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WAVE PROPAGATION DUE TO LASER IRRADIATION IN VISCOELASTIC THIN METAL FILM WITH FRACTIONAL RELAXATION OPERATOR

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Abstract. In the present paper, the model of generalized thermo-viscoelastic theory with fractional relaxation operators is used to capture the microscale responses of viscoelastic metal film irradiated uniformly by a laser pulse heat flux with non-Gaussian form. Employing Laplace transform as a tool, the problem has been solved analytically in the transformed domain. The inverse transforms are obtained by using a numerical method based on Fourier expansion techniques. The effects of fractional relaxation operators and viscoelastic property on the responses of the metal film are discussed and illustrated graphically.

Keywords: thermo-viscoelastic material, fractional relaxation operators, thin metal film, Laplace transforms, numerical results.

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1. Introduction

Heat conduction is critical in solids analysis. Any influence on a solid is connected with heat and mass transit alterations, and vice versa. The change in the warmth conduction condition from diffusive to wave type may also be impacted by a minute consideration of the warmth transport miracle or in a phenomenological route by strategies for changing the conventional Fourier law of heat conduction [1]. The first is owed to Cattaneo [2], who established a wave-type warming condition by suggesting another law of warmth conduction to replace the classic Fourier law. Lord and Shulman [3], as well as Green and Lindsay [4], were the first to develop a thermoelasticity model that ensured wave velocity. Summaries and presentations of wider concepts are provided by Ignaczak [5], Chandrasekharaiah [6], Sherief and Ezzat [7], and Ezzat et al. [8-12].

Viscoelasticity is the property of materials that show both gooey and versatile attributes when experiencing disfigurement. Gooey materials, similar to nectar, oppose shear stream and strain straightly with time when pressure is connected. Flexible materials strain when

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extended and rapidly come back to their unique state once the pressure is evacuated. Viscoelastic materials have components of both of these properties and, in that capacity, show the time-subordinate strain. While versatility is normally the aftereffect of bond extending along crystallographic planes in an arranged strong, consistency is the outcome of the dispersion of particles or atoms inside a nebulous material [13]. Direct viscoelastic materials are rheological materials that show time-temperature pace of-stacking dependence. At the point when their reaction isn't just an element of the present information, yet in addition to the present and past information history, the portrayal of the viscoelastic reaction can be communicated utilizing the convolution (innate) basic. The mechanical-model depiction of direct viscoelastic lead results was analyzed by Gross [14]. One can imply Atkinson and Craster [15] for a review of split mechanics and theories to the viscoelastic materials. A general survey of time-subordinate material properties has been displayed in Refs. [16-22].

The fractional investigation has been used viably to change many existing models of physical methods. Caputo and Mainardi [23] and Caputo [24] found incredible simultaneousness with preliminary outcomes when using incomplete backups for the depiction of viscoelastic materials and developed the relationship between fragmentary subordinates and the speculation of straight viscoelasticity. Adolfsson et al. [25] developed a more to date halfway demand model of viscoelasticity. One can imply Podlubny [26] for an outline of usages of partial examination. Sherief et al. [27] displayed a partial condition of warmth conduction and showed a uniqueness theory and decided a correspondence association and a variational standard. Ezzat [28] established a new model of fractional heat conduction equation utilizing the Taylor-Riemann series expansion of time-fractional order. In continuum mechanics, the partial order hypothesis of thermoelasticity and thermo-viscoelasticity was determined by many authors in works [29-40].

Understanding the physics of laser-matter interactions in ultrashort pulses is important for many well-acknowledged applications from material modifications to biology. The ultra-short lasers are those with pulse duration ranging from nanoseconds to femtoseconds in general. In the case of ultra-short-pulsed laser heating, the high-intensity energy flux and ultra-short duration laser beam, have introduced situations where very large thermal gradients or an ultra-high heating speed may exist on the boundaries. Ultrafast lasers, even the novel laser burst development, have been for the most part used in different applications, particularly in scaled-down scale machining. The part of ultrafast laser and matter correspondence, for instance, heat move, misshaping of a nanostructure, has gotten different speculative and exploratory research interests. In such cases the old-style models of thermo-adaptability may be tried to give exact responses: directly off the bat, Fourier's law of warmth conduction law may isolate under the high warmth change and low-temperature conditions; likewise, old-style adaptability may bomb as the outside trademark length (or time) approaches to manage within trademark length (or time) [41]. One can refer to Wang et al. [42], Inogamov et al. [43,44], and Khokhlov et al. [45, 46] for a survey of applications of a thin metal film irradiated by a laser pulse.

In our work, the thermo-viscoelastic qualities in a metal film illuminated by a femtosecond laser beat with a non-Gaussian worldly profile by thinking about the partial unwinding administrators impacts of the volume are explored. Some fundamental hypotheses on the straight-coupled hypothesis, Biot [1] and generalized speculations of thermo-viscoelasticity Lord-Shulman [3] with one relaxation time and Green-Lindsay [4] with two relaxation times are built up. Laplace transform is utilized to acquire a logical answer for a slight film of viscoelastic material. The outcomes have been introduced graphically.

2. Unified mathematical model

The unified model of thermo-viscoelasticity with fractional relaxation operator's hypothesis is comprised as:

1. The constitutive equation

$$S_{ij}(\boldsymbol{x},t)=\int_0^t R_\beta(t-\xi)\frac{\partial e_{ij}(\boldsymbol{x},\xi)}{\partial\xi}\mathrm{d}\xi=\hat{\boldsymbol{R}}_\beta(e_{ij}), \tag{1}$$

$$\sigma(\boldsymbol{x},t)=\frac{\sigma_{kk}(\boldsymbol{x},t)}{3}=\int_0^t R_\alpha(t-\xi)\frac{\partial}{\partial\xi}(e_{ij}-3\alpha_T\hat{T})(\boldsymbol{x},\xi)\mathrm{d}\xi=\hat{\boldsymbol{R}}_\alpha(e_{ij}-3\alpha_T\hat{T}), \tag{2}$$

where $R_\beta(t)$ and $R_\alpha(t)$ are the relaxation modules functions such that $R(\infty)>0$,

$$S_{ij}=\sigma_{ij}-\frac{\sigma_{kk}}{3}\delta_{ij},\ \varepsilon_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}),\quad \mathrm{e}_{ij}=\varepsilon_{ij}-\frac{e}{3}\delta_{ij},\quad e=\varepsilon_{kk},\quad \boldsymbol{x}=(x_1,x_2,x_3),$$

$$R_\beta(t,\beta)=\frac{R_1}{\Gamma(1-\beta)}(t/\tau)^{-\beta},\qquad 0<\beta\le 1, \tag{3}$$

$$R_\alpha(t,\alpha)=\frac{R_2}{\Gamma(1-\alpha)}(t/\tau)^{-\alpha},\qquad 0<\alpha\le 1, \tag{4}$$

$$\hat{T}(\boldsymbol{x},t)=\Theta(\boldsymbol{x},t)+\upsilon\dot{\Theta}(\boldsymbol{x},t),\quad \Theta(\boldsymbol{x},t)=T(\boldsymbol{x},t)-T_0. \tag{5}$$

Hence,

$$S_{ij}(\boldsymbol{x},t)=\hat{\boldsymbol{R}}_\beta(e_{ij}),\qquad 0<\beta\le 1, \tag{6}$$

$$\sigma(\boldsymbol{x},t)=\hat{\boldsymbol{R}}_\alpha(e-3\alpha_T\hat{T}),\qquad 0<\alpha\le 1, \tag{7}$$

where

$$\hat{\boldsymbol{R}}_\beta(f)=\frac{R_1\tau^\beta}{\Gamma(1-\beta)}\int_0^t(t-\xi)^{-\beta}\frac{\partial f(\boldsymbol{x},\xi)}{\partial\xi}d\xi,\quad 0<\beta\le 1, \tag{8}$$

$$\hat{\boldsymbol{R}}_\alpha(f)=\frac{R_2\tau^\alpha}{\Gamma(1-\alpha)}\int_0^t(t-\xi)^{-\alpha}\frac{\partial f(\boldsymbol{x},\xi)}{\partial\xi}d\xi,\quad 0<\alpha\le 1. \tag{9}$$

2. The stress-strain temperature relation:

$$\sigma_{ij}=\hat{\boldsymbol{R}}_\beta(\varepsilon_{ij}-\frac{e}{3}\delta_{ij})+\hat{\boldsymbol{R}}_\alpha(e-3\alpha_T\hat{T})\delta_{ij}. \tag{10}$$

3. The equation of motion

$$\rho\frac{\partial^2 u_i}{\partial t^2}=\hat{\boldsymbol{R}}_\beta(\frac{1}{2}\nabla^2 u_i+\frac{1}{6}e_{,i})+\hat{\boldsymbol{R}}_\alpha(e_{,i}-\gamma\hat{T}_{,i}). \tag{11}$$

4. The heat equation

$$k\,\Theta_{,ii}=\rho C_E\left(\frac{\partial\Theta}{\partial t}+\tau_o\frac{\partial^2\Theta}{\partial t^2}\right)+3\alpha_T T_o\hat{\boldsymbol{R}}_\alpha(\frac{\partial e}{\partial t}+n_0\tau_0\frac{\partial^2 e}{\partial t^2}). \tag{12}$$

In the above conditions, a comma means material subordinates, and the summation show is utilized.

3. Problem formulation

Consider local irradiation of a metal film’s front surface by a non-Gaussian temporal profile laser pulse. The size of the film spot and its sides are far greater than the thickness of the film. They can therefore be considered as a one-dimensional issue.

At that point, the administering one-dimensional conditions of a homogeneous, isotropic, thermo-viscoelastic medium with fractional relaxation operators in the missing of a body force is given by:

The displacement components have the form:

$$u_x = u(x,t), \quad u_y = u_z = 0. \tag{13}$$

The strain-displacement relation:

$$e = \frac{\partial u}{\partial x}. \tag{14}$$

The equation of motion takes the form

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{2}{3}\hat{\boldsymbol{R}}_\beta \left(\frac{\partial^2 u}{\partial x^2}\right) + \hat{\boldsymbol{R}}_\alpha \left(\frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial \hat{T}}{\partial x}\right), \qquad 0 < \alpha, \beta \le 1. \tag{15}$$

The x-component of normal stress is given by the constitutive equation:

$$\sigma_{xx} = \frac{2}{3}\hat{\boldsymbol{R}}_\beta \left(\frac{\partial u}{\partial x}\right) + \hat{\boldsymbol{R}}_\alpha \left(\frac{\partial u}{\partial x} - 3\alpha_T \hat{T}\right) \qquad 0 < \alpha, \beta \le 1. \tag{16}$$

The heat conduction equation is given by:

$$k \frac{\partial^2 \Theta}{\partial x^2} = \rho C_E \left(\frac{\partial \Theta}{\partial t} + \tau_o \frac{\partial^2 \Theta}{\partial t^2}\right) + 3\alpha_T T_o \hat{\boldsymbol{R}}_\alpha \left(\frac{\partial^2 u}{\partial x \partial t} + n_o \tau_o \frac{\partial^3 u}{\partial x \partial t^2}\right), \; 0 < \alpha \le 1. \tag{17}$$

Let us introduce the following non-dimensional variables:

$$x^* = \frac{x}{L}, \; u^* = \frac{x}{L}, \; t^* = \frac{c_o}{L}t, \; t_p^* = \frac{c_o}{L}t_p, \; \theta^* = \frac{\gamma}{K}\theta, \; \sigma_{xx}^* = \frac{\sigma_{xx}}{K}, R_1^* = \frac{2}{3K}R_1,$$
$$R_2^* = \frac{1}{K}R_2, L = \frac{1}{c_o \eta_o}, \; \varepsilon = \frac{\gamma^2 T_o}{kK\,\eta_o}. \tag{18}$$

Substituting from Eq. ((18) into Eqs. (15)-((17), and dropping the primes for accommodation, we acquire the accompanying arrangement of non-dimensional conditions:

$$\frac{\partial^2 \Theta}{\partial x^2} = \left(\frac{\partial \Theta}{\partial t} + \tau_o \frac{\partial^2 \Theta}{\partial t^2}\right) + \frac{\varepsilon R_1 \tau^\alpha}{\Gamma(1-\alpha)} \int_0^t (t-\xi)^{-\alpha} \frac{\partial}{\partial \xi}\left(\frac{\partial^2 u(x,\xi)}{\partial x \partial \xi} \mathrm{d}\xi\right.$$
$$\left. + n_o \tau_o \frac{\partial^3 u(x,\xi)}{\partial x \partial \xi^2}\right) \mathrm{d}\xi \quad , \tag{19}$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{R_2 \tau^\alpha}{\Gamma(1-\alpha)} \int_0^t (t-\xi)^{-\alpha} \frac{\partial}{\partial \xi}\left(\frac{\partial^2 u(x,\xi)}{\partial x^2} - \frac{\partial \hat{T}(x,\xi)}{\partial x}\right) \mathrm{d}\xi$$
$$+ \frac{R_1 \tau^\beta}{\Gamma(1-\beta)} \int_0^t (t-\xi)^{-\beta} \frac{\partial}{\partial \xi}\left(\frac{\partial^2 u(x,\xi)}{\partial x^2}\right) \mathrm{d}\xi \quad , \tag{20}$$

$$\sigma_{xx} = \frac{R_2 \tau^\alpha}{\Gamma(1-\alpha)} \int_0^t (t-\xi)^{-\alpha} \frac{\partial}{\partial \xi}\left(\frac{\partial u(x,\xi)}{\partial x} - \hat{T}(x,\xi)\right) \mathrm{d}\xi$$
$$+ \frac{R_1 \tau^\beta}{\Gamma(1-\beta)} \int_0^t (t-\xi)^{-\beta} \frac{\partial}{\partial \xi}\left(\frac{\partial u(x,\xi)}{\partial x}\right) \mathrm{d}\xi. \tag{21}$$

Limiting cases

(i) In the theory of coupled thermoelasticity

1- The equations (19)-(21) in the limiting case $n_o = \beta = \alpha = 0, \tau_o = \upsilon = 0, R_1 = 2\mu,$ $R_2 = K$ transforms to the works of Biot [1].

(ii) In the theory of coupled thermo-viscoelasticity

1- The equations (19)-(21) in the limiting case $n_o = \beta = \alpha = 0, \tau_o = \upsilon = 0, \; R_1, R_2 > 0$ we obtain the works of Atkinson and Craster [15].

(iii) In the theory of generalized thermoelasticity with one relaxation time

The equations (19)-(21) in the limiting case $\beta = \alpha = \upsilon = 0,\ \tau_o > 0, n_0 = 1,\ R_1 = 2\mu, R_2 = K$ lead to the works of Lord-Shulman [3] and Sherief and Ezzat [7].

(iv) In the theory of generalized thermoelasticity with two relaxation times

The equations (19)-(21) in the limiting case $n_o = \beta = \alpha = 0,\ \tau_o > 0, \upsilon > 0,\ R_1 = 2\mu, R_2 = K$ lead to the works of Green-Lindsay [4] and Ezzat [9].

(v) In the theory of generalized thermo-viscelasticity with one relaxation time

The equations (19)-(21) in the limiting case $\beta = \alpha = \upsilon = 0,\ \tau_o > 0, n_0 = 1,\ R_1, R_2 > 0$ lead to the works of Ezzat et al. [16].

(vi) In the theory of generalized thermo-viscelasticity with two relaxation times

The equations (19)-(21) in the limiting case $\beta = \alpha = 0, \tau_o > 0, \upsilon > 0, n_o = 1, R_1, R_2 > 0$ yield the work of Ezzat [21].

(vii) In the theory of generalized thermo-viscoelasticity with fractional relaxation operator

The equations (19)-(21) in the limiting case $\beta > 0, \alpha > 0, \tau_o > 0, \upsilon = 0,\ \ n_0 = 1$, transform to the work of Ezzat et al. [32].

4. A problem for a thin film layer

As shown in Figure 1, we consider a homogenous, isotropic thermo-viscoelastic metal film with fractional relaxation operators occupying the region $0 \le x \le L$ and resting on a nonconducting rigid foundation at the plane $x = L$. The surface $x = 0$ is irradiated uniformly by a laser pulse heat flux with non-Gaussian temporal profile and traction free as follows [47]:

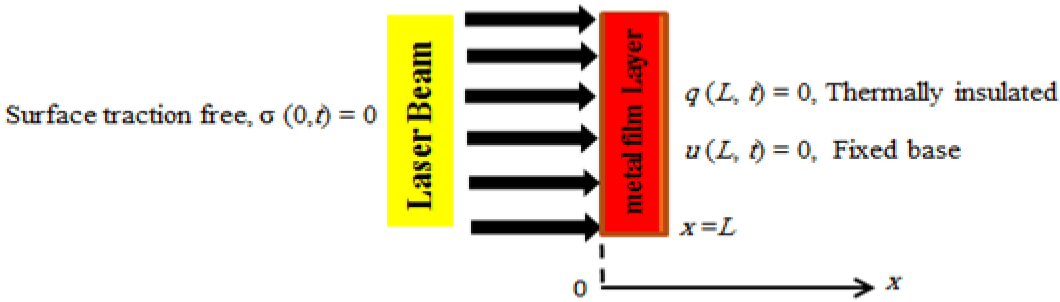

Fig. 1. Schematic geometry of metal film subjected to a laser-pulse heat flux

$$-k\,\frac{\partial \Theta(0,t)}{\partial x} = q_o \frac{t^2}{16 t_p^3} e^{-\frac{t}{t_p}}, \quad \sigma(0,t) = 0, \tag{22}$$

where q_o is a constant and t_p is the pulsing heat flux characteristic time.

At the rigid base $x = L$

$$q(L,t) = 0, \qquad u(L,t) = 0, \tag{23}$$

where q denotes the component of the heat flux vector perpendicular to the film surface.

Condition (22) means that the film surface is acted on by a laser pulse heat flux at the time t = 0, while condition (23) means that the rigid base of the film layer is thermally insulated.

5. The analytical solutions in the Laplace transform domain

We assume the initial state of the medium is quiescent, i.e.

$u(x,0) = \dot{u}(x,0) = \sigma(x,0) = \dot{\sigma}(x,0) = \Theta(x,0) = \dot{\Theta}(x,0) = 0$.

Performing the Laplace change with parameter s characterized by the relation $L\{\psi(x,t)\} = \overline{\psi}(x,s) = \int_0^\infty e^{-st}\psi(x,t)\,dt$,

of both sides Eqs. (14) and (19)-(23), we get a coupled framework of the taking after equations, we obtain:

$$\bar{e} = \frac{\partial \bar{u}}{\partial x}, \tag{24}$$

$$\left(D^2 - s - \tau_o s^2\right)\overline{\Theta} = \varpi_2 \varepsilon s(1 + n_o \tau_o s) D\bar{u}, \tag{25}$$

$$\left(D^2 - \varpi_1 s^2\right)\bar{u} = \varpi_1 \varpi_2 \left(1 + \upsilon s\right) D\,\overline{\Theta}, \tag{26}$$

$$\bar{\sigma}_{xx} = \frac{1}{\varpi_1} D\,\bar{u} - \varpi_2 \left(1 + \upsilon s\right)\overline{\Theta}, \tag{27}$$

where $D = \frac{\partial}{\partial x}$, $\varpi_1 = \frac{1}{R_1(\tau s)^\beta + R_2(\tau s)^\alpha}$, $\varpi_2 = R_2(\tau s)^\alpha$, since $L\left\{t^{-\beta}\right\} = \Gamma(1-\beta)/s^{1-\beta}$,

$L\left\{R_1(t,\beta)\right\} = \frac{R_1(\tau s)^\beta}{s}$ and $L\left\{R_2(t,\alpha)\right\} = \frac{R_2(\tau s)^\alpha}{s}$.

The mechanical boundary conditions

$\bar{\sigma}(0,s) = 0$, (28)

$\bar{u}(L,s) = 0$. (29)

The thermal boundary conditions

$$\overline{\Theta}'(0,s) = -\frac{q_o t_p}{8(st_p + 1)^3}, \tag{30}$$

$$\bar{q}(L,s) = 0. \tag{31}$$

The general solution of Eq. (25) for a bounded layer is assumed to be

$$\bar{u}(x,s) = A\cosh k_1 x + B\sinh k_1 x + C\cosh k_2 x + D\sinh k_2 x\ , \tag{32}$$

where *A*, *B*, *C*, and *D* are some parameters depending on *s* and *L* and k_1, k_2 are the roots with positive real parts of the characteristic equation

$$k^4 - \left[\varpi_1 s^2 + s(1 + \tau_o s) + \varepsilon \varpi_1 \varpi_2^2 s\left(1 + \upsilon s\right)\left(1 + n_o \tau_o s\right)\right]k^2 + \varpi_1 s^3 (1 + \tau_o s) = 0, \tag{33}$$

where *k* are the roots of the biquadratic equation (33) are k_1^2 and k_2^2 satisfy the relations:

$$k_1^2 + k_2^2 = \varpi_1 s^2 + s(1 + \tau_o s) + \varepsilon \varpi_1 \varpi_2^2 s\left(1 + \upsilon s\right)\left(1 + n_o \tau_o s\right), \tag{34a}$$

$$k_1^2\ k_2^2 = \varpi_1 s^3 (1 + \tau_o s). \tag{34b}$$

From Eqs. (26) and (32), we get

$$\overline{\Theta}(x,s) = \frac{1}{\varpi_1 \varpi_2 \left(1 + \upsilon s\right)}\left[\frac{A\left(k_1^2 - \varpi_1 s^2\right)}{k_1}\sinh k_1 x + \frac{B\left(k_1^2 - \varpi_1 s^2\right)}{\xi_1}\cosh k_1 x + \frac{C\left(k_2^2 - \varpi_1 s^2\right)}{k_2}\sinh k_2 x + \frac{D\left(k_2^2 - \varpi_1 s^2\right)}{k_2}\cosh k_2 x\right]. \tag{35}$$

Using Fourier law of heat conduction and Eq. (31), we obtain

$$\overline{\Theta}'(L,s) = 0. \tag{36}$$

Equations (27) and (28) can be combined to give

$$\bar{u}'(0,s)=\frac{q_o t_p\,\varpi_1\varpi_2\left(1+\upsilon s\right)}{8(st_p+1)^3}. \tag{37}$$

Using the boundary conditions (30), (36), and (37), in Eqs. (32) and (35), the parameters *A*, *B*, *C*, and *D* can be deduced as

$$\begin{aligned} A&=-\frac{k_1 q_o t_p\varpi_1\varpi_2(1+\upsilon s)}{8(st_p+1)^3(k_1^2-k_2^2)}\tanh k_1L, & B&=\frac{k_1 q_o t_p\varpi_1\varpi_2(1+\upsilon s)}{8(st_p+1)^3(k_1^2-k_2^2)} \\ C&=\frac{k_2 q_o t_p\varpi_1\varpi_2(1+\upsilon s)}{8(st_p+1)^3(k_1^2-k_2^2)}\tanh k_2L, & D&=-\frac{k_2 q_o t_p\varpi_1\varpi_2(1+\upsilon s)}{8(st_p+1)^3(k_1^2-k_2^2)} \end{aligned}. \tag{38}$$

Replacement from Eq. (38) in (32) and (35), we have

$$\bar{u}(x,s)=-\frac{q_o t_p\varpi_1\varpi_2(1+\upsilon s)}{8(st_p+1)^3(k_1^2-k_2^2)}\left[k_1\frac{\sinh k_1(L-x)}{\cosh k_1L}-k_2\frac{\sinh k_2(L-x)}{\cosh k_2L}\right], \tag{39}$$

$$\begin{aligned}\bar{\Theta}(x,s)=\frac{q_o t_p}{8(st_p+1)^3(k_1^2-k_2^2)}&\left[\frac{\left(k_1^2-\varpi_1 s^2\right)}{k_1}\frac{\sinh k_1(L-x)}{\cosh k_1L}\right.\\ &\left.-\frac{\left(k_2^2-\varpi_1 s^2\right)}{k_2}\frac{\sinh k_2(L-x)}{\cosh k_2L}\right].\end{aligned} \tag{40}$$

Substituting Eqs. (39) and (40) into Eq. (27), one obtains

$$\bar{\sigma}(x,s)=\frac{q_o t_p\varpi_1\varpi_2(1+\upsilon s)s^2}{8(st_p+1)^3(k_1^2-k_2^2)}\left[\frac{\cosh k_1(L-x)}{\cosh k_1L}-\frac{\cosh k_2(L-x)}{\cosh k_2L}\right]. \tag{41}$$

Those complete solutions in the Laplace transform domain.

6. Inversion of the Laplace transforms

We shall now outline the method used to invert the Laplace transforms in the above equations. Let $\bar{f}(s)$ be the Laplace transform of a function $f(t)$. The inversion formula for Laplace transforms can be written as Honig and Hirdes [48]

$$f(t)=\frac{e^{\varsigma t}}{2\pi}\int_{-\infty}^{\infty}e^{ity}\,\bar{f}(\varsigma+iy)\,\mathrm{d}y,$$

where ς is an arbitrary real number greater than all the real parts of the singularities of $\bar{f}(s)$.

Expanding the function $h(t)=\exp(-\varsigma t)f(t)$ in a Fourier series in the interval $[0,2\nu]$, we obtain the approximate formula [48]:

$$f(t)\approx f_N(t)=\frac{1}{2}c_0+\sum_{k=1}^{N}c_k,\ \text{ for }\ 0\le t\le 2\nu, \tag{42}$$

where

$$c_k=\frac{e^{\varsigma t}}{\nu}\mathrm{Re}\left[e^{ik\pi t/\nu}\,\bar{f}\left(\varsigma+ik\pi/\nu\right)\right]. \tag{43}$$

Two methods are used to reduce the total error. First, the 'Korrektur' method is used to reduce the discretization error. Next, the ε-algorithm is used to reduce the truncation error and therefore to accelerate convergence.

The Korrektur-method uses the following formula to evaluate the function $f(t)$

$$f(t)=f_{NK}(t)=f_N(t)-e^{-2\varsigma m}f_{N'}(2\nu+t). \tag{44}$$

We shall now describe the ε-algorithm that is used to accelerate the convergence of the series in (42). Let N be an odd natural number and let $s_m = \sum_{k=1}^{m} c_k$ be the sequence of partial sums of (42). We define the ε-sequence by

$\varepsilon_{0,m} = 0, \varepsilon_{1,m} = s_m, \quad m = 1, 2, 3, ...$

and $\varepsilon_{n+1,m} = \varepsilon_{n-1,m+1} + 1/\left(\varepsilon_{n,m+1} - \varepsilon_{n,m}\right), \quad n, m = 1, 2, 3, ...$

It can be shown from Refs. [48,49] that the sequence $\varepsilon_{1,1}, \varepsilon_{3,1}, ..., \varepsilon_{N,1}, ...$ converges to $f(t) - c_0/2$ faster than the sequence of partial sums.

7. Calculation results and discussions

The technique dependent on a Fourier series extension proposed by Honig and Hirdes [48] is received to alter the Laplace change in the previous section. The numerical code has been readied utilizing Fortran 77 programming language. For the numerical program, the accuracy maintained was five digits.

For the sake of simplicity, all the diagrams are plotted by taking the mechanical and thermophysical properties of the film as follows [50,51]. So as to translate the numerical calculations, a gold film of 1.2 nm thickness is subjected to femtosecond pulsed laser heating, and the mechanical and thermophysical properties of the film are shown in Table 1.

Table 1. Values of the constants [54,55]

$\rho = 1.93\times10^3 kg\ /\ m^3$	$k = 0.55\ J\ /\ m.sec.K$	$E = 525x10^7 N\ /\ m^2$
$C_E = 1.4x10^3\ J\ /\ kg.K$	$\lambda = 453.7\times10^7\ N\ /\ m^2$	$\mu = 194\times10^7\ N\ /\ m^2$
$\gamma = 210\times10^4 N\ /\ m^2 K$	$\eta_o = 3.36\times10^6\ sec/\ m^2$	$C_o = 2200\ m\ /\ sec$
$t_p = 100\,\mathrm{fs}$	$\alpha_T = 14.2\times10^{-6} K^{-1}$	$T_o = 300\ K$
$\tau_o = 0.02\ s$	$\upsilon = 0.03\ s$	$\varepsilon = 0.12$

The numerical system laid out above was utilized to get the dimensionless temperature, stress, and displacement distributions of a gold film versus dimensionless distance x , the femtosecond beat laser warming was switched on throughout the interval $(0 \le x \le 1.2)$. The outcomes have appeared in Figs. 2-4. In the present area, we have endeavored to demonstrate the impact of the fractional-order parameter, time among the idea of different physical fields. The computations were carried out for one value of time, namely t = 0.1 and two values of α and β, namely, $\alpha = \beta = 0.0$ (thermo-viscoelasticity theory with relaxation operators) and $\alpha = \beta = 0.5$ (generalized thermo-viscoelasticity theory with fractional relaxation operators). The temperature, stress, and displacement distributions are obtained and plotted. In these figures, solid lines represent the solution obtained in the frame of Biot theory and dashed lines represent Green-Lindsay theory, while dotted lines represent Lord-Shulman theory.

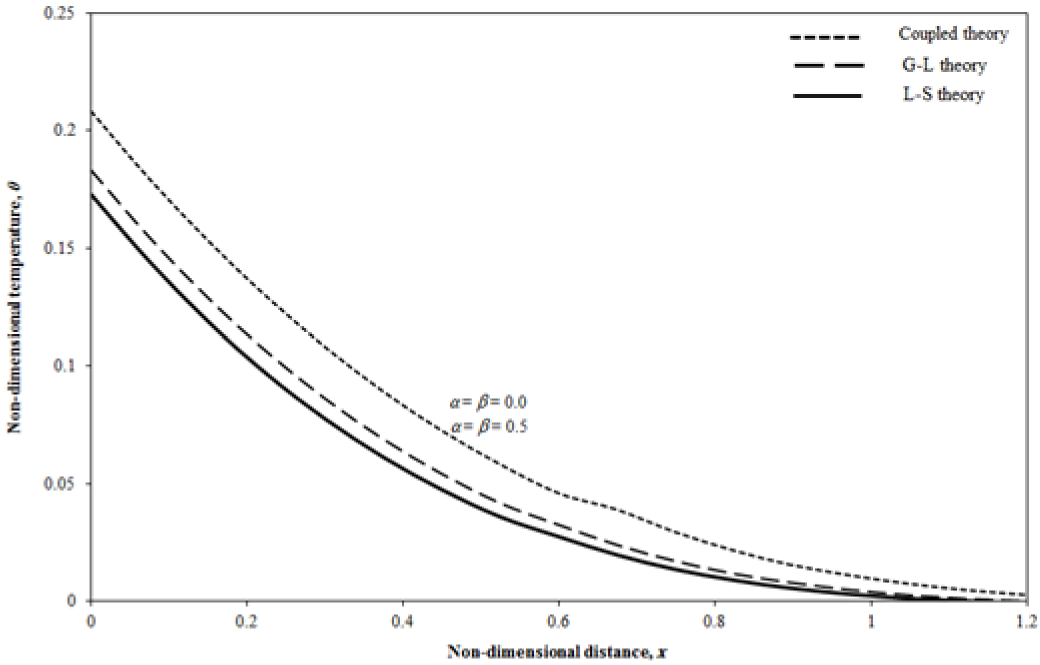

Fig. 2. Effect of fractional parameters α and β on temperature distribution for the different theories for t = 0.1

We noticed that the temperature fields have not been affected when postponement α, β increased.

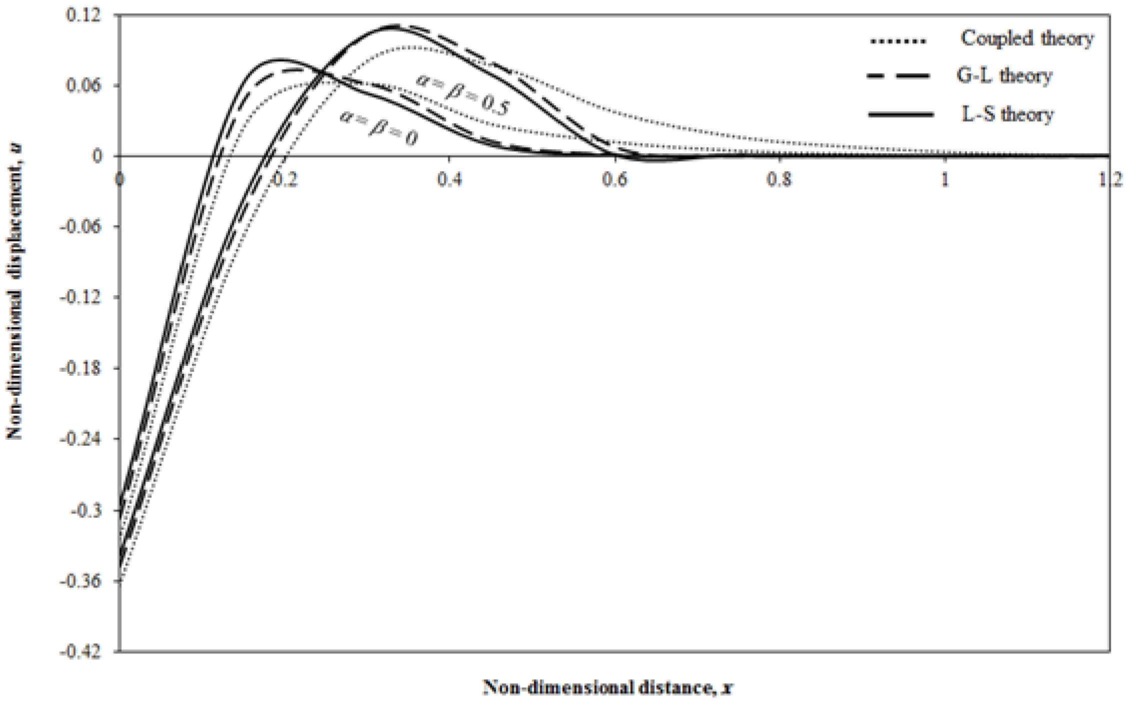

Fig. 3. Effect of fractional parameters α and β on displacement distribution for the different theories for t = 0.1

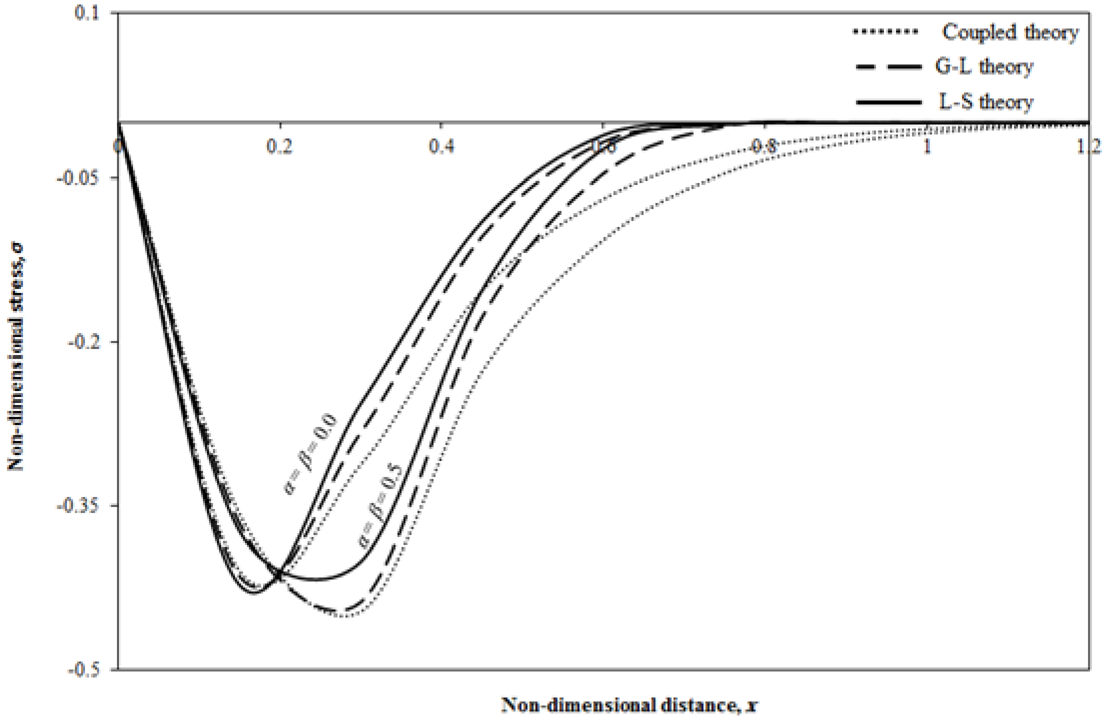

Fig. 4. Effect of fractional parameters α and β on stress distribution for the different theories for t = 0.1

Figures 3 and 4 portray the variety of displacement u and thermal stress σ on the front surfaces of metal with L thickness versus distance x for three theories. For $\alpha = \beta = 0.0$, the results are in line with all the previous results of the combined applications, the widespread thermo-viscoelasticity with one period of relaxation (L-S theory) [3] and the widespread thermal-viscoelasticity with two periods of relaxation (G-L theory) in the different fields [4]. For $\alpha = \beta = 0.5$ the solution appears like the widespread thermo-viscoelasticity theories; this is a very interesting finding because the current hypothesis will retain the benefit of the generalized theory, i.e. thermal and mechanical effect reaction does not instantly exceed infinity but is restricted to the small region of space that spans over time.

8. Concluding remarks

1) A mathematical model of the thermo-viscoelastic metal film with fractional relaxation operators, irradiated uniformly by a laser pulse with non- Gaussian form is constructed.
2) In contrast to fractional models for relaxation operators in thermo-viscoelastic materials, it is more practical to measure the current one using integer-order differentials and integrals.
3) We may describe the materials by fractional relaxation operators in compliance with the proposed model, and the results of the study.

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Nomenclature

x	one-dimensional space variable
λ, μ	Lame' constants
C_E	specific heat at constant strains
K	$= \lambda + (2/3)\mu$, bulk modulus
C_o^2	$= \dfrac{K}{\rho}$, longitudinal wave speed
ε_{ij}	components of strain tensor
e_{ij}	components of strain deviator tensor
σ_{ij}	components of stress tensor
S_{ij}	components of stress deviator tensor
e	$= \varepsilon_{ii}$, Dilatation
Q	energy source of the laser pulse
q	heat flux
E	modulus of elasticity
k	thermal conductivity
t_p	time duration of the laser pulse
x_o	absorptive depth of the heat energy
R_o	rejectivity of the irradiated surface
L	non-dimensional film thickness

R	relaxation functions
T	absolute temperature
u_i	components of displacement vector
α_T	coefficient of linear thermal expansion
γ	$= 3K\,\alpha_T$
δ_{ij}	Kronecker's delta.
T_o	reference temperature
ν	Poisson's ratio
ε	$= \dfrac{\gamma^2 T_0}{k\,\eta_o\,\rho C_0^2}$, Thermal coupling parameter
Θ	$= T - T_0$, such that $\lvert \Theta / T_0 \rvert << 1$,
ρ	mass density
τ	the ratio of the shear viscosity to Young's modulus
τ_o, υ	two relaxation times
α, β	fractional orders
t	time
$\Gamma(.)$	Gamma function

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