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ON THE ISSUE OF ANALYTICAL DERIVATION OF STRESS STATE IN A CYLINDRICAL SHELL WITH A CIRCULAR HOLE UNDER AXIAL TENSION

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Abstract. The stress field in the cylindrical shell with the circular hole is considered. Thoroughly analyzing previous works we concluded that up to the present there are no explicit formulae useful for applications despite numerous works on this subject. Moreover, the classical analytical approach lacks a rigorous mathematical foundation. Its applicability is limited to cases that slightly differ from the plane Kirsch problem. The numerical results of various researches show a significant discrepancy. This paper proposes a new analytical approach based on a different form of representation for the fundamental system of solutions of governing equations. Since, in contrast to previous works, cutting of series is not required the boundary conditions are satisfied with a very high degree of accuracy. Our model works in all ranges that mechanics allows and there are no mathematical restrictions. Some numerical results are presented.

Keywords: cylindrical shell, cutouts, stress state, circular hole, elasticity theory

1. Introduction

Motivation. The purpose of this article is to develop a new approach to the analytical derivation of the stress state in a cylindrical shell with a circular hole under axial tension. The buckling problem, nonlocal fracture mechanics, and some other problems require a full stress field, but not just the values of stresses at the boundary.

The problems of determining the stress field in cylindrical shells with a hole for various loading conditions are considered in a number of recent works in connection with numerous engineering applications [1,2,11].

Analysis of numerous papers of 1940-70 as Lurie, Guz, Savin, Lekkerkerker, Eringen, and Naghdi, Murthy, Van Dyke [3-10] revealed that there are no explicit formulas that can be used for further investigations or for engineering applications. The problem was formulated by Lurie [3]. His followers Guz, Savin, Naghdi, Murthy [4,5,7,9], and some others tried to get an analytical solution but without proper mathematical justification. Besides that their approach is applicable in a very narrow range of parameters that will be discussed below in more detail. Some other researchers as Eringen, Lekkerkerker, and Van Dyke [8,6,10] used numerical procedures based on the collocation method. There were significant discrepancies in the results obtained in different papers and we estimate the results of Van Dyke [10] as more realistic since they are in good agreement with the results obtained in this paper.

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Therefore we assume that it is time to reconsider some approaches with more high accuracy in ideas and calculations.

2. Formulation of the Problem

Government equation. The cylindrical shell with a circular hole under axial tension (in $\boldsymbol{x}$-coordinate) is considered. A dimensionless parameter that takes into account the curvature of the circular cylinder is $\boldsymbol{\beta^2} = \frac{\boldsymbol{r_0^2\sqrt{3\,(1-\nu^2)}}}{\boldsymbol{4Rh}}$ [3]. The limit case $\boldsymbol{\beta \to 0}$ corresponds to the plane Kirsch problem. In classical works, in the frame of analytical approach, the ratio between the size of the hole and dimensions of the shell is considered to be $\boldsymbol{\beta \ll 1}$ while numerical approach allows us to consider $\boldsymbol{\beta > 1}$. E.g., in the paper of Van Dyke parameter $\boldsymbol{\beta}$ varies up to 4 [10].

The solution of this problem is represented as a composition of two solutions – perturbated and unperturbated. The first one is responsible for the problem with a hole and the second one – for axial tension applied along x-coordinate at the edge of the infinite cylinder that can be expressed through shell forces as

$$T_x = p, \quad T_{xy} = 0, \quad T_y = 0. \tag{1}$$

The system of equilibrium equations of the membrane shell for the search of the perturbated problem solution can be reduced to the equation [3,12]

$$\Delta\Delta\Phi + 8i\beta^2 \frac{\partial^2\Phi}{\partial x^2} = 0, \tag{2}$$

where $\Phi = \frac{Eh}{8\beta^2 R} w - iU$, w – displacement normal to the middle surface, U – stress function. The connection between shell forces in Cartesian coordinates and stress function is given by

$$\begin{pmatrix} T_x & T_{xy} \\ T_{xy} & T_y \end{pmatrix} = \begin{pmatrix} \frac{\partial^2 U}{\partial y^2} & -\frac{\partial^2 U}{\partial y \partial x} \\ -\frac{\partial^2 U}{\partial y \partial x} & \frac{\partial^2 U}{\partial x^2} \end{pmatrix}. \tag{3}$$

In order to split an operator into conjugate let us introduce substitution

$$\alpha = (1+i)\beta \quad => \quad \alpha^2 = 2i\beta^2 \quad => \quad 8i\beta^2 = -(2i\alpha)^2 \tag{4}$$

that follow us to the next equation:

$$\left(\Delta\Delta - (2i\alpha)^2 \frac{\partial^2}{\partial x^2}\right)\Phi = 0.$$

Since Δ and $\frac{\partial}{\partial x}$ commute, we get

$$\left(\Delta - 2i\alpha\frac{\partial}{\partial x}\right)\left(\Delta + 2i\alpha\frac{\partial}{\partial x}\right)\Phi = 0. \tag{5}$$

It should be noted that operators $L_1 = \left(\Delta - 2i\alpha\frac{\partial}{\partial x}\right)$ and $L_2 = \left(\Delta + 2i\alpha\frac{\partial}{\partial x}\right)$ are commutative:

$$L_1L_2 = L_2L_1, \quad \mathrm{Ker}L_1 + \mathrm{Ker}L_2 \subseteq \mathrm{Ker}L_1L_2\,.$$

In this problem $\mathrm{Ker}L_1 + \mathrm{Ker}L_2$ are being found which means that some solutions of equation (5) can be missed. However, obtained solutions are enough to satisfy boundary conditions.

Function Φ can be introduced in form $\Phi = e^{i\alpha x}\cdot\Psi$. Since the symmetry of equation (5) relative to transform $\alpha \leftrightarrow (-\alpha)$ the function $\Phi = \mathrm{e}^{-i\alpha x}\cdot\Psi$ is also the solution. Hence equation (5) can be rewritten in the following form

$$e^{i\alpha x}\cdot(\Delta\Psi - (i\alpha)^2\Psi) = 0 \quad => \quad \Delta\Psi + \alpha^2\,\Psi = 0. \tag{6}$$

Polarizing and separating variables in (6) $\Psi(x,y) = \Psi(r,\vartheta) = z(r)\cdot f(\vartheta)$ we obtain

$$\frac{r^2\left[z''(r)+\frac{1}{r}\cdot z'(r)+\alpha^2\cdot z(r)\right]}{z(r)} = -\frac{f''(\vartheta)}{f(\vartheta)} = C = +n^2. \tag{7}$$

Due to 2π-periodicity of the solution regarding ϑ we get the condition $C = +n^2$ that brings us integers n:

$$f'' + Cf = 0, \qquad C = n^2, \qquad f_1 = \cos\sqrt{C}\vartheta, \qquad f_2 = \sin\sqrt{C}\vartheta.$$

Considering the left part of the equation (7) regarding r we got Bessel equation

$$r^2\cdot z''(r) + r\cdot z'(r) + (\alpha^2 r^2 - n^2)\cdot z(r) = 0, \tag{8}$$

that after substitutes $z(r) = \tilde{z}(\alpha r)$ and $t = \alpha r$ appears in its classical form:

$$\tilde{z}''(t) + \frac{1}{t}\cdot\tilde{z}'(t) + \left(1 - \frac{n^2}{t^2}\right)\cdot\tilde{z}(t) = 0. \tag{9}$$

The fundamental system of solutions for the equation (9) is expressed through Hankel's functions that are linear combinations of Bessel and Neiman functions:

$$H_n^{(1)}(t) = J_n(t) + iY_n(t),\ H_n^{(2)}(t) = J_n(t) - iY_n(t). \tag{10}$$

The choice of Hankel functions here is due to the fact that they are the only functions from Bessel functions class that tends to zero with an unlimited increase of their complex argument modulus

$$\lim_{r\to+\infty} H_n^{(1)}\left(r\,e^{i\vartheta}\right) = \lim_{r\to+\infty} H_n^{(2)}\left(r\,e^{-i\vartheta}\right) = 0\,, \quad \vartheta \in [\varepsilon;\ \pi - \varepsilon]. \tag{11}$$

Since $\alpha = (1+i)\beta$ corresponds to the value $\vartheta = \frac{\pi}{4} \in [0;\pi]$ we should use only $H_n^{(1)}(t)$ function.

Thus similar to the paper [4] the solution of equation (5) can be written as

$$\Phi = \Phi_1 + \Phi_2 \tag{12}$$

$$\Phi_1 = e^{i\alpha x}\cdot\Psi = e^{i\alpha x}\sum_{n=-\infty}^{+\infty} C_n\cdot e^{in\vartheta}\cdot H_n^{(1)}(\alpha r) = \left[C_n = i^{-n}a_n^{(1)}\right] =$$

$$= e^{i\alpha x}\sum_{n=-\infty}^{+\infty} i^{-n}\cdot e^{in\vartheta}\cdot H_n^{(1)}(\alpha r)\cdot a_n^{(1)},$$

$$\Phi_2 = e^{-i\alpha x}\cdot\Psi = e^{-i\alpha x}\sum_{n=-\infty}^{+\infty} i^{n}\cdot e^{in\vartheta}\cdot H_n^{(1)}(\alpha r)\cdot a_n^{(2)}.$$

Due to the symmetry of the problem, it is convenient to change the basis of linear independent solutions

$$\begin{pmatrix} e^{i\alpha x}\cdot H_n^{(1)}(\alpha r)\cdot\cos(n\vartheta) \\ e^{i\alpha x}\cdot H_n^{(1)}(\alpha r)\cdot\sin(n\vartheta) \\ e^{-i\alpha x}\cdot H_n^{(1)}(\alpha r)\cdot\cos(n\vartheta) \\ e^{-i\alpha x}\cdot H_n^{(1)}(\alpha r)\cdot\sin(n\vartheta) \end{pmatrix} = \begin{pmatrix} 1 & 0 & i & 0 \\ 0 & 1 & 0 & i \\ 1 & 0 & -i & 0 \\ 0 & 1 & 0 & -i \end{pmatrix}\begin{pmatrix} \cos(\alpha x)\cdot H_n^{(1)}(\alpha r)\cdot\cos(n\vartheta) \\ \cos(\alpha x)\cdot H_n^{(1)}(\alpha r)\cdot\sin(n\vartheta) \\ \sin(\alpha x)\cdot H_n^{(1)}(\alpha r)\cdot\cos(n\vartheta) \\ \sin(\alpha x)\cdot H_n^{(1)}(\alpha r)\cdot\sin(n\vartheta) \end{pmatrix}$$

where only $\cos(\alpha x)\cdot H_n^{(1)}(\alpha r)\cdot\cos(n\vartheta)$ with even n и $\sin(\alpha x)\cdot H_n^{(1)}(\alpha r)\cdot\cos(n\vartheta)$ with odd n are remained. Thus the form of solution for Φ is obtained in the following form

$$\Phi = -i\frac{py^2}{2} + \sum_{n=0}^{\infty}(A_n + iB_n)\cdot\begin{bmatrix}\cos(\alpha x)\cdot H_n^{(1)}(\alpha r)\cdot\cos(n\vartheta) \\ \sin(\alpha x)\cdot H_n^{(1)}(\alpha r)\cdot\cos(n\vartheta)\end{bmatrix}, \tag{13}$$

$$\alpha x = (1+i)\beta\cdot r\cos\vartheta,$$

term $-i\frac{py^2}{2}$ added from unperturbated solution and satisfies the boundary condition

$$T_x = -\frac{\partial^2 U}{\partial y^2} = p.$$

Lurie and his followers [3,4,7,9] used the expansion of basic functions $\cos(\alpha x)\cdot H_n^{(1)}(\alpha r)\cdot\cos(n\vartheta)$ and $\sin(\alpha x)\cdot H_n^{(1)}(\alpha r)\cdot\cos(n\vartheta)$ into power series in βr. The attempts to determine coefficients A_n and B_n were based on the expansions

$$A_n=\sum_{k=0}^{\infty}a_n^k\,\beta^{2k},\; B_n=\sum_{k=0}^{\infty}b_n^k\,\beta^{2k} \tag{14}$$

using stress-free boundary conditions

$$\begin{cases} T_{rr}|_{r=r_0}=0\\ T_{r\vartheta}|_{r=r_0}=0\\ M_r|_{r=r_0}=0\\ Q_r|_{r=r_0}=0. \end{cases} \tag{15}$$

It should be mentioned that a reasonable algorithm had not been proposed. Eringen and Naghdi [7] continued this approach with correct boundary conditions; the work of Murthy is an extension of Lurie's ideas for an elliptic case [9].

The other approach to the determination of coefficients in formulae (13-14) used the collocation procedure. Lekkerkerker, Eringen and Naghdi, Van Dyke [6,8,10] got some numerical results.

Classical approaches discussion. The idea of the expansion by small parameter $\boldsymbol{\beta}$ leads to the representation of the solution through a linear combination of Kirsch's solution and terms depending on $\boldsymbol{\beta}$:

$$\sigma=\sigma_0+\beta^2\sigma_2+\beta^4\sigma_4+\cdots \tag{16}$$

where σ_0 – Kirsch stresses. The authors of this paper have repeated this way and have got the explicit second term σ_2 in polar coordinates as final formulae for stress field are not yet prescribed anywhere:

$$\begin{aligned} \sigma_{2_{rr}}&=\frac{p\pi}{8}\left(1-\frac{4}{r^2}+\frac{3}{r^4}\right)\cos(2\vartheta),\\ \sigma_{2_{r\vartheta}}&=-\frac{p\pi}{8}\left(1+\frac{2}{r^2}-\frac{3}{r^4}\right)\sin(2\vartheta),\\ \sigma_{2_{\vartheta\vartheta}}&=-\frac{p\pi}{8}\left(1+\frac{3}{r^4}\right)\cos(2\vartheta). \end{aligned} \tag{17}$$

The term $\beta^4\sigma_4$ and following terms contain Poison's coefficient and are very heavy for prescribing however for small β they do not improve the solution as it was previously expected (see Fig. 1).

For small β just a very small difference with the solution of the Kirsch problem is observed. For bigger β the analytical theory based on expansion in a small parameter is not valid hence some other methods like the collocation procedure should be used. In the works of Lekkerkerker, Eringen, Naghdi, Van Dyke [6,8,10] numerical solutions were presented in form of tables. However, there were contradictions between the results presented in different papers of the same authors.

The general feature of this approach is an expansion of expressions that contain the Hankel function in parameter β. Since the argument of Hankel functions also contains parameter β it leads to a significant loss of accuracy. So it is more reasonable to calculate values of these functions at certain points.

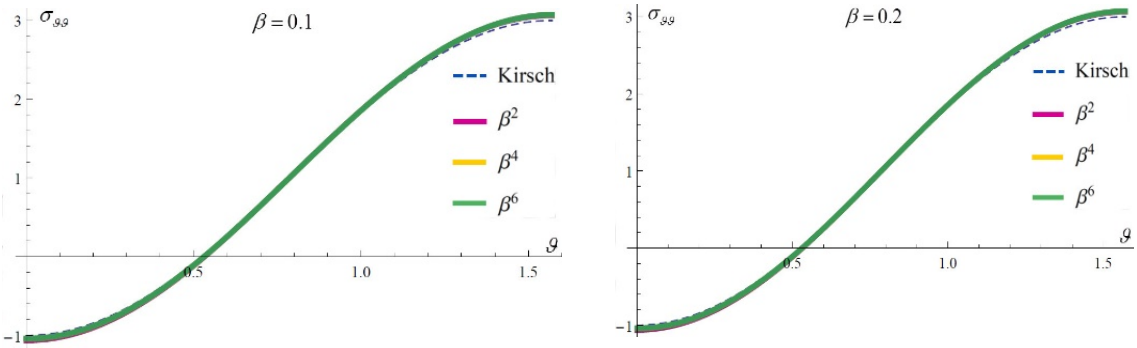

Fig. 1. Membrane stress $\sigma_{\vartheta\vartheta}$ depending on ϑ on the boarder up to $\beta = 0.1$ (a) and $\beta = 0.2$ (b)

It is also worth mentioning that normal displacement which is given by the formula

$$w = \left(\frac{Eh}{8\beta^2 R}\right)^{-1} \text{Re}\,\Phi$$

in papers [3,9] does not vanish with $\beta \to 0$ as one would expect for the plane problem. This contradiction is eliminated by adding some constant to the solution [3,9].

3. Our Approach

We propose to consider the fundamental system of solutions of equation (2) in the following form

$$\begin{bmatrix} \cos(\alpha x) \cdot H_n^{(1)}(\alpha r) \cdot \cos(n\vartheta) \\ \sin(\alpha x) \cdot H_n^{(1)}(\alpha r) \cdot \cos(n\vartheta) \end{bmatrix} = \\ = (-1)^{\left[\frac{n}{2}\right]} \cdot \frac{H_n^{(1)}((1+i)\beta r)}{H_n^{(1)}((1+i)\beta)} \cdot \Big[J_n\big((1+i)\beta r\big) + \\ + \sum_{l=1}^{\infty} (-1)^l \cdot \Big(J_{n+2l}\big((1+i)\beta r\big) + J_{n-2l}\big((1+i)\beta r\big)\Big) \cdot \cos 2l\vartheta\Big]. \quad (18)$$

The full derivation of this formula demands special attention and is published in a separate paper [13].

Thus the general solution is presented as

$$\Phi = -i\frac{py^2}{2} + \sum_{n=0}^{\infty} (a_n + ib_n) \cdot f_n, \quad (19)$$

where f_n is a right-hand side of the equation (18), a_n, b_n – unknown coefficients that should be found by using boundary conditions. There is no need to expand these coefficients into series.

The advantage of this representation is in the separation of variables. It should be noted that in this case there are no functions that depend on both ϑ and β. It is easy to see that Bessel and Hankel's functions depend only on βr while cosine depends only on ϑ. The validity of the equality (18) is proved in a separate paper. The combination of Bessel functions with different indexes in this expansion has a deep mechanical sense since the frequency and amplitude of perturbed solution are observable.

One more advantage of this approach is the more convenient representation of boundary conditions. Since no cutting of series is needed we get high accuracy for the satisfaction of boundary conditions and a wide range of parameter β can be considered.

Let us introduce

$$g_{n,l} = (-1)^{\left[\frac{n}{2}\right]+l} \cdot \frac{H_n^{(1)}\big((1+i)\beta r\big)}{H_n^{(1)}\big((1+i)\beta\big)} \cdot \Big(J_{n+2l}\big((1+i)\beta r\big) + J_{n-2l}\big((1+i)\beta r\big)\Big)$$

which are trigonometric coefficients of Fourier in (18).

Expansion into Fourier series in ϑ of four boundary conditions gives us the next algebraic system.

Table 1. First boundary condition $T_{rr}|_{r=r_0} = 0$

cos 0	$\frac{p}{2} - \mathrm{Im}\sum_{n=0}^{\infty}(a_n + ib_n)\cdot\frac{g'(n,0)}{2}$
cos 2ϑ	$\frac{p}{2} + \mathrm{Im}\sum_{n=0}^{\infty}(a_n + ib_n)\cdot\big(4g(n,1) - g'(n,1)\big)$
cos 4ϑ	$\mathrm{Im}\sum_{n=0}^{\infty}(a_n + ib_n)\cdot\big(16g(n,2) - g'(n,2)\big)$
cos 6ϑ	$\mathrm{Im}\sum_{n=0}^{\infty}(a_n + ib_n)\cdot\big(36g(n,3) - g'(n,3)\big)$
…	…
cos 2lϑ	$\mathrm{Im}\sum_{n=0}^{\infty}(a_n + ib_n)\cdot\big(4l^2g(n,l) - g'(n,l)\big)$

Table 2. Second boundary condition $T_{r\vartheta}|_{r=r_0} = 0$

sin 2ϑ	$-\frac{p}{2} - \mathrm{Im}\sum_{n=0}^{\infty}2(a_n + ib_n)\cdot\big(g'(n,1) - g(n,1)\big)$
sin 4ϑ	$-\mathrm{Im}\sum_{n=0}^{\infty}4(a_n + ib_n)\cdot\big(g'(n,2) - g(n,2)\big)$
***sin* 6ϑ**	$-\mathrm{Im}\sum_{n=0}^{\infty}6(a_n + ib_n)\cdot\big(g'(n,3) - g(n,3)\big)$
…	…
sin 2lϑ	$-\mathrm{Im}\sum_{n=0}^{\infty}2l(a_n + ib_n)\cdot\big(g'(n,l) - g(n,l)\big)$

Table 3. Third boundary condition $M_r|_{r=r_0} = 0$

cos 0	$\mathrm{Re}\sum_{n=0}^{\infty}(a_n + ib_n)\cdot\left(\frac{\nu g'(n,0) + g''(n,0)}{2}\right)$
cos 2ϑ	$\mathrm{Re}\sum_{n=0}^{\infty}(a_n + ib_n)\cdot\big(-4\nu g(n,1) + \nu g'(n,1) + g''(n,1)\big)$
cos 4ϑ	$\mathrm{Re}\sum_{n=0}^{\infty}(a_n + ib_n)\cdot\big(-16\nu g(n,2) + \nu g'(n,2) + g''(n,2)\big)$
cos 6ϑ	$\mathrm{Re}\sum_{n=0}^{\infty}(a_n + ib_n)\cdot\big(-36\nu g(n,3) + \nu g'(n,3) + g''(n,3)\big)$
…	…

cos 2lϑ	$\mathrm{Re}\sum_{n=0}^{\infty}(a_n+ib_n)\cdot\left(-4l^2\nu g(n,l)+\nu g'(n,l)+g''(n,l)\right)$

Table 4. Forth boundary condition $Q_r|_{r=r_0}=0$

cos 0	$\mathrm{Re}\sum_{n=0}^{\infty}(a_n+ib_n)\cdot\left(\frac{-g'(n,0)+g''(n,0)+g'''(n,0)}{2}\right)$
cos 2ϑ	$\mathrm{Re}\sum_{n=0}^{\infty}(a_n+ib_n)\cdot(4(3-\nu)g(n,1)-(9-4\nu)g'(n,1)+g''(n,1)+g'''(n,1))$
cos 4ϑ	$\mathrm{Re}\sum_{n=0}^{\infty}(a_n+ib_n)\cdot(16(3-\nu)g(n,2)-(33-16\nu)g'(n,2)+g''(n,2)+g'''(n,2))$
cos 6ϑ	$\mathrm{Re}\sum_{n=0}^{\infty}(a_n+ib_n)\cdot(36(3-\nu)g(n,3)-(73-36\nu)g'(n,3)+g''(n,3)+g'''(n,3))$
…	**…**
cos 2lϑ	$\mathrm{Re}\sum_{n=0}^{\infty}(a_n+ib_n)\cdot\left(4l^2(3-\nu)\cdot g(n,l)-\left(1+4l^2(2-\nu)\right)g'(n,l)+g''(n,l)+g'''(n,l)\right)$

For convenience in this paper, we take $r_0=1$. Thus we got a system that has a block structure and all elements can be found easily. A more detailed analysis of this system and technique of founding coefficients is written in a separate paper [13].

4. Results and Discussion

Stresses can be found using formulae (3) and (19) with coefficients a_n, b_n that are determined as a solution of the linear system presented in Tables 1-4. According to [4] from the point of view of mechanics, such model works for small and middle cutouts for $0\le\beta\le 3{,}5-4{,}5$. This paper shows the first analytical approach that allows getting results for all acceptable range of parameter β. Membrane circumferential stresses obtained for various curvature parameters are presented below. The reason to choose the value of parameter $\beta=0.3/\sqrt{2}$ (Fig. 1) is determined by comparison with the results obtained in the work of Eringen and Naghdi [8]. Their data have a significant difference from those of Van Dyke [10] with whom we have almost 100 % agreements in the results (Fig. 5).

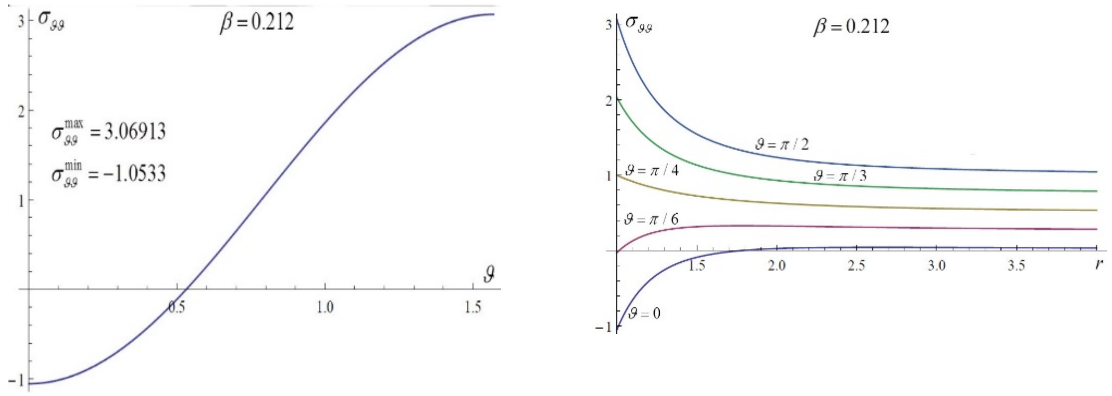

Fig. 2. Membrane stress $\sigma_{\vartheta\vartheta}$ for $\beta = \frac{0.3}{\sqrt{2}} \approx 0.212$ and $\nu = 0.3$

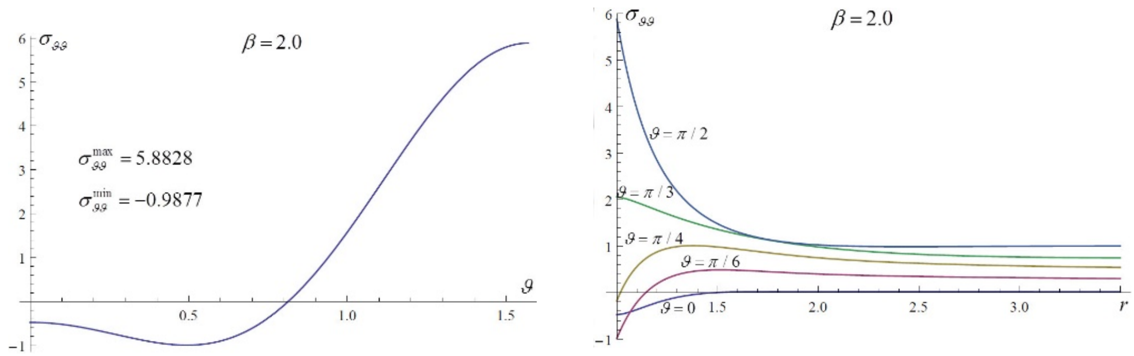

Fig. 3. Membrane stress $\sigma_{\vartheta\vartheta}$ for $\beta = 2.0$ and $\nu = 0.3$

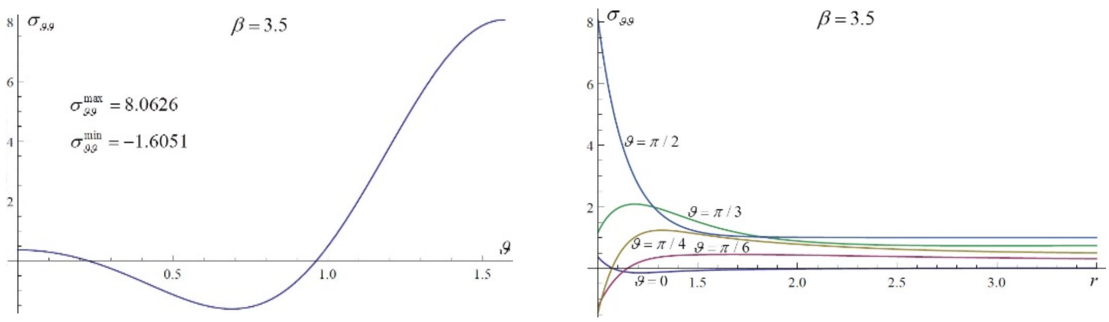

Fig. 4. Membrane stress $\sigma_{\vartheta\vartheta}$ for $\beta = 3.5$ and $\nu = 0.3$

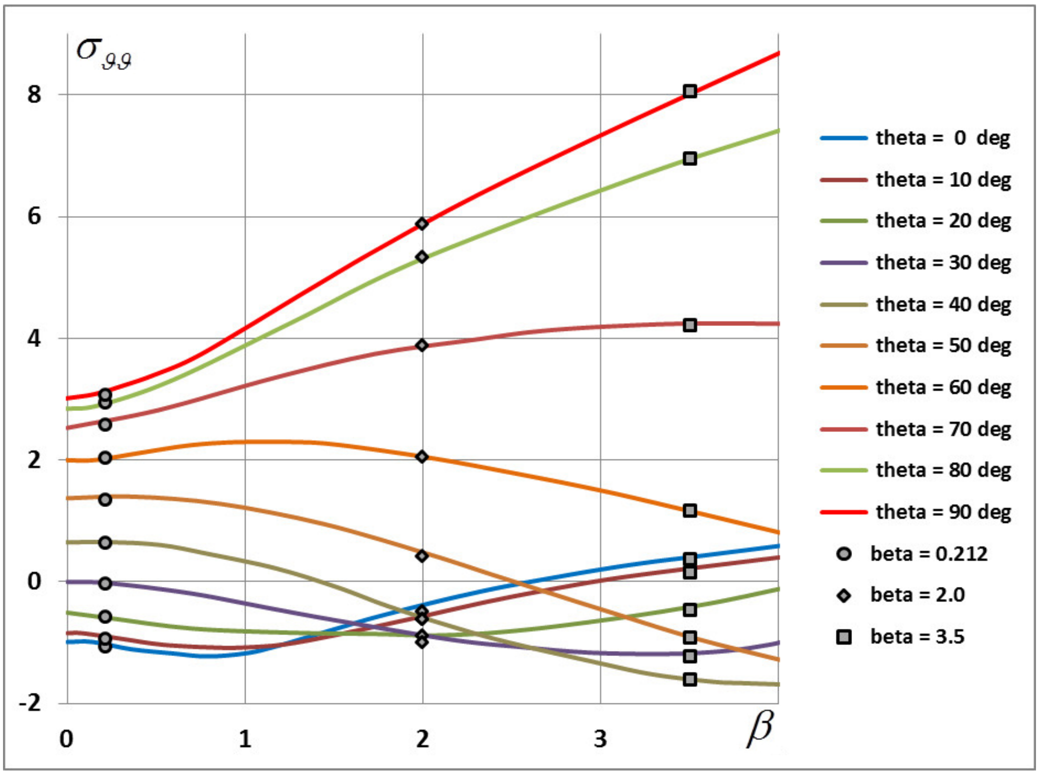

Fig. 5. Comparison with data of paper [10]: lines correspond to results of P.Van Dyke, dots for $\beta = 0.212; 2.0; 3.5$ are results of current paper

Considering researches of recent years it is observed the prevalence of approaches based on Finite Elements Method (FEM) [2,11]. However, results obtained by FEM need to be compared with analytical results and exact solutions. This explains the importance and value of the development of theoretical approaches.

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Appendix A

Nomenclature

R, h – radius and thickness of the cylindrical shell
r_0 – radius of the hole
$r_0 = 1$ in the present paper
ν – Poisson's ratio
E – Young's modulus
$\beta^2 = \frac{r_0^2\sqrt{3\,(1-\nu^2)}}{4Rh}$ – curvature parameter
w – displacement normal to the shape
U – stress function
T_x, T_{xy}, T_y – forces of the shell in Cartesian coordinates
$T_{rr}, T_{r\vartheta}, T_{\vartheta\vartheta}$ – forces of the shell in polar coordinates
$\sigma = T/h$ – middle surface stresses
p – axial tension applied at the edge of an infinite cylinder along x-coordinate
$\alpha = (1+i)\beta$
$t = \alpha r$
$H_n^{(1)}, H_n^{(2)}$ – Hankel function
J_n – Bessel function
Y_n – Neiman function
M_r – bending moment
Q_r – Kirchhoff shear