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DEFORMATION INSTABILITY IN CRYSTALLINE ALLOYS: LUDERS BANDS

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Abstract. A mechanism of localization of plastic deformation at high temperatures is investigated in the framework of the autowave model. A model of the formation and propagation of Luders bands is proposed. It is established that a Luders band is a wavefront of the plastic deformation rate. From this article's point of view, the conditions for the formation of a Luders band are decisive for the interpretation of the fluidity serration. The critical value of the deforming stress is determined, at which the homogeneous deformation becomes unstable with respect to the localized flow in the form of Luders bands. In the present consideration, we interpret the Luders band as a wave of switching of the plastic deformation rate. At low stresses, dislocations surrounded by clouds of impurity atoms move slowly and cannot provide a deformation comparable to that, which is produced by a testing machine. As the stress increases, the generation of "fast" dislocations begins, which are formed either due to the detachment of dislocations from the clouds of impurity atoms. As a result, this state, having arisen, propogates as a running front forming a shear band of "fast" dislocations.

Keywords: localization of deformation, plastic deformation, high temperatures, autowaves, Luders bands

1. Introduction

It is known [1,2] that the instability of plastic flow of poly- and monocrystals in alloys at elevated temperatures is typically accompanied by localization of macroplastic deformation. The last one is realized in the form of a localized plastic deformation wave, which is usually named Chernov-Luders band (or simply Luders band). The emergence of Luders bands is concominant phenomenon to different unstable regimes of plastic deformation in alloys [3-5]. For example, in BCC-alloys, where the stress-strain curve has the upper and lower yield stresses, the band originates in domains, where there are stress concentrators, on the sample surface. In the stage of being formed, the shear band increases until the plastic shear zone covers the whole sample cross-section. Luders bands are also observed under conditions of the jump-like deformation (the Portevin-LeChatelier effect) [6-9]. In this case, the irregularity of load fluctuations is due to the stochastic occurrence and propagation of localized shear bands in a sample.

After Cottrell's works [2], it seems reasonable that the appearance of Luders bands is related directly to the alloy dynamical aging, that is, to the formation of dissolved atoms "atmospheres" on mobile dislocations. This leads to the nonlinear dependence of the dislocation braking force on the velocity and, as a consequence, to the instability and inhomogeneity of the plastic flow.

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This paper attempts to provide a possible explanation of the Luders bands formation based on the autowave model. In a sense, a distributed model, proposed below, generalizes the discrete model [4].

2. Autowave model

Let us consider the behavior of an ensemble of dislocations in a slip band of width L. We choose the axis $0x$ in the direction of a given dislocation slip system. Let the distribution of dislocations in the slip band be characterized by their densities $\rho_+(x,t)$ and $\rho_-(x,t)$, and such that $\rho_+^0 = \rho_-^0 = \rho_0/2$ in the equilibrium state. Let us denote by $v(x,t)$ the average velocity of dislocations of the same sign (for example, positive ones).

We assume that the deviation of the dislocation density from the stationary value is insignificant, then the plastic deformation in a shear band in active loading regime can be described by the following system of equations

$$m^*(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}) = b(\tau + \tau^{int}) - F(v), \quad (1)$$

$$\frac{\partial \tau^{int}}{\partial t} = -\frac{\tau^{int}}{t_a} + \gamma_1 \frac{\partial^2 \dot{\varepsilon}}{\partial x^2}, \quad (2)$$

$$\frac{\partial \varepsilon}{\partial t} = b\rho_0 v, \quad (3)$$

$$\frac{\partial \tau}{\partial t} = G^*[\dot{\varepsilon}_0 - \frac{b\rho_0}{L_p}\int_0^{L_p} v(x,t)dx]. \quad (4)$$

The equation (1) is the equation of motion of the dislocation with effective mass m^*, τ is the shear stress in the acting slipping system determined by the external stress σ according to the relation $\tau = (\sigma - \sigma_D)/\overline{m}$ ($\overline{m}$ is the Taylor factor, σ_D is the Hall-Petch stress), τ^{int} is internal stress field from the dislocation charge system, $F(v)$ is the braking force per unit dislocation length, caused by the dynamical aging of alloys [10]. If the aging time is small (compared with the time scale of load jump), then the description can be limited to the stationary distribution of impurity atoms around a moving dislocation. In this case, $F(v)$ has an N -shaped form. Let us restrict our consideration to this case.

We believe that the internal stresses in the slipping system, in the case of a polycrystal, are entirely determined by the interaction of dislocations with grain boundaries. Correspondingly, equation (2) takes into account the fact that the elastic fields induced at the boundaries (in accordance with the continual limit of the Ballou-Bilby formula $\tau^{int} = \gamma_1 \partial_{xx}^2 \varepsilon$) relax due to fine turning. The parameter $\gamma_1 \approx \alpha_g G d^2$ serves as a measure of the elastic correlation of grains ($\alpha_g \approx 1$, d is grain size), t_a is the characteristic time of plastic accommodation, i.e., the relaxation time to a stationary value of the Burgers vector induced by plastic deformation at the grain boundaries.

Equation (4) is nothing else than the Gilman-Johnston equation for the active loading mode [3], which takes into account the dynamics of the load change σ under the condition that the stretching rate of the crystalline sample is constant. Here $\dot{\varepsilon}_0$ is a given strain rate in the slip band, $G^* = \kappa h_0/\overline{m}\zeta_1 S$ is effective modulus of elasticity, κ is rigidity of the machine-sample system, h_0 and S are height and cross-section of the sample, L_p is length of the plastic deformation zone, ζ_1 is a geometric factor of the order of 1.

Equations (1)-(4) of the source system belong to the class of equations describing autowave processes in nonequilibrium media [11]. Solutions in the form of running excitation fronts and running pulses are the basic solutions that are most often implemented in experiments on the observation of autowaves in such media. The goal is to match the autowave process of plastic deformation with solutions of the specified type. Therefore, we focus on the conditions for the implementation of such solutions for a specific system of equations (1)–(4).

Analysis of equations (1)–(4) shows that, in addition to the homogeneous stationary solution $v_0 = \dot{\varepsilon}_0 / b\rho_0$, $b\tau_0 = F(v_0)$, other types of solutions are possible, if the function $F(v)$ has a non-linear N-shaped form, that is, there is an interval with negative friction. Such dependence of the braking force on velocity of dislocations was observed in alloys and it is due to the interaction mechanisms of dislocations with atoms of the dissolved substance [12,13].

It is convenient to distinguish between the following three ranges of dislocation velocities. At velocities $v < V_{max} = \alpha^2 v_t / e$ ($v_t = D_s kT / bW_m$, D_s is diffusion coefficient of impurity atoms) the dislocation moves with an almost saturated atmosphere. At $v > V_{min} = D_s \alpha / b$, the dislocation moves without an atmosphere and the braking is due to statistically distributed impurity atoms. In the intermediate region $V_{max} < v < V_{min}$ a dislocation, having originally a saturated atmosphere, lose it with the increase of the velocity resulting in a decrease of braking.

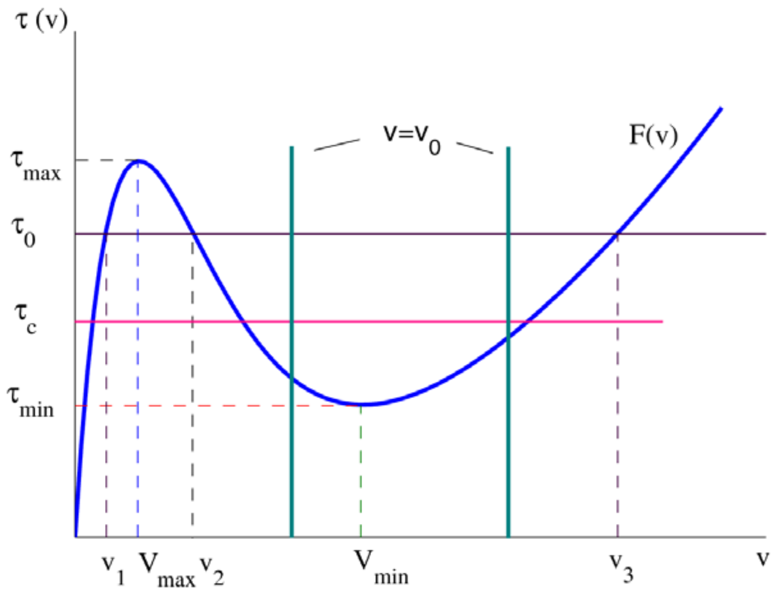

Fig. 1. The phase plane of the variables τ, v. The curve $F(v)$ corresponds to the dependence of the braking force of dislocation by the Cottrell's impurity atmosphere on the dislocation velocity. Here $\tau_{max} = \pi c_0 W_m (1 + \alpha^2 / 2e)$, $\tau_{min} = \pi c_0 W_m$, $\alpha = W_m / kT$, W_m is maximal binding energy of the dislocation with impurity atoms, c_0 is equilibrium concentration of the impurity atoms [12]. τ_0 is deforming shear stress, τ_c is the critical stress above which the occurrence and propagation of Luders bands becomes possible

On phase plane of the variables τ and v the straight line $v - v_0 = 0$ can intersect the curve $b\tau = F(v)$ in different ways depending on the loading conditions (Fig. 1). If it intersects the graph $F(v)$ in the stable domain ($F'(v)|_{v=v_0} > 0$), then the plastic deformation process is developed by the propagation of the excitation fronts of the deformation rate, interpreted as the formation and propagation of the Luders bands. If the intersection occurs in the unstable domain ($F'(v)|_{v=v_0} < 0$), then, additionally with the propagation of the wavefronts, a jump-like self-oscillating mode of the unstable flow is established, which manifests itself as the Portevin-LeChatelier effect. Next, we consider the first case.

3. The Luders band

We assume that the deformation conditions of the test sample are such that the load τ changes quasi-statically. In this case, the system of equations (1)–(4) reduces to the system (1)–(3) with $\tau = const$.

Next, we make some assumptions about the nature of the dislocation ensemble evolution. Usually, there is a hierarchy of relaxation times in complex dynamical systems. So let us assume that the plastic accommodation time τ_a is significantly less than the typical time scale of changes of the variables v and ε. In this approximation

$$\tau^{int} \approx \tau_a \gamma_1 \frac{\partial^2 \dot{\varepsilon}}{\partial x^2} = \tau_a \gamma_1 b \rho_0 \frac{\partial^2 v}{\partial x^2}, \tag{5}$$

then the system of equations (1)–(3) reduces to the equation

$$m^* \left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right) = \eta \frac{\partial^2 v}{\partial x^2} + b\tau - F(v), \tag{6}$$

where $\tau = const$, $\eta = b^2 \rho_0 \tau_a \gamma_1$.

The line $b\tau = b\tau_0 = const$ can intersect the curve $F(v)$ in different ways. We are interested in the case when the intersection occurs at three points (see Fig. 1), for example, v_1, v_2 and v_3 ($v_1 < v_2 < v_3$). Then the equation (6) has three equilibrium states: two stable, corresponding to the velocities $v = v_1$ and $v = v_3$, and one unstable corresponding to the velocity $v = v_2$. Approximating $F(v)$ by a piecewise linear function, one can obtain approximate values of the intersection points of $b\tau_0$ and $F(v)$:

$$\begin{aligned} v_1 &= \frac{2\alpha D_s}{\pi c_0 b^2 W_m (2e + \alpha^2)}, \quad v_3 = \frac{\tau D_s}{\alpha^3 \pi c W_m}, \\ v_2 &= \frac{D_s}{b}\left[\alpha - \frac{2(e-1)}{\alpha}\left(\frac{\tau}{\pi c_0 W_m} - 1\right)\right]. \end{aligned} \tag{7}$$

To find stationary running solutions to the equation (6), let us go to the special variable $\xi = x - ct$, assuming $v = v(\xi)$. To get the results in an analytical form, we approximate $f(v) = b\tau - F(v)$ by the cubic trinomial $f(v) = \kappa(v - v_1)(v - v_2)(v - v_3)$. Then it is easy that the velocity of the wavefronts propagation is expressed as

$$c = c_\pm = v_2 + (\delta\eta/m^*)(v_1 + v_3 - 2v_2), \tag{8}$$

where

$$\delta = \delta_\pm = \frac{m^*}{4\eta}\left(1 \pm \sqrt{1 + 8\eta\kappa/m^{*2}}\right). \tag{9}$$

Since the type of wave solutions depends on the parameter δ, then as it follows from (8),(9) there are two types of solitary waves corresponding to two values of δ ($\delta_+ > 0$, $\delta_- < 0$) and having different velocities c_+ and c_- of propagating wavefronts. Accordingly, the profile of running wave solutions has the step-like form

$$v(x,t)=\frac{v_3+v_1}{2}+\frac{v_3-v_1}{2}th\frac{\xi-\xi_0}{\Lambda_\delta}, \tag{10}$$

where $\Lambda_\delta=\delta(v_3-v_1)^{-1}$, $\Lambda_\pm=|\Lambda_\delta|$ is a characteristic front width of the corresponding wave, ξ_0 is an arbitrary constant depending on the initial conditions.

It follows from the expression (10), that the positive value $\delta=\delta_+$ corresponds to the switching wave from the state v_1 to the state v_3 (softening wave), and the negative value $\delta=\delta_-$ corresponds to the drop wave from the state v_3 to the state v_1 (hardening wave).

The width of the wavefronts depends on the value of the parameter $\psi=8\eta\kappa/m^{*2}$. If $\psi<<1$, then the front width of the softening wave is $\Lambda_+\approx 2\psi m^*/4\kappa(v_3-v_1)$, while for the hardening wave we have the front width $\Lambda_-\approx m^*/\kappa(v_3-v_1)$. So, in this case, $\Lambda_->>\Lambda_+$. If $\psi>>1$, then $\Lambda_+\approx\Lambda_-\approx\sqrt{2\eta/\kappa}/(v_3-v_1)$.

The velocity of the softening wave c_+ is greater than the velocity of the hardening wave c_- under the condition $v_1+v_3-2v_2>0$. In this case, the fronts run in the same direction with a relative velocity

$$\Delta c=c_+-c_-\approx\frac{\sqrt{1+8\eta\kappa/m^{*2}}}{4}(v_1+v_3-2v_2). \tag{11}$$

In the case of $v_1+v_3-2v_2<0$, the softening wave does not appear, since it is annihilated by the hardening wave. Thus, there is a critical value of the stress $\tau=\tau_c$, corresponding to the condition $\Delta c=0$, below which ($\tau<\tau_c$) the perturbation damps and the deformation of the material occur macroscopically smoothly without the appearance of inhomogeneous wave structures.

When a crystal is loaded and the plastic deformation rate is maintained constant, the boundary conditions in the considered problem have the form: $v(-\infty,t)=v_3$, $v(\infty,t)=v_1$. These conditions are satisfied only by the softening wave. This wave, when passing through the entire crystal, creates a localized zone of plastic deformation with an increased dislocations velocity v_3 there.

From the condition $\Delta c=v_1+v_3-2v_2=0$ and the relation $\sigma=\sigma_D+\overline{m}\tau$ it is not difficult to obtain the critical load value corresponding to τ_c:

$$\sigma_c\approx\sigma_D+\frac{\overline{m}\pi c_0W_m(2W_m^2+4(e-1)k^2T^2)}{W_m^2+4kT^2(e-1)+2k^2T^2(1+2ek^2T^2/W_m^2)}. \tag{12}$$

The graph of the temperature dependence $\sigma_c=\sigma_c(T)$ at typical values for some alloys ($W_m=0.1\text{eV},\overline{m}=3$), $\sigma_D=10$ MPa) is shown in Fig. 2 at different values of the impurity concentration. By comparison with experimental curves for the physical yield point of a number of alloys [14,15], it can be noted that the dependence (12) finds experimental confirmation if σ_c is considered as a physical yield point.

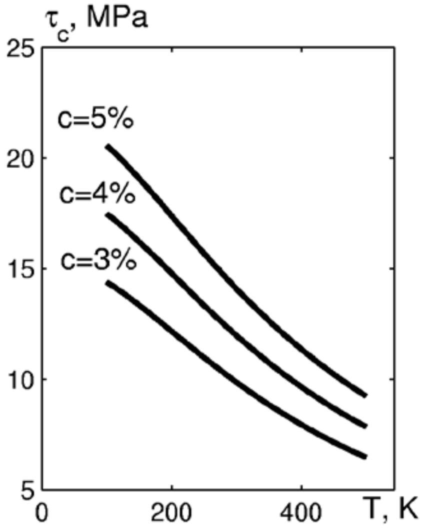

Fig. 2. Graph of the temperature dependence $\sigma_c = \sigma_c(T)$. The value σ_c is interpreted as the physical yield point

4. Conclusions

In the present consideration, we interpret the Luders band as a wave of switching of the plastic deformation rate. At low stresses, dislocations surrounded by clouds of impurity atoms move slowly and cannot provide a deformation comparable to that, which is produced by a testing machine. This causes an increase in stress. As the stress σ increases, the generation of "fast" dislocations begins, which are formed either due to the detachment of dislocations from the clouds of impurity atoms. However, when $\sigma < \sigma_c$ "fast" dislocations are captured by impurity atmospheres (this state is unstable with respect to the switching wave with a relative velocity $\Delta c < 0$), so their share in plastic deformation at stress below the critical one is relatively small and the load continues to increase. When the condition $\sigma = \sigma_c$ is realized in some domain of the sample, a switching wavefront is formed there. At stresses above critical $(\sigma > \sigma_c)$, nascent "fast" dislocations having the velocity $v = v_3(\sigma)$ are now in a stable state relative to their capture by clouds of impurity atoms. As a result, this state, having arisen, propagates as a running front forming a shear band of "fast" dislocations.

From these positions, the conditions for Chernov-Luders band formation are crucial for the interpretation of the sharp yield point observed in BCC crystals. The critical value σ_c, at which the homogeneous deformation becomes unstable with respect to the localized flow in the form of Chernov - Luders bands, should be considered as the lower yield stress. On the other hand, the upper yield stress is the value of the applied stress, which is reached by the test machine as the equilibrium state $\sigma = \sigma_0$, $v = v_0$ is approached. Since the value $v_0 = \dot{\varepsilon}_0 / b\rho_0$ is large at the beginning of the deformation (the initial dislocation density ρ_0 is small), then $v_0 > v_3$ and the test machine passes the critical value σ_c quickly. With increasing of deformation ρ_0 increases, v_0 decreases and, correspondingly, the equilibrium value $\sigma_0 = \sigma(v_0)$ tends asymptotically to σ_c. As a result, the load drops and a yield point appears. Under the condition $\sigma_0 \approx \sigma_c$, a stationary flow mode is established (observed as a "plateau" on the tension curve), characterized either by a further expansion of the Chernov-Luders band or by the appearance of new shear bands.

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