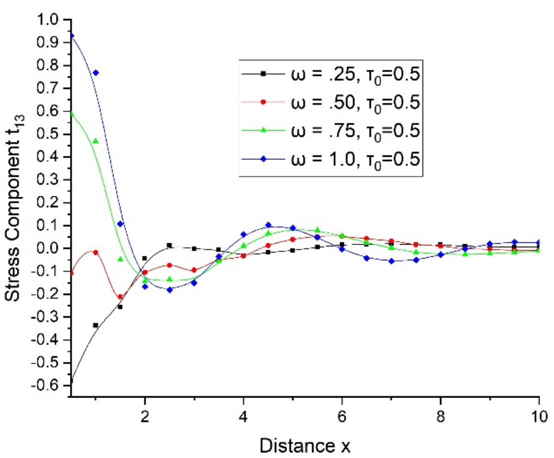

Fig. 5. Variations of the stress component t_{13} with distance x

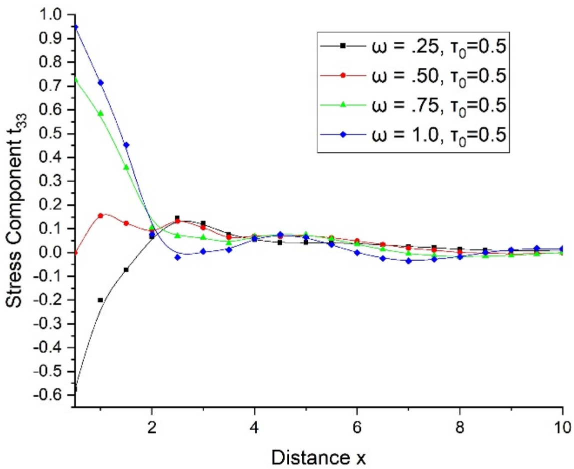

Fig. 6. Variations of the stress component t_{33} with distance x

Case II: Mechanical force and linearly distributed load with time-harmonic source frequency, rotation

Figure 7 to Figure 12 show the variations of the displacement components (*u* and *w)*, temperature T and stress components (t_{11}, t_{13} and t_{33}) for a transversely isotropic magneto-thermoelastic medium with mechanical force and linearly distributed load and with combined effects of relaxation time, rotation, time-harmonic source frequency in generalized thermoelasticity without energy dissipation respectively. The displacement components (*u* and *w)* and temperature T illustrate the variation in the initial range of distance and then shows the small oscillatory pattern. Stress components (t_{11}, t_{13} and t_{33}) in Fig. 10 to Fig. 12 vary (increase or decrease) during the initial range of distance near the loading surface of the time-harmonic source and follow a small oscillatory pattern for the rest of the range of distance. A small value of ω e shows more stress near the loading surface.

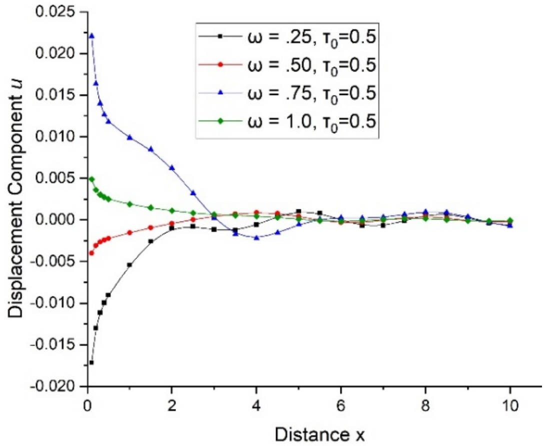

Fig. 7. Variations of displacement component u with distance x

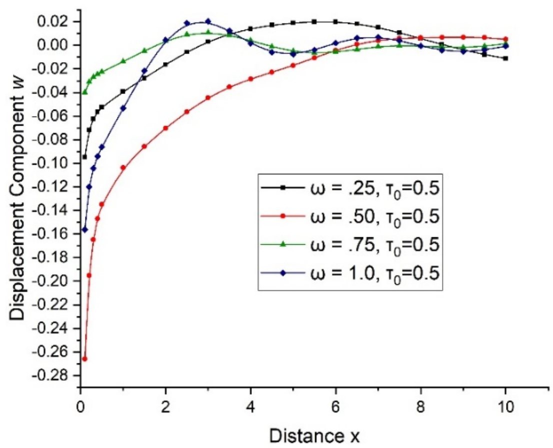

Fig. 8. Variations of displacement component w with distance x

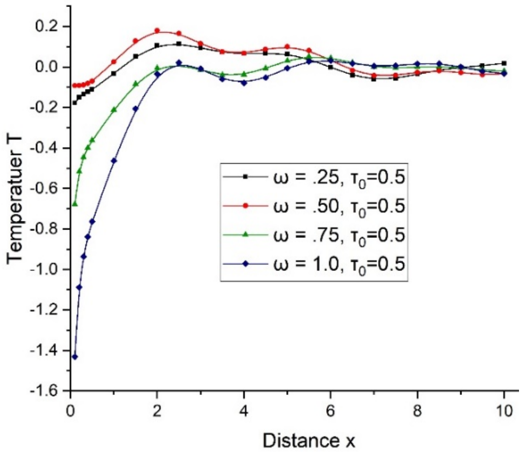

Fig. 9. Variations of temperature T with distance x

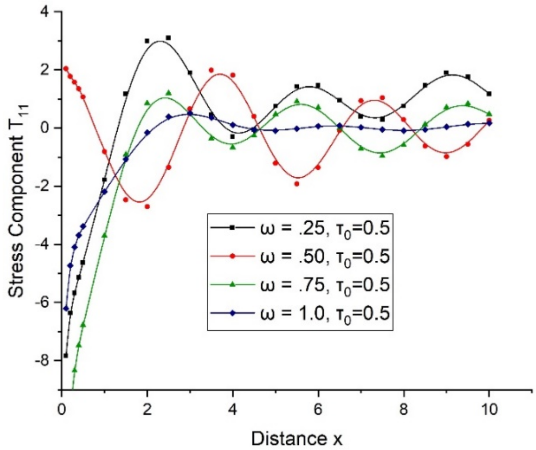

Fig. 10. Variations of stress component t_{11} with distance x

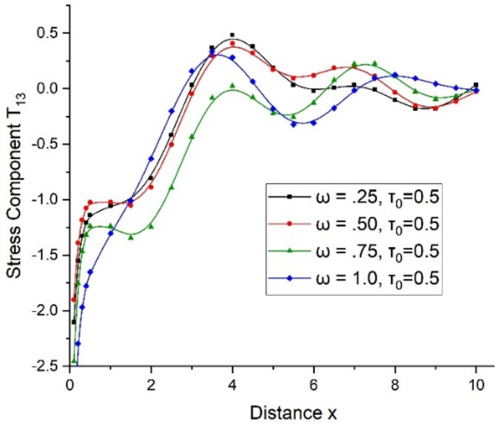

Fig. 11. Variations of the stress component t_{13} with distance x

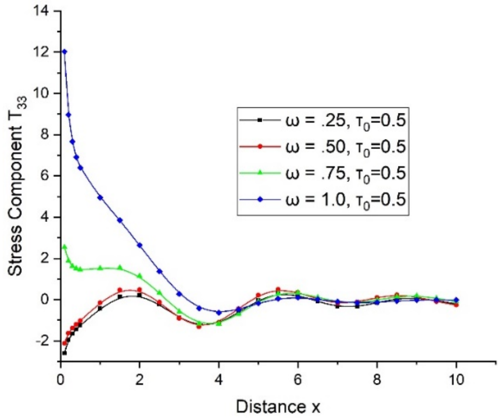

Fig. 12. Variations of the stress component t_{33} with distance x

Case III: Mechanical force and uniformly distributed load with time-harmonic source frequency and rotation

Figure 13 to Figure 18 show the variations of the displacement components (*u* and *w)*, temperature T and stress components (t_{11}, t_{13} and t_{33}) for a transversely isotropic magneto-thermoelastic medium with mechanical force and uniformly distributed load and with combined effects of relaxation time, rotation, time-harmonic source in generalized thermoelasticity without energy dissipation respectively. The displacement components (*u* and *w)*, temperature T and Stress components t_{13} illustrate the same pattern but having different magnitudes with frequency. Stress components (t_{11} and t_{33}) in Fig. 16 to Fig. 18 vary (increase or decrease) during the initial range of distance near the loading surface of the time-harmonic source and follow the small oscillatory pattern for the rest of the range of distance. Higher the value of ω higher the stress near the loading surface.

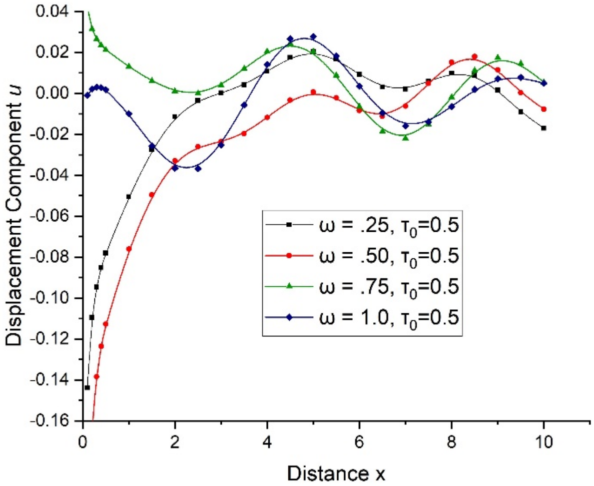

Fig. 13. Variations of displacement component u with distance x

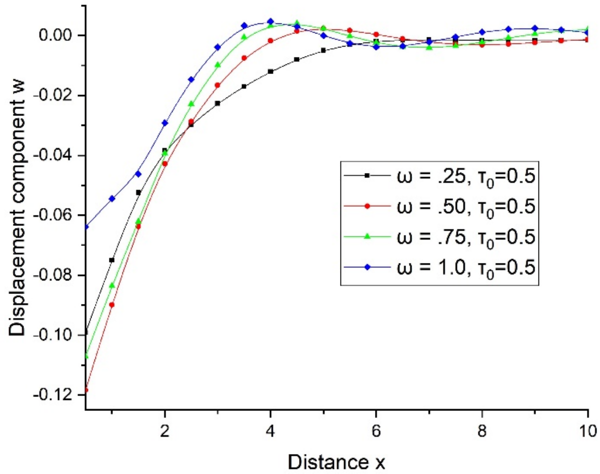

Fig. 14. variations of displacement component w with distance x

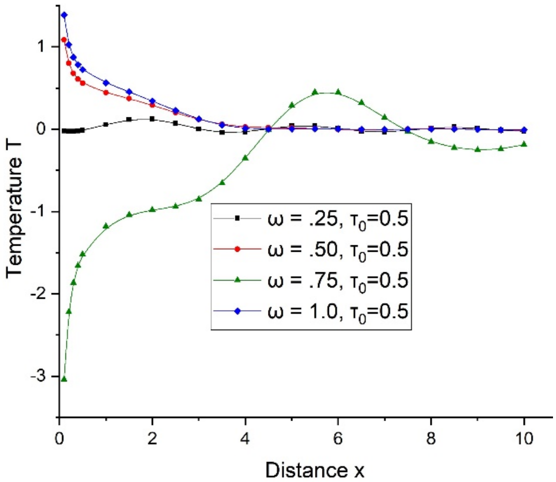

Fig. 15. Variations of temperature T with distance x

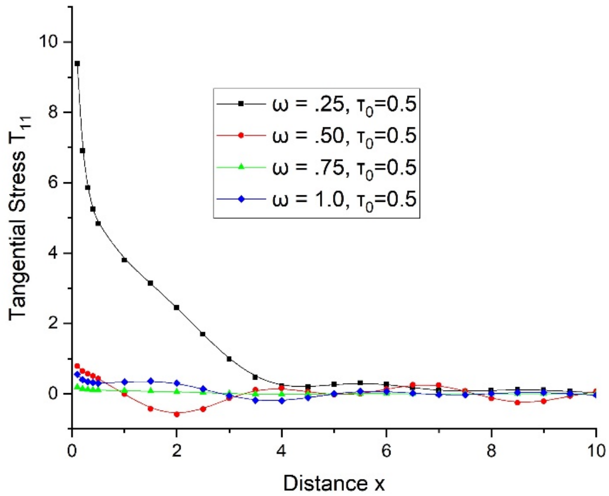

Fig. 16. Variations of stress component t_{11} with distance x

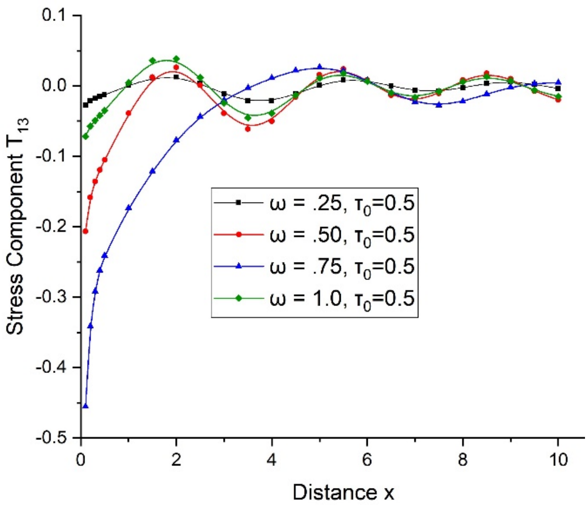

Fig. 17. Variations of the stress component t_{13} with distance x

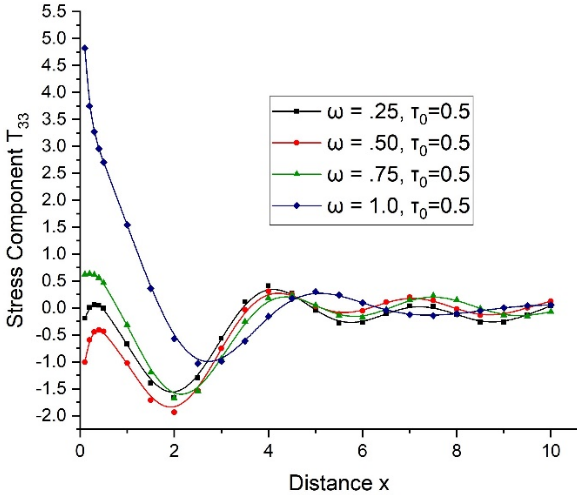

Fig. 18. Variations of the stress component t_{33} with distance x

Case IV: Thermal source and concentrated load with time-harmonic source frequency and rotation

Figure 19 to Figure 24 show the variations of the displacement components (u and w), temperature T and stress components (t_{11}, t_{13} and t_{33}) for a transversely isotropic magneto-thermoelastic medium with thermal source and concentrated load and with combined effects of relaxation time, rotation, a time-harmonic source in generalized thermoelasticity without energy dissipation respectively. The displacement components (u and w) and Stress component t_{13} illustrate the same pattern but having different magnitudes of frequency. Temperature T and Stress components (t_{11}, t_{33}) show the same behaviour for different values of ω.

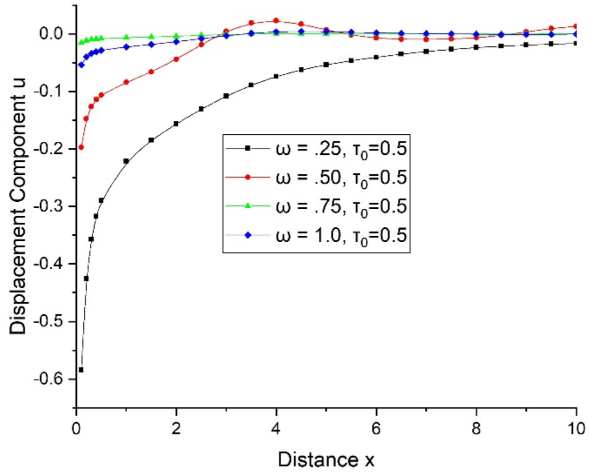

Fig. 19. Variations of displacement component u with distance x

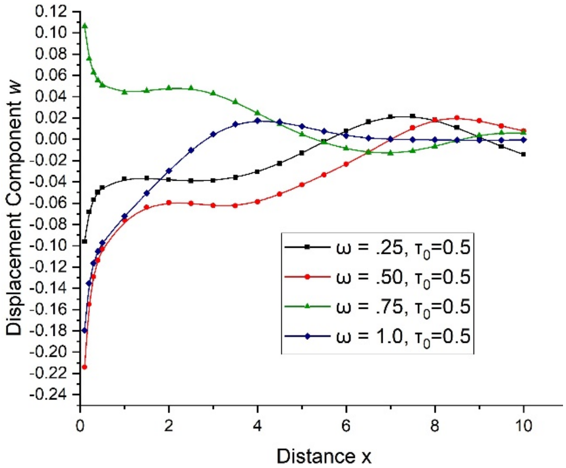

Fig. 20. Variations of displacement component w with distance x

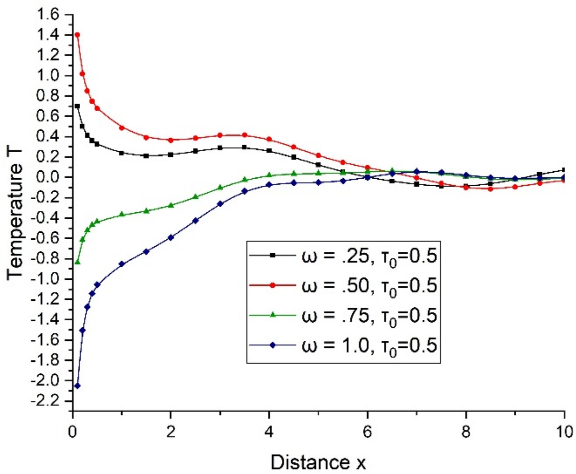

Fig. 21. Variations of temperature T with distance x

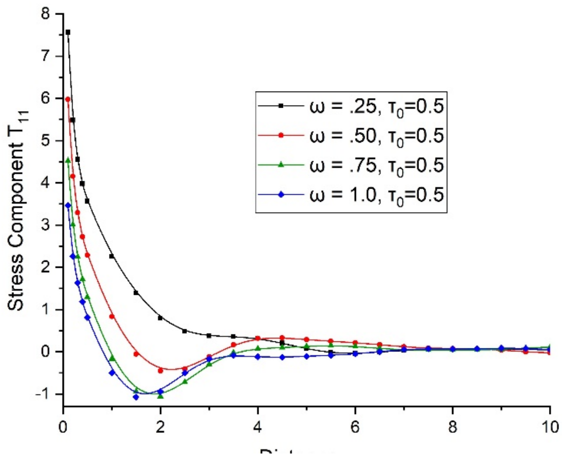

Fig. 22. Variations of stress component t_{11} with distance x

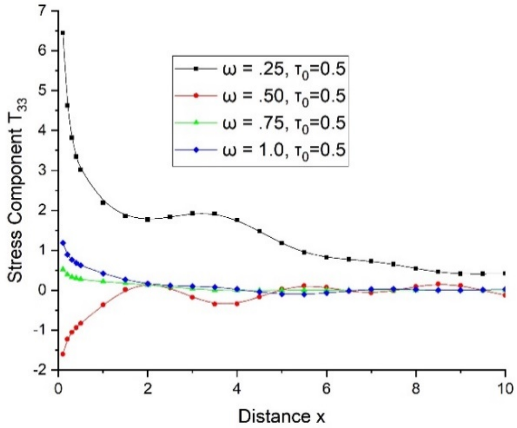

Fig. 23. Variations of the stress component t_{13} with distance x

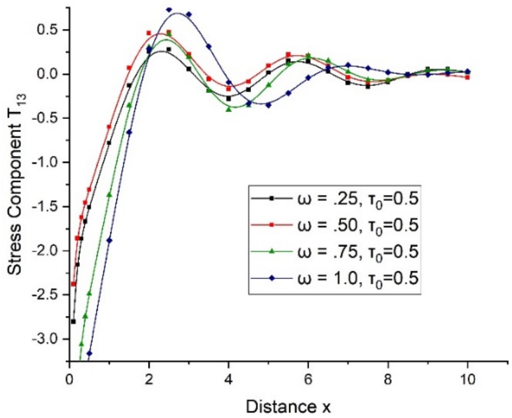

Fig. 24. Variations of the stress component t_{33} with distance x

Case V: Thermal Source and linearly distributed load with time-harmonic source frequency and rotation

Figure 25 to Figure 30 show the variations of the displacement components (*u* and *w)*, temperature T and stress components (t_{11}, t_{13} and t_{33}) for a transversely isotropic magneto-thermoelastic medium with thermal source and linearly distributed load and with combined effects of relaxation time, rotation, time-harmonic source in generalized thermoelasticity without energy dissipation respectively. The displacement components (*u* and *w)* illustrate the same pattern but having different magnitudes with frequency. Temperature T decreases during the initial range of distance near the loading surface of the time-harmonic source. Stress components (t_{11}, t_{13} and t_{33}) in Fig. 28 to Fig. 30 show the different behaviour with different frequencies.

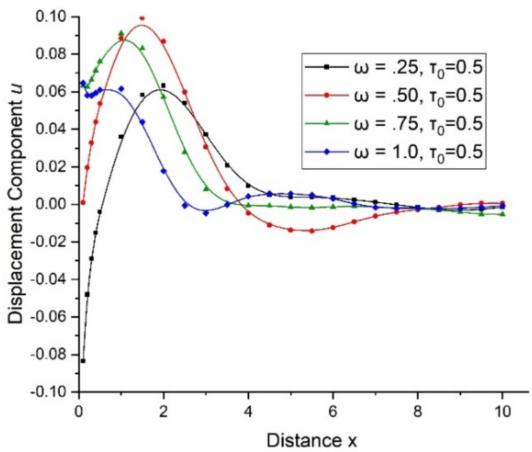

Fig. 25. Variations of displacement component u with distance x

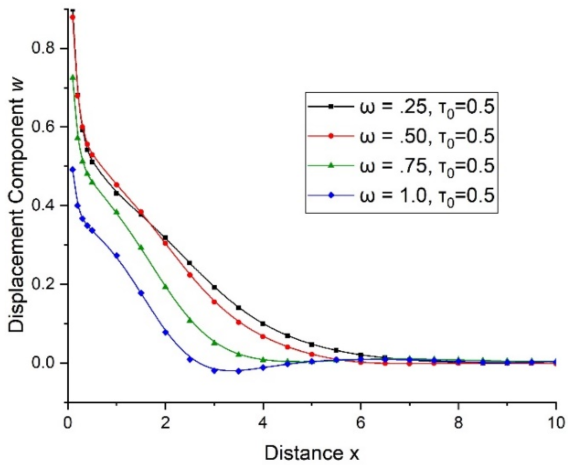

Fig. 26. Variations of displacement component w with distance x

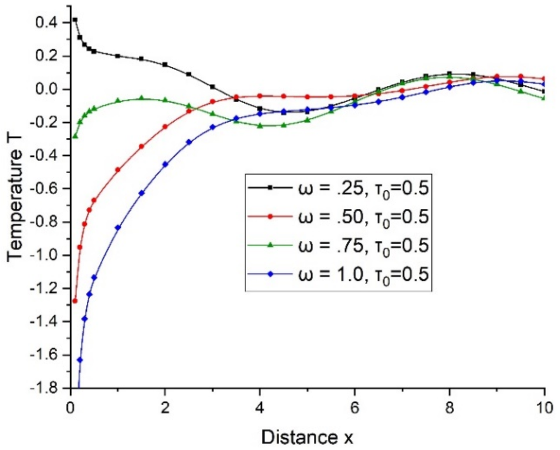

Fig. 27. Variations of temperature T with distance x

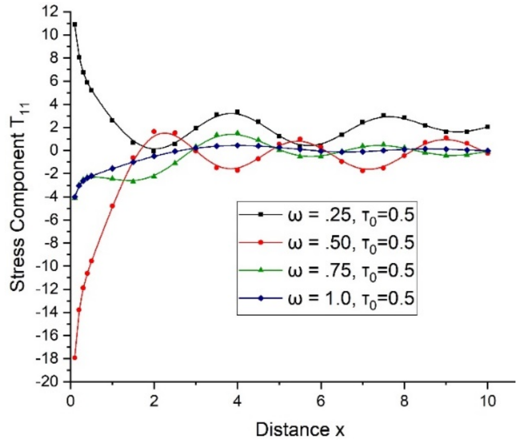

Fig. 28. Variations of stress component t_{11} with distance x

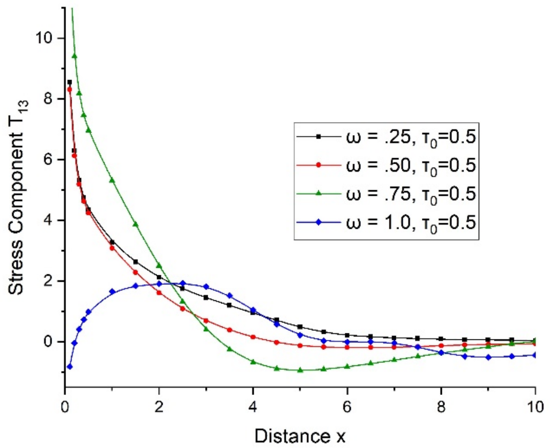

Fig. 29. Variations of the stress component t_{13} with distance x

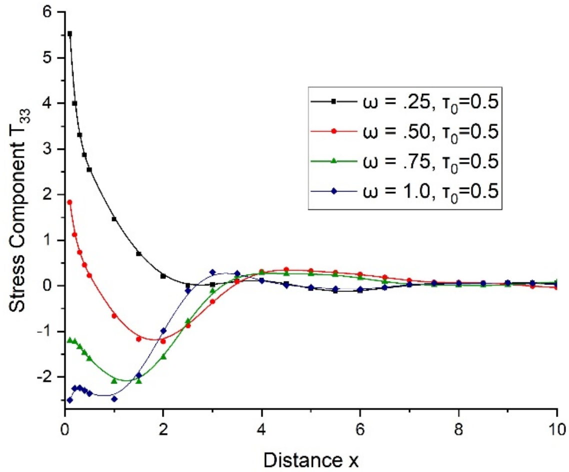

Fig. 30. Variations of the stress component t_{33} with distance x

Case VI: Thermal source and uniformly distributed load with time-harmonic source frequency and rotation

Figure 31 to Figure 36 show the variations of the displacement components (u and w), temperature $T$and stress components (t_{11}, t_{13} and t_{33}) for a transversely isotropic magneto-thermoelastic medium with thermal source and uniformly distributed load and with combined effects of relaxation time, rotation, time-harmonic source in generalized thermoelasticity without energy dissipation respectively. The displacement components (u and w) and temperature T illustrate the different patterns with all values of frequencies. Stress components (t_{11}, t_{13} and t_{33}) in Fig. 34 to Fig. 36 show the different patterns with different frequencies. Stress component shows the same oscillatory pattern for $\omega = 0.5, 0.75 \text{ } and \text{ } 1.0$ and the opposite pattern for $\omega = 0.25$.

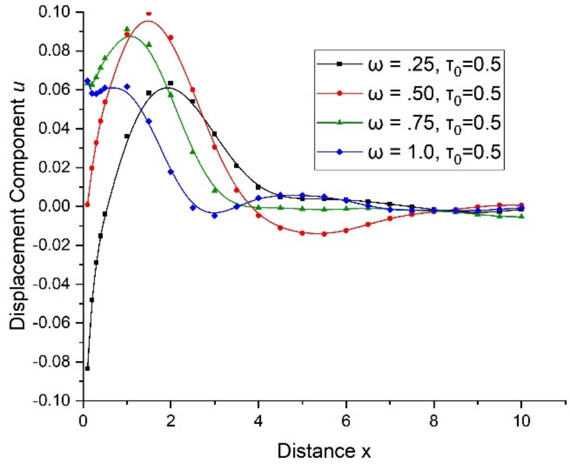

Fig. 31. Variations of displacement component u with distance x

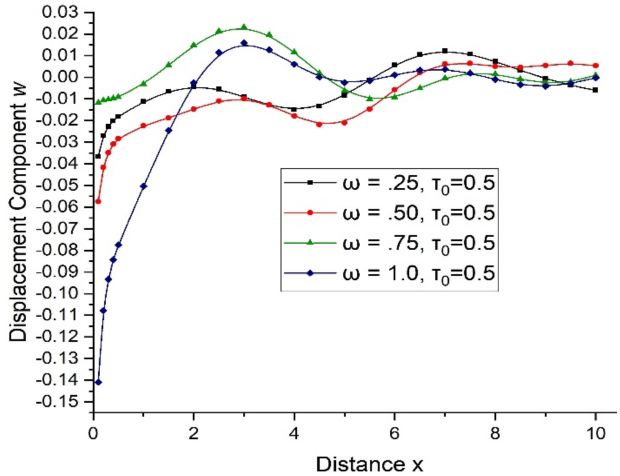

Fig. 32. Variations of displacement component w with distance x

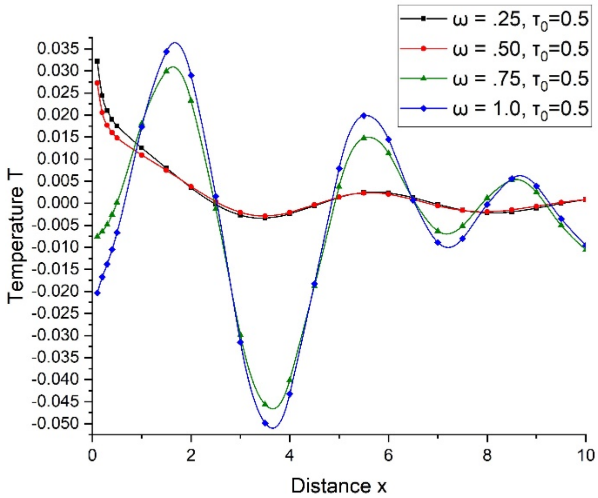

Fig. 33. Variations of temperature T with distance x

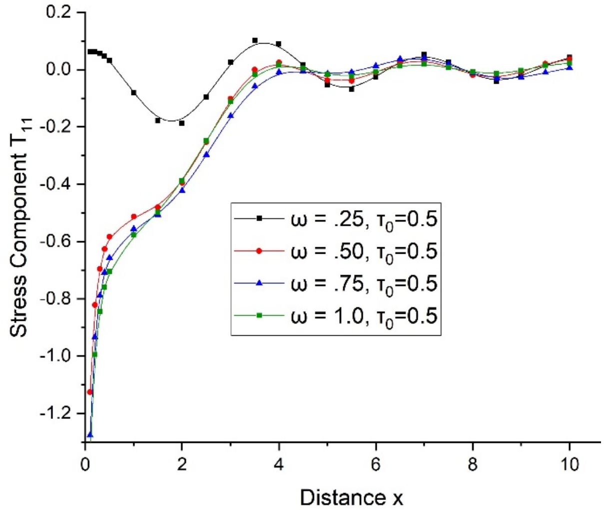

Fig. 34. Variations of stress component t_{11} with distance x

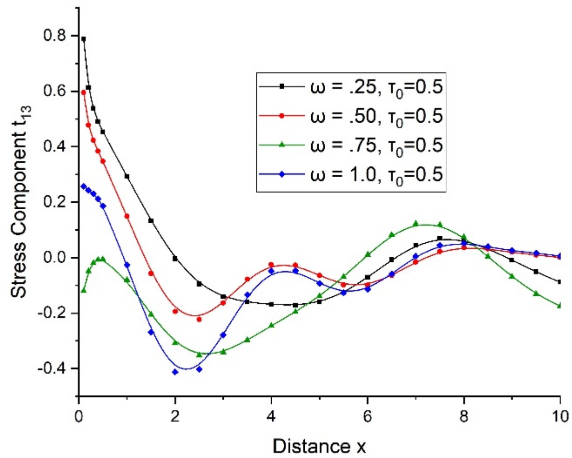

Fig. 35. Variations of the stress component t_{13} with distance x

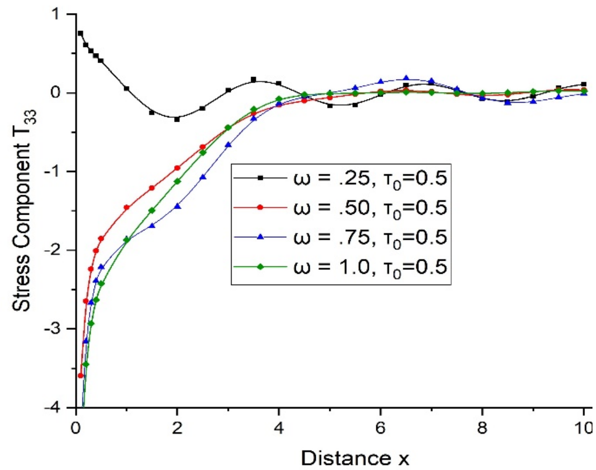

Fig. 36. Variations of the stress component t_{33} with distance x

7. Conclusions

From the above investigation, it is observed that frequency of time-harmonic source with LS-theory plays a key role in the oscillatory behaviour of the physical quantities both near as well as just as far from the source. The physical quantities differ with the change in angular frequency. The result gives the inspiration to study magneto-thermoelastic materials as an innovative domain of applicable thermoelastic solids. The shape of curves shows the impact of different angular frequencies and fixed relaxation time and rotation on the body and fulfills the purpose of the study. The outcomes of this research are extremely helpful in the dynamic response of time-harmonic sources in transversely isotropic magneto-thermoelastic medium with rotation which is beneficial to detect the deformation field such as geothermal engineering; advanced aircraft structure design, thermal power plants, composite engineering, geology, high-energy particle accelerators and in real life as in geophysics, auditory range, geomagnetism, etc. The proposed model in this research is relevant to different problems in thermoelasticity and thermodynamics.

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