Materials Physics and Mechanics 44 (2020) 306-315 Received: March 18, 2020

ON THE INFLUENCE OF MULTIPLE SURFACE DEFECTS ON THE BEHAVIOR OF MEDIA OF DIFFERENT RHEOLOGY

V.A. Babeshko*, O.V. Evdokimova, O.M. Babeshko, I.S. Telyatnikov

Kuban State University, 149 Stavropolskiya st. Krasnodar, 350040, Russia

Southern Scientific Center RAS, Rostov-on-Don, 344006, Chekhov st., 41, Russia

*e-mail: babeshko41@mail.ru

Abstract. The behavior of coated media is investigated using the block element method under the assumption that the coating is exposed to a liquid substance that gradually leads to the destruction of the coating. It is assumed that the coating contains many defects that are parallel to each other. A similar situation occurs when the body is bent around the axis in a liquid medium, resulting in high tensile stresses in the coating. Such problems occur in bearing pairs. Another example of such a situation occurs in seismology, in the subduction zone, which represents the advance of oceanic lithospheric plates under the continental ones.

Keywords: cracks, block elements, coatings with defects, Kirchhoff plates, liquid layer, set of defects, stress concentration, subduction, tsunami

1. Introduction

The block element method, originally developed as a convergent approach that combines several traditional methods for solving boundary value problems for partial differential equations, has found a fairly wide range of practical applications [1]. In particular, it has helped to detect a new type of earthquake, called the starting one, to identify another mechanism of environment destruction, having identified a new type of crack, to find an approach for the most important aspect of tsunami forecasting. The most important was the possibility of using the block element method to study boundary value problems for partial differential equations with variable coefficients, which makes it possible to design new materials of mosaic structure [2].

The block element method is applied to study the behavior of the coated media under the assumption that the coating surface is exposed to a liquid medium and contains N defects. Assuming that the liquid layer can be modeled using shallow water equations, we study a block structure that includes a body in the form of a three-dimensional deformable layer, a defective coating modeled by Kirchhoff plates. The distribution of stress concentration in such a block structure has been studied. The analysis showed that even in the presence of a liquid, the features of the stress-strain state of the material that is inherent in the case of its absence are preserved. The presence of multiple cracks leads to the necessity to compare the parameters describing the stress concentration and search for the most vulnerable of them when predicting the medium destruction.

In problems related to the study of bearings, this method highlights the most vulnerable part. In subduction problems, the zone is defined in this way as the nearest earthquake and the possibility of a tsunami. Some zone adjacent to the contact between the oceanic and continental plates, called the Benioff seismofocal zone, has a set of parallel faults in the oceanic plate. During the subduction process, the plate goes through several rheological States

http://dx.doi.org/10.18720/MPM.4432020_3

until it melts at a great depth. This phenomenon provokes earthquakes and tsunamis in the coastal zone (Indonesian 2004 and Fukushima 2011). Boundary value problems for both of the problems described above are identical. The developed method allows to predict destructive events, evaluate the expected consequences and ways to anticipate them.

The theory of cracks, including those located parallel to the boundaries of the areas occupied by the material, as well as on the border between the coating and the material, has been developed in numerous works [3-15]. In this paper, we consider defects in the form of new type of cracks, the most destructive in problems with coatings [1]. The result of the work shows the importance of their detection, since even single defects can lead to irreversible consequences in such structures, and multiplicity – to complete destruction.

2. Basic equation

Let's select one defect from the existing N set in the coverage. Let's set a boundary value problem for it. The result of this study is then transferred to the study of multiple parallel defects using the method proposed in [16].

We assume that the liquid layer and the plate are affected by external time-harmonic forces directed vertically. In a local coordinate system $x_1x_2x_3$ with the origin x_1x_2 at the plane coincident with the median plane of the plate, ox_3 – axis directed upwards along the normal to the plate, the axis ox_1 is directed tangentially to the edge of the end plate, the axis ox_2 – normal to its boundary. The area of the left plate is marked λ and described by the ratio $|x_1| \le \infty, \quad x_2 \le -\theta$, and of the right – r -index and coordinates. For plates, the Kirchhoff equation for fragments b, $b=\lambda,r$ of regions Ω_b with boundaries $\partial\Omega_b$, with vertical harmonic stress effects $t_{3b}e^{-i\omega t}$ from above and $g_{3b}e^{-i\omega t}$ below, after excluding the time parameter, has the form

$$\mathbf{R}_b(\partial x_1,\partial x_2)u_{3b}+\boldsymbol{\varepsilon}_{53b}(t_{3b}-g_{3b})\equiv(\frac{\partial^4}{\partial x_1^4}+2\frac{\partial^2}{\partial x_1^2}\frac{\partial^2}{\partial x_2^2}+\frac{\partial^4}{\partial x_2^4}-\boldsymbol{\varepsilon}_{43b})u_{3b}+\boldsymbol{\varepsilon}_{53b}(g_{3b}-t_{3b})=0,$$

$$\mathbf{R}_b(-i\boldsymbol{\alpha}_1,-i\boldsymbol{\alpha}_2)U_{3b}=[\;(\boldsymbol{\alpha}_1^2+\boldsymbol{\alpha}_2^2)^2-\boldsymbol{\varepsilon}_{43b}\;]U_{3b},$$

$$U_{3b}=\mathbf{F_2}u_{3b},\quad G_{3b}=\mathbf{F_2}g_{3b},\quad T_{3b}=\mathbf{F_2}t_{3b},\quad b=\boldsymbol{\lambda},r,$$

$$m_b=-D_{b1}(\frac{\partial^2 u_{3b}}{\partial x_2^2}+\nu_b\frac{\partial^2 u_{3b}}{\partial x_1^2})=f_{3b}(\partial\Omega_b),\quad D_{b1}=\frac{D_b}{H^2},\quad D_{b2}=\frac{D_b}{H^3},\quad x_{k0}=Hx_k,\quad k=1,2,$$

$$q_b=-D_{b2}(\frac{\partial^3 u_{3b}}{\partial x_2^3}+(2-\nu_b)\frac{\partial^3 u_{3b}}{\partial x_1^2\partial x_2})=f_{4b}(\partial\Omega_b),$$

$$u_{3b}=f_{1b}(\partial\Omega_b),\quad \frac{\partial u_{3b}}{H\partial x_2}=f_{2b}(\partial\Omega_b),\;D_b=\frac{E_b h_b^{\,3}}{12(1-\nu_b^{\,2})},$$

$$\boldsymbol{\varepsilon}_{43b}=\boldsymbol{\omega}^2\boldsymbol{\rho}_b\frac{(1-\boldsymbol{\nu}_b^{\,2})12H^4}{Eh_b^{\,2}},\quad \boldsymbol{\varepsilon}_{53b}=\frac{(1-\boldsymbol{\nu}_b^{\,2})12H^4}{E_b h_b^{\,3}},\quad \boldsymbol{\varepsilon}_6^{-1}=\frac{(1-\boldsymbol{\nu})H}{\boldsymbol{\mu}}.$$

Here $\boldsymbol{\nu}_b$ – Poisson's ratio, E_b – Young's modulus, h_b – plate thickness, $\boldsymbol{\rho}_b$ – density, $\boldsymbol{\omega}$ – frequency of vibrations, g_{3b}, t_{3b} – values of contact stresses from the base side and pressures on the plates of the liquid layer from above, acting along the axis x_3 in the region Ω_b. $\mathbf{F}_2\equiv\mathbf{F}_2(\alpha_1,\alpha_2)$ and $\mathbf{F}_1\equiv\mathbf{F}_1(\alpha_1)$ both the two-dimensional and one-dimensional Fourier transform operators, respectively, m_b and q_b – bending moment and cutting force, $f_1(\partial\Omega_b)$ vertical displacement at the boundary; $f_2(\partial\Omega_b)$ angle of rotation of the median plane around the axis x_1, in the coordinate system x_1ox_2; h_b – plate thickness, H – size parameter of the substrate, for example, the thickness of the deformable material layer.

The behavior of a block element, which is a layer Ω_0 of incompressible liquid thickness on the surface, is described by H_1 shallow water equations of the following type [16]

$$p=(i\omega\rho\varphi+\rho g\frac{ih_b}{\omega H_1^2}\Delta\varphi)e^{-i\omega t}-we^{-i\omega t}.$$

Here p – the pressure in the liquid layer, ρ – the density of the liquid, g – the acceleration of gravity, φ – the speed potential in the liquid, w – the external effect on the layer. Given that the upper boundary of the plate is under the pressure of a liquid layer, taking into account the model taken, it is necessary to take

$$t_{3b}=p,\ u_{3b}=\frac{h_b}{i\omega H_1^2}\Delta\varphi_b.$$

As a result, the differential equation to the velocity potential takes the form

$$\Delta^3\boldsymbol{\varphi}_b+(\boldsymbol{\varepsilon}_{53b}\boldsymbol{\rho}g\text{-}\boldsymbol{\varepsilon}_{43b})\Delta\boldsymbol{\varphi}_b+\boldsymbol{\varepsilon}_{53b}\boldsymbol{\rho}\frac{\boldsymbol{\omega}^2H_1^2}{h_b}\boldsymbol{\varphi}_b-i\boldsymbol{\varepsilon}_{53b}\frac{\boldsymbol{\omega}H_1^2}{h_b}(g_{3b}-w_b)=0.$$

To use the block element method, it is required to apply its algorithm, which includes the stages of external algebra, external analysis, and factor topology, which will be explained in detail below. At the stage of external algebra, the boundary value problem is reduced to a functional equation of the following form

$$N_b(\alpha_1,\alpha_2)\Phi_b(\alpha_1,\alpha_2)=\int\limits_{\partial\Omega_b}\omega_b(\alpha_1,\alpha_2)+S_b(\alpha_1,\alpha_2),$$

$$N_b(\alpha_1,\alpha_2)=(\alpha_1^2+\alpha_2^2)^3+(\alpha_1^2+\alpha_2^2)(\varepsilon_{53b}\rho g-\varepsilon_{43b})-\varepsilon_{53b}R_b,$$

$$S_b(\alpha_1,\alpha_2)=i\varepsilon_{53b}\frac{\omega H_1^2}{h_b}\mathbf{F}_2(\alpha_1,\alpha_2)(g_{3b}-w_b),\quad \Phi_b(\alpha_1,\alpha_2)=\mathbf{F}_2(\alpha_1,\alpha_2)\varphi_b,\quad R_b=\rho\frac{\omega^2H_1^2}{h_b}.$$

Here $\omega_b(\alpha_1,\alpha_2)$, $b=\lambda,r$ are external forms, which are quite simple to construct, corresponding to the boundary value problem under consideration. The correlation between boundary stresses and displacements on the surface of the elastic medium on which the plates are located has the form

$$u_{3s}(x_1,x_2)=\varepsilon_6^{-1}\sum_{n=1}^{2}\iint\limits_{\Omega_n}k(x_1-\xi_1,x_2-\xi_2)g_{3n}(\xi_1,\xi_2)d\xi_1d\xi_2,\quad x_1,x_2\in\Omega_s,\ s=1,2$$

$$u_{31}=u_{3\lambda},\quad u_{32}=g_{3r},\quad u_{33}=u_{3\theta},\quad g_{31}=g_{3\lambda},\quad g_{32}=g_{3r},$$

$$\Omega_1\equiv\Omega_\lambda(|x_1|\le\infty;\quad x_2\le-\theta),\quad \Omega_2\equiv\Omega_r(|x_1|\le\infty;\quad \theta\le x_2),\quad \Omega_3\equiv\Omega_\theta(|x_1|\le\infty;\quad -\theta\le x_2\le\theta)$$

or

$$u_{3s}(x_1,x_2)=\frac{1}{4\boldsymbol{\pi}^2\boldsymbol{\varepsilon}_6}\sum_{n=1}^{2}\int_{-\infty}^{\infty}\int K(\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2)G_{3n}(\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2)e^{-i\langle\boldsymbol{\alpha},x\rangle}d\boldsymbol{\alpha}_1d\boldsymbol{\alpha}_2,\quad K(\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2)=\mathbf{F}_2(\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2)k(x_1,x_2)$$

$$\langle\boldsymbol{\alpha},x\rangle=\boldsymbol{\alpha}_1x_1+\boldsymbol{\alpha}_2x_2,\quad K(\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2,0)=O(A^{-1}),\quad A=\sqrt{\boldsymbol{\alpha}_1^2+\boldsymbol{\alpha}_2^2}\to\infty.$$

$K(\alpha_1,\alpha_2)$ – analytical function of two complex variables $\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2$, in particular, meromorphic, various examples of it are given in numerous publications [17].

We apply the external analysis stage to the study of functional equations, called so, because differential operations are performed on external forms. For this purpose, we present the functional equations as

$U_{3b}(\alpha_1,\alpha_2) = N_b^{-1}(\alpha_1,\alpha_2)[\ \omega_b(\alpha_1,\alpha_2) + \varepsilon_{53b} S_{3b}(\alpha_1,\alpha_2)\ ]$.

We require the implementation of an automorphism, one of the ways to implement which is to turn to zero the Leret form-residues only in those zeros $\alpha_{2m\pm} = \alpha_{2m\pm}(\alpha_1)$ of the determinant of the function $N_b(\alpha_1,\alpha_2)$ that provides each of the boundary problems as carriers only their own plates. Pseudo-differential equations degenerate into algebraic ones in a given boundary value problem. Taking into account the accepted notation, the equation for the left plate can be represented in the form

$$-e^{-i\alpha_2 \cdot \theta)}\{\ B_{1\lambda}(\alpha_1,\alpha_{2m-})Q_\lambda(\alpha_1,-\theta) + B_{2\lambda}(\alpha_1,\alpha_{2m-})M_\lambda(\alpha_1,-\theta) + B_{3\lambda}(\alpha_1,\alpha_{2m-})U_{3\lambda\partial x_2}(\alpha_1,-\theta) +$$

$$+B_{4\lambda}(\alpha_1,\alpha_{2m-})U_{3\lambda}(\alpha_1,-\theta) + B_{5\lambda}(\alpha_1,\alpha_{2m-})P(\alpha_1,-\theta) + B_{6\lambda}(\alpha_1,\alpha_{2m-})V_{x_2}(\alpha_1,-\theta)\ \} + S_\lambda(\alpha_1,\alpha_{2m-}) = 0,\ m = 1,2,3$$

The second pseudo-differential equation has a similar form.

3. Simplify the task statement

The solution of the problem stated in this article in the case of one defect, performed in [16], showed that the decisive role in the evaluating the zone of maximum stress concentration will be relatively thin layer of liquid, and the parameters of the defects distribution in the deformed coating. Therefore, in the future, without loss of accuracy, we simplify the task by focusing on the study of the effect of multiple faults. The behavior of a thin layer of liquid, as a result of the movement of coating fragments outside the defect or the surface of plates, is described by the piston theory of hydraulics. With that in mind, we will give a statement of the problem for elastically deformable objects, and after solving this problem, we will take into account the behavior of a thin layer of liquid.

Thus, we consider a coating that lies on a deformable base, representing the Kirchhoff plate, which has several types of parallel defects infinite in length, such that they divide the coating into several fragments. In cases where the defect edges do not close, there are two positions – when there is a distance between them that is different from zero, and when there is no distance between them. In the first case, the edges of the plates are removed from each other and are located on a linearly deformable base, in the second case, the distance is absent. We believe that the surface between the edges of the defect is free from stress, and external forces act on the ends of the plates, directed by the rule of external vectors. Boundary problems are considered in a coordinate system $x_1x_2x_3$ with the origin in the plane x_1x_2 coinciding with the median plane of the plate, the axis ox_3 directed up the normal to the plate, the axis ox_1 directed tangentially to the defect boundary, and the axis ox_2 – along the normal to its border. Plates representing the fragments of the coating is the region Ω_n $|x_1| \le \infty,\ \ c_{2n-1} \le x_2 \le c_{2n}, n = 1,2,...,N,\ \ c_{2N} = \infty$ with $\partial\Omega_{2n-1}$ left and $\partial\Omega_{2n}$ right borders. The defect numbers follow the numbers of the coating fragments and there are two possible states for the defects n: $c_{2n} - c_{2n+1} < 0,\ \ c_{2n} - c_{2n+1} = 0$. Let's limit ourselves to the case of only vertical impacts on the plates, assuming that bending moments and cutting forces that differ from zero can be set at the ends. The Kirchhoff equation for plate fragments n, $n = 1,2,...,N$ plates, that occupy areas Ω_n under the specified vertical harmonic stress $t_{3n}e^{-i\omega t}$ effects from above and $g_{3n}e^{-i\omega t}$ -below, after reducing the harmonic component, has the form

$$\mathbf{R}_n(\partial x_1,\partial x_2)u_{3n} + \varepsilon_{53n}(t_{3n} - g_{3n}) \equiv (\frac{\partial^4}{\partial x_1^4} + 2\frac{\partial^2}{\partial x_1^2}\frac{\partial^2}{\partial x_2^2} + \frac{\partial^4}{\partial x_2^4} - \varepsilon_{43n})u_{3n} + \varepsilon_{53n}(t_{3n} - g_{3n}) = 0, \quad (1)$$

$$\mathbf{R}_n(-i\boldsymbol{\alpha}_1,-i\boldsymbol{\alpha}_2)U_{3n} \equiv R_n(-i\boldsymbol{\alpha}_1,-i\boldsymbol{\alpha}_2)U_{3n} \equiv (\boldsymbol{\alpha}_1^2+\boldsymbol{\alpha}_2^2)^2 U_{3n},$$

$$U_{3n}=\mathbf{F}_2 u_{3n}, \quad G_{3n}=\mathbf{F}_2 g_{3n}, \quad T_{3n}=\mathbf{F}_2 t_{3n} \quad n=1,2,...,N,$$

$$M_n=-D_{n1}\left(\frac{\partial^2 u_{3n}}{\partial x_2^2}+\boldsymbol{\nu}_b\frac{\partial^2 u_{3n}}{\partial x_1^2}\right)=f_{3n}(\partial\Omega_n), \quad D_{n1}=\frac{D_n}{H^2}, \quad D_{n2}=\frac{D_n}{H^3},$$

$$Q_n=-D_{n2}\left(\frac{\partial^3 u_{3n}}{\partial x_2^3}+(2-\boldsymbol{\nu}_n)\frac{\partial^3 u_{3n}}{\partial x_1^2\partial x_2}\right)=f_{4n}(\partial\Omega_n),$$

$$u_{3n}=f_{1n}(\partial\Omega_n), \quad \frac{\partial u_{3n}}{H\partial x_2}=f_{2n}(\partial\Omega_n), \; D_n=\frac{E_n h_n^3}{12(1-\boldsymbol{\nu}_n^2)}.$$

The relationship between boundary stresses and displacements on the surface of the elastic medium on which the plates are located is given by the relations

$$u_3(x_1,x_2)=\varepsilon_6^{-1}\sum_{n=1}^{N}\iint_{\Omega_n} k(x_1-\xi_1,x_2-\xi_2)g_{3n}(\xi_1,\xi_2)d\xi_1 d\xi_2, \quad -\infty\le x_1,x_2\le\infty,$$

$$k(x_1,x_2)=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int K(\alpha_1,\alpha_2)e^{-i\langle\alpha,x\rangle}d\alpha_1 d\alpha_2, \tag{2}$$

$$U_3(\alpha_1,\alpha_2)=\varepsilon_6^{-1}K(\alpha_1,\alpha_2)\sum_{n=1}^{N}G_{3n}(\alpha_1,\alpha_2).$$

The above designations are transferred to the case of multiple defects.

Here $\boldsymbol{\nu}_n$, $\boldsymbol{\nu}$ – Poisson coefficients of the blocks and the base, respectively; μ – base shear modulus; E_n – Young's modulus; h_n – block thickness; $\boldsymbol{\rho}_n$ – density; $\boldsymbol{\omega}$ – frequency of vibrations; g_{3n}, t_{3n} – values of contact stresses and external pressures acting along the axis x_3 in the area Ω_n, respectively; M_b and Q_b – bending moment and cutting force; – vertical movement at the boundary; $f_2(\partial\Omega_n)$ – angle of rotation of the median plane around the axis x_1, in the coordinate system $x_1 o x_2$; H – the dimensional parameter of the substrate, for example, the thickness of the layer; u_{3n} – vertical displacement under the block elements; u_{3n0} – vertical displacement of the lower base outside the block elements; u_3 – vertical movement of the lower base along the entire length. The designations are borrowed from [18-20].

4. Method of research

Considering the plates and the base (1) as a block structure consisting of three deformable blocks, we apply the block element method for its study. This method, as described in [25], assumes, as the first step, the immersion of the boundary value problem in the topological structure by means of the external algebra. As a result, a functional equation of the boundary value problem for a block structure is constructed. The authors call the multi-step algorithm for further studies of the functional equation, which no longer has any relation to the external algebra apparatus, an external analysis in the block element theory [25]. It includes differential factorization of matrix functions with elements from several complex variables, and implementation. An automorphism of the composed in the calculation of Lehrer's form-deductions, or incomplete Wiener-Hopf functional equations, construction of pseudo-differential equations, extraction of integral equations from them, dictated by specific boundary conditions of the boundary problem, solution of integral equations, and obtaining an integral representation of the boundary problem in each block in the form of a "Packed" block element. Finally, "gluing" the solutions of each block, consisting of constructing a factor topology of some topological spaces that are Cartesian products of topological spaces of carriers and solutions. Using the external algebra stage, we write the functional equation of the boundary value problem as

$$R_n(-i\alpha_1,-i\alpha_2)U_{3n}\equiv[\ (\alpha_1^2+\alpha_2^2)^2-\varepsilon_{43n}\ ]U_{3n}=-\int_{\partial\Omega_n}\omega_n-\varepsilon_{53n}S_{3n}(\alpha_1,\alpha_2), \quad (3)$$

$$S_{3n}(\alpha_1,\alpha_2)=\mathbf{F}_2(\alpha_1,\alpha_2)(t_{3n}-g_{3n}).$$

Here are the external forms that participate in the representation [18-20], which, taking into account the choice of the coordinate system, look like

$$\omega_n(x_1,x_2)=e^{i\langle\alpha,x\rangle}\Big\{\ -\Big[\ \frac{\partial^3u_{3n}}{\partial x_2^3}-i\alpha_2\frac{\partial^2u_{3n}}{\partial x_2^2}-\alpha_2^2\frac{\partial u_{3n}}{\partial x_2}+i\alpha_2^3u_{3n}+2\frac{\partial^3u_{3n}}{\partial x_1^2\partial x_2}-$$

$$-2i\alpha_2\frac{\partial^2u_{3n}}{\partial x_1^2}\ \Big]dx_1+\Big[\ \frac{\partial^3u_{3n}}{\partial x_1^3}-i\alpha_1\frac{\partial^2u_{3n}}{\partial x_1^2}-\alpha_1^2\frac{\partial u_{3n}}{\partial x_1}+i\alpha_1^3u_{3n}\ \Big]dx_2\ \Big\},\quad n=1,2....N$$

and in the special case of straight-line boundaries are represented by formulas

$$\omega_n=e^{i\langle\alpha,x\rangle}\Big\{\ -\Big[\ i\alpha_2M_nD_n^{-1}-Q_nD_n^{-1}-(\alpha_2^2+\nu_n\alpha_1^2)\frac{\partial u_{3n}}{\partial x_2}+i\alpha_2\big[\ \alpha_2^2+(2-\nu_n)\alpha_1^2\big]u_{3n}\ \Big]\ \Big\}dx_1.$$

In formula (2), when integrating, in the case of a finite-size plate, the border of the right plate $\partial\Omega_n$ is two ends – the left and right. Since the area occupied by the slab is considered as a topological variety with an edge, local coordinates are entered at the boundary, whose orientation is consistent with the orientation of the interior of the variety.

As a result, the following pseudo-differential equations are obtained for each block of the block system

$$\mathbf{F}_1^{-1}(\xi_1)\Big\langle\ \int_{\partial\Omega_{2n-1}}\Big\{\ i\alpha_{21-}D_n^{-1}M_n-D_n^{-1}Q_n-(\alpha_{21-}^2+\nu_n\alpha_1^2)\frac{\partial u_{32n-1}}{\partial x_2}+$$

$$+i\alpha_{21-}\big[\ \alpha_{21-}^2+(2-\nu_n)\alpha_1^2\ \big]u_{32n-1}\Big\}e^{i(\alpha_1x_1+\alpha_{22-}c_{2n-1})}dx_1-$$

$$-\int_{\partial\Omega_{2n}}\Big\{\ i\alpha_{21-}D_n^{-1}M_n-D_n^{-1}Q_n-(\alpha_{21-}^2+\nu_n\alpha_1^2)\frac{\partial u_{32n}}{\partial x_2}+$$

$$+i\alpha_{21-}\big[\ \alpha_{21-}^2+(2-\nu_n)\alpha_1^2\ \big]u_{32n}\Big\}e^{i(\alpha_1x_1+\alpha_{22-}c_{2n})}dx_1+\varepsilon_{53n}\big[\ G_{3n}(\alpha_1,\alpha_{21-})-T_{3n}(\alpha_1,\alpha_{21-})\ \big]\ \Big\rangle=0$$

$$\mathbf{F}_1^{-1}(\xi_1)\Big\langle\ \int_{\partial\Omega_{2n-1}}\Big\{\ i\alpha_{22-}D_n^{-1}M_n-D_n^{-1}Q_n-(\alpha_{22-}^2+\nu_n\alpha_1^2)\frac{\partial u_{32n-1}}{\partial x_2}+$$

$$+i\alpha_{22-}\big[\ \alpha_{22-}^2+(2-\nu_n)\alpha_1^2\ \big]u_{32n-1}\Big\}e^{i(\alpha_1x_1+\alpha_{22-}c_{2n-1})}dx_1-$$

$$-\int_{\partial\Omega_{2n}}\Big\{\ i\alpha_{22-}D_n^{-1}M_n-D_n^{-1}Q_n-(\alpha_{22-}^2+\nu_n\alpha_1^2)\frac{\partial u_{32n}}{\partial x_2}+$$

$$+i\alpha_{22-}\big[\ \alpha_{22-}^2+(2-\nu_n)\alpha_1^2\ \big]u_{32n}\Big\}e^{i(\alpha_1x_1+\alpha_{22-}c_{2n})}dx_1+\varepsilon_{53n}\big[\ G_{3n}(\alpha_1,\alpha_{22-})-T_{3n}(\alpha_1,\alpha_{22-})\ \big]\ \Big\rangle=0$$

The same system of pseudo-differential equations is added to it by replacing α_{21-} with α_{21+} and α_{22-} with α_{22+}.

The system of pseudo-differential equations under the number N has the form

$$\int\limits_{\partial\Omega_{2N-1}} \{ \ i\alpha_{21+}D_N^{-1}M_N - D_N^{-1}Q_N - (\alpha_{21+}^2 + \nu_N\alpha_1^2)\frac{\partial u_{32N-1}}{\partial x_2} +$$

$$+i\alpha_{21+}[\ \alpha_{21+}^2 + (2-\nu_N)\alpha_1^2\ ]u_{32N-1}\} e^{i(\alpha_1 x_1 + \alpha_{21+}c_{2N-1})}dx_1 + \varepsilon_{53N}[\ G_{3N}(\alpha_1,\alpha_{21+}) - T_{3N}(\alpha_1,\alpha_{21+})\ ]\ \rangle$$

$$\int\limits_{\partial\Omega_{2N-1}} \{ \ i\alpha_{22+}D_N^{-1}M_N - D_N^{-1}Q_N - (\alpha_{22+}^2 + \nu_N\alpha_1^2)\frac{\partial u_{32N-1}}{\partial x_2} +$$

$$+i\alpha_{22+}[\ \alpha_{22+}^2 + (2-\nu_N)\alpha_1^2\ ]u_{32N-1}\} e^{i(\alpha_1 x_1 + \alpha_{22+}c_{2N-1})}dx_1 + \varepsilon_{53N}[\ G_{3N}(\alpha_1,\alpha_{22+}) - T_{3N}(\alpha_1,\alpha_{22+})\ ]\ \rangle$$

In integrals, the notation is accepted

$$\alpha_{21-} = -i\sqrt{(\alpha_1)^2 - \sqrt{\varepsilon_{b43}}}, \quad \alpha_{22-} = -i\sqrt{(\alpha_1)^2 + \sqrt{\varepsilon_{b43}}},$$

$$\alpha_{21+} = i\sqrt{(\alpha_1^r)^2 - \sqrt{\varepsilon_{b43}}}, \quad \alpha_{22+} = i\sqrt{(\alpha_1^r)^2 + \sqrt{\varepsilon_{b43}}}.$$

The equations for the left semi-infinite plate remain the same as those obtained earlier. Analyzing built pseudo-differential equations for the case of a plate of limited length, you can see an increase in the number of unknowns generated by an additional boundary compared to the case of a single boundary at a semi-bounded plate. Studies of this case show that the presence of another border at the plate of finite length does not change the type features, both in the case of the remoteness of the fault banks, and when approaching, however, affects the values of coefficients for features.

Using the approach described in [11-13], pseudo-differential equations, taking into account the shape of regions Ω_b in the form of bands of infinite length, we apply the Fourier transform on the coordinate x_1.

Let's introduce the following notation system

$$\mathbf{Y}_{2n-1} = \{y_{12n-1}, y_{22n-1}, y_{12n}, y_{22n}\}, \quad \mathbf{Z}_{2n-1} = \{z_{12n-1}, z_{22n-1}, z_{12n}, z_{22n}\}, \quad n = 1,2,...N$$

$$y_{1s} = D_n^{-1}\mathbf{F}_1 M_s, \quad y_{2s} = D_n^{-1}\mathbf{F}_1 Q_s, \quad z_{1s} = \mathbf{F}_1\frac{\partial u_{3s}}{\partial x_2}, \quad z_{2s} = \mathbf{F}_1 u_{3s}, \quad s = 1,2,...,N$$

$$\mathbf{F}_1 g = \mathbf{F}_1(\alpha_1)g, \quad \mathbf{F}_2 g = \mathbf{F}_2(\alpha_1,\alpha_2)g,$$

$$\mathbf{K}_{2n-1} = \{k_{12n-1-}, k_{22n-1-}, k_{12n-1+}, k_{22n-1+}\}, \qquad k_{1s-} = \varepsilon_{53s}\mathbf{F}_2(\alpha_1,\alpha_{21-})(t_{3s} - g_{3s}),$$

$$k_{2s-} = \varepsilon_{53s}\mathbf{F}_2(\alpha_1,\alpha_{22-})(t_{3s} - g_{3s}), \quad k_{1s+} = \varepsilon_{53s}\mathbf{F}_2(\alpha_1,\alpha_{21+})(t_{3s} - g_{3s}),$$

$$k_{2s+} = \varepsilon_{53s}\mathbf{F}_2(\alpha_1,\alpha_{22+})(t_{3s} - g_{3s}).$$

As a result, we obtain systems of four algebraic equations for each block element, which in matrix form have the representation

$$\mathbf{A}_{2n-1}\mathbf{Y}_{2n-1} + \mathbf{B}_{2n-1}\mathbf{Z}_{2n-1} + \mathbf{K}_{2n-1} = 0, \quad n = 1,2,...,N.$$

Let's consider the case when the bending moment and the cutting force are equal to zero, that is, the ends of the plates are free from stresses, $\mathbf{Y}_{2n-1} = 0$.

In this case, systems of algebraic equations are solved and the solution representable in the form

$$\mathbf{Z}_{2n-1} = -\mathbf{B}_{2n-1}^{-1}\mathbf{K}_{2n-1}.$$

By adding the solutions found to the external forms (3), we get

$$U_{3n} = -R_n^{-1}(-i\alpha_1, -i\alpha_2)[\ \int\limits_{\partial\Omega_n}\omega_n + \varepsilon_{53n}\mathbf{F}_2(g_{3n} - t_{3n})\ ], \quad \mathrm{n} = 1,2,...,N.$$

By conjugating block elements with the base (1), (2), that is, by building a factor topology [18-20], we arrive at a system of functional equations of the form

$$\sum_{b=1}^{N} U_{3b} + \sum_{b=0}^{N-1} U_{3b0} = U_3,$$

$$-\sum_{b=1}^{N} R_b^{-1}(-i\alpha_1,-i\alpha_2)\Big[\int_{\partial\Omega_b} \omega_b + \varepsilon_{53b}(G_{3b}-T_{3b})\Big] = \varepsilon_6^{-1} K(\alpha_1,\alpha_2)\sum_{n=1}^{N} G_{3n}(\alpha_1,\alpha_2).$$

This system of functional equations is solved by factorization by reducing to the Riemann boundary system for pairs of analytical functions. This approach is implemented in [17,21] and allows describing solutions of the functional equation for all unknowns simultaneously. However, we are interested in the stress concentration zones that occur in the defect zones.

In order to identify the parameters of stress concentrations in the complex block structure under consideration, we will sequentially identify defects in the constructed functional equation, considering them in individual local coordinates x directed by x_2 with the beginning in the center of the fault. In the boundary value problem under study, two positions of defects edges are possible: when the distance between the defects edges is different from zero, we denote it $2\theta > 0$, and when it is absent, that is, $\theta = 0$. Let's denote the function of the left block $G_{32n-1}(\alpha_1,\alpha_2) = G^-(\alpha_1,\alpha_2)$ and the right block $G_{32n+1}(\alpha_1,\alpha_2) = G^+(\alpha_1,\alpha_2)$ in the selected coordinate system.

Then the functional equations take the form studied in [18-20]

$$\begin{aligned}
&\Big[\varepsilon_{532n-1}(\alpha_1^2+\alpha_2^2)^{-2} + \varepsilon_6^{-1}K(\alpha_1,\alpha_2)\Big]G^+(\alpha_1,\alpha_2) = \\
&= -\Big[\varepsilon_{532n+1}(\alpha_1^2+\alpha_2^2)^{-2} + \varepsilon_6^{-1}K(\alpha_1,\alpha_2)\Big]G^-(\alpha_1,\alpha_2) + U_{3\theta}(\alpha_1,\alpha_2) + \\
&+(\alpha_1^2+\alpha_2^2)^{-2}\Big[A_{2n-1}k_{12n-10} + B_{2n-1}k_{22n-10} + A_{2n+1}k_{12n+10} + B_{2n+1}k_{22n+10} + \\
&+\varepsilon_{532n-1}T^+(\alpha_1,\alpha_2) + \varepsilon_{532n+1}T^-(\alpha_1,\alpha_2)\Big], \quad \theta > 0,
\end{aligned} \tag{4}$$

$$U_{3\theta}(\alpha_1,\alpha_2) = \int_{-\infty}^{\infty}\int_{-\theta}^{\theta} u_3(x_1,x)e^{i\langle\alpha,x\rangle}dx_1dx.$$

$$\begin{aligned}
&\Big[\varepsilon_{532n+1}(\alpha_1^2+\alpha_2^2)^{-2} + \varepsilon_6^{-1}K_1(\alpha_1,\alpha_2)\Big]G^+(\alpha_1,\alpha_2) = \\
&= -\Big[\varepsilon_{532n-1}(\alpha_1^2+\alpha_2^2)^{-2} + \varepsilon_6^{-1}K_1(\alpha_1,\alpha_2)\Big]G^-(\alpha_1,\alpha_2) + \\
&+(\alpha_1^2+\alpha_2^2)^{-2}\Big[A_{2n-1}k_{12n-10} + B_{2n-1}k_{22n-10} + A_{2n+1}k_{12n+10} + B_{2n+1}k_{22n+10} + \\
&+\varepsilon_{532n-1}T^+(\alpha_1,\alpha_2) + \varepsilon_{532n+1}T^-(\alpha_1,\alpha_2)\Big], \quad \theta = 0.
\end{aligned} \tag{5}$$

Here A_{2n-1}, B_{2n-1}, A_{2n+1}, B_{2n+1} are complex expressions that are omitted for the sake of brevity. Note that the presented functional equations have not only functions $G^+(\alpha_1,\alpha_2)$, $G^-(\alpha_1,\alpha_2)$, but also functionals $G^+(\alpha_1,\alpha_{2+})$, $G^-(\alpha_1,\alpha_{2-})$, $G^+(\alpha_1,\alpha_{2+})$, $G^{-\prime}(\alpha_1,\alpha_{2-})$, as unknowns. They are linearly included in k_{sn0} and need to be defined. We obtained two different functional Wiener-Hopf equations. The first, (4), is the generalized Wiener-Hopf functional equation, in relation to the presence of a function $U_{3\theta}(\alpha_1,\alpha_2)$. It is solved by the method described in [17], which consists of inverting a system of two integral equations of the second kind with a completely continuous function in a certain space of continuous with weight. After transformations, it looks like

$$X^+ - \{ \ -\frac{M_1^+}{M_2^-} Y^- e^{-i2\alpha_2\theta} \ \}^+ = \{ \ \frac{1}{M_2^-} \Phi e^{-i\alpha_2\theta} \ \}^+ ,$$
$$Y^- + \{ \ \frac{M_2^-}{M_1^+} X^+ e^{i2\alpha_2\theta} \ \}^- = \{ \ \frac{1}{M_1^+} \Phi e^{i\alpha_2\theta} \ \}^- ,$$
$$M_1 = M_1^+ M_1^- , \quad M_2 = M_2^+ M_2^- ,$$
$$M_2^+ G^+ = X^+ , \quad M_1^- G^- = Y^- ,$$
$$M_1 = [\ \varepsilon_{53n-1} (\alpha_1^2 + \alpha_2^2)^{-2} + \varepsilon_6^{-1} K(\alpha_1, \alpha_2) \ ],$$
$$M_2 = [\ \varepsilon_{53n} (\alpha_1^2 + \alpha_2^2)^{-2} + \varepsilon_6^{-1} K(\alpha_1, \alpha_2) \ ].$$

The work designations are accepted from [18].

After solving the boundary value problem, defining the functions and, you need to find the values of the functionals $G^+(\alpha_1,\alpha_2)$ and $G^-(\alpha_1,\alpha_2)$, and, as well as the functionals $G'_+(\alpha_1,\alpha_{2+})$ and $G'_-(\alpha_1,\alpha_{2-})$.

To determine them, a system of linear algebraic equations described in [18,19] is constructed.

It is proved quite simply that the solution of the first functional equation for $\theta > 0$ leads to the following properties of contact stresses between plates and the substrate at the edges in the local coordinate system

$$g_{32n-1}(x_1,x) = \sigma_{12n-1}(x_1,x)(-x-\theta)^{-1/2}, \quad x < -\theta,$$
$$g_{32n+1}(x_1,x) = \sigma_{12n+1}(x_1,x)(x-\theta)^{-1/2}, \quad x > \theta, \tag{6}$$

for fairly smooth ones t_{3n}. The second functional equation is the Wiener-Hopf equation. Inversion of the second equation, (5), leads for $x \to 0$ to the following properties of solutions

$$g_{32n-1}(x_1,x) \to \sigma_{22n-1}(x_1,x_2)x^{-1},$$
$$g_{32n+1}(x_1,x) \to \sigma_{22n+1}(x_1,x)x^{-1}. \tag{7}$$

Functions σ_{22n} are continuous in both parameters.

5. Conclusions

The analysis of the obtained results allows us to draw the following conclusions. A relatively thin layer of liquid with vertical movements of the coating fragments will repeat their movement according to the piston rule of hydraulics and will not have any noticeable effects on the coating. The main destruction of the coated material will come from the destruction of areas of defects or micro cracks. It is very important that the paper identifies coefficients for features in the defect zones, which are not difficult to calculate using the obtained formulas The zone with the maximum values of the coefficients $\sigma_{22n}(x_1,x)$ bears the risk of destruction of materials with defective coatings.

Acknowlegements. *This work was supported by the Russian Foundation for Basic Research, projects nos* ***(****19-41-230003****),*** ***(****19-41-230004****),*** ***(****19-48-230014****),*** *(18-08-00465), (18-01-00384), (18-05-80008), the GZ UNC RAS, project № 01201354241, (00-19-13), Ministry of the science Russian Federation, project (FZEN-2020-0020) charged on 2020 year.*

References

[1] Babeshko VA, Evdokimova OV, Babeshko OM. The external analysis in the problem of hidden defects and in the prediction of earthquakes. *Ecological bulletin of BSEC scientific centers.* 2016;13(2): 19-28.

[2] Babeshko VA, Evdokimova OV, Babeshko OM, Ryadchikov IV. A Method for the Design of Inhomogeneous Materials and Block Structures. *Doklady Physics*. 2018;63(10): 402-406.
[3] Griffith A. The Phenomena of Rupture in Solids. *Trans. Roy. Soc. London*. 1920;221A: 163-197.
[4] Irwin G. Fracture dynamics. In: *Fracture of metals*. Cleveland: ASM; 1948. p.147-166.
[5] Morozov NF. *Mathematical problems in the crack theory*. Moscow: Nauka; 1984. (In Russian)
[6] Sator C, Becker W. Closed-form solutions for stress singularities at plane bi- and trimaterial junctions. *Arch. Appl. Mech.* 2012;82: 643-658.
[7] Cherepanov GP. *Brittle fracture mechanics.* Moscow: Nauka; 1974. (In Russian)
[8] Allabadl MH. *Dynamic fracture mechanics.* Southampton: Computational Mechanics Publications; 1995.
[9] Balars J, Sladek J, Sladek V. *Stress Analysis by Boundary Element Methods.* Amsterdam: Elsevier; 1989.
[10] Comninou M. The Interface Crack. *Trans. ASME J. Appl. Mech.* 1977;44(4): 631-636.
[11] Comninou M. The Interface Crack in a Shear Field. *Trans. ASME J. Appl. Mech*. 197;45(2): 287-290.
[12] Comninou M, Schmuser D. The Interface Crack in a Combined Tension-Compression and Shear Field. *Trans. ASME J. Appl. Mech*. 1979;46(2): 345-348.
[13] Dugdale DS. Yielding of Steel Sheets Containing Slits. *J. Mech. And Phys. Solids.* 1960;8(2): 100-108.
[14] Sih GC. *Elastodynamic crack problems.* Leyden: Noordhoff; 1977.
[15] Gautesen AK, Dunders J. The interface crack in a tension field. *Trans. ASME J. Appl. Mech*. 1987;54: 93-98.
[16] Babeshko OM, Evdokimova OV, Babeshko VA, Shestopalov VL, Evdokimov VS. Mechanical aspects of the behavior of the surface of coated bodies when exposed to corrosive liquid. *Materials Physics and Mechanics*. 2019;42(5): 558-563.
[17] Vorovich II, Babeshko VA. *Dynamic Mixed Problems of the Theory of Elasticity for Nonclassical Regions.* Moscow: Nauka; 1979. (In Russian)
[18] Babeshko VA, Evdokimova OV, Babeshko OM. On the possibility of predicting some types of earthquake by a mechanical approach. *Acta Mechanica.* 2018;229(5): 2163-2175.
[19] Babeshko VA, Evdokimova OV, Babeshko OM. On a mechanical approach to the prediction of earthquakes during horizontal motion of lithospheric plates. *Acta Mechanica.* 2018;229: 4727-4739.
[20] Babeshko VA, Evdokimova OV, Babeshko OM. A starting earthquake with harmonic effects. *Doklady Physics.* 2016;61: 551-554.
[21] Babeshko VA, Evdokimova OV, Babeshko OM, Pavlova AB, Telatnikov IS, Fedorenko AG. The theory of block structures in problems on the strength of galleries and constructions with multiple connections. *Doklady Physics.* 2019;64(11): 4-8.