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Analytical description of the Bauschinger effect using experimental data and the generalized Masing principle

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ABSTRACT
The research is devoted to the problem of obtaining an analytical expression for the dependence of stresses on strains during unloading and subsequent loading by the reverse sign force, taking into account the Bauschinger effect. The assessment of the deformed state was carried out using the use of Hencky strains. The mathematical model was developed under the assumption of a cyclically ideal material. To process the experimental data, the generalized Masing principle was applied, which is used to describe the ideal Bauschinger effect. On the basis of experimental data for the 45HGMA material, curves of changes in the coefficients of the Bauschinger effect were obtained using the least squares method. The results obtained showed sufficient convergence with experimental data. The results of the study can be used in solving elastic-plastic problems for various processes of alternating loading using the deformation theory of plasticity, when a description of the deformation diagram of the material is required, using the analytical dependence of stresses on strains according to the hypothesis of a single curve.
KEYWORDS
Bauschinger effect • generalized Masing principle • deformation diagram • alternating loading stress • logarithmic strain

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Introduction

The issues of determining stresses and strains in the presence of finite areas of plastic strains, in cases where external loads are applied once, are currently well studied, especially in the framework of deformation theory and in the theory of ideal plasticity. However, in engineering practice, numerous cases can be found when external forces are applied repeatedly (including with a sign change) and the behavior of the elastic-plastic system differs significantly from the case of a single loading.

If, during the first loading, plastic strains occurred in the entire body or in some of its finite areas, then after the removal of external forces, it will not return to its original state, certain residual strains and stresses will occur in it. With subsequent loading by an arbitrary system of forces, the body will behave differently than in the case of its loading from the initial state. For example, if a sample, previously stretched beyond the elastic limit, is compressed, then plastic strains will appear at a lower axial load value than at the previous stretching. The elastic limit during subsequent compression decreases to a greater extent, the higher the stress of the previous stretching was. Similar changes are observed in the case of stretching after pre-compression. Hysteresis loops are also observed during unloading and re-stretching. This behavior of the material is called the Bauschinger effect.

The first successful attempt to explain the Bauschinger effect and the hysteresis loops observed during repeated loading was made by G. Masing, who proceeded from the fact that individual grains in a polycrystalline body, due to their different orientations and the anisotropy of crystals, have different mechanical characteristics and deform differently [1]. He proposed an interesting scheme of elastic-plastic deformation of a sample made of polycrystalline material, which is quite fully described in the work of Moskvitin V.V. [1]. The Masing model or principle is widely used by various authors in the analysis of alternating and cyclic loading [2–4].

According to the analysis of the current state of the research issue, most of the scientific works aimed at studying the Bauschinger effect are of an applied nature, in particular, taking into account the Bauschinger effect during experimental research was considered in [5]. Experimental studies of the behavior of the material, as well as its hardening under cyclic loading are reflected in [6,7]. The issues of deformation under cyclic loading, alternating loading in the case of loading and unloading were studied in [8–11]. Dynamic models of elastic-plastic deformation, which were studied in the works [12,13], also seem to be very relevant in recent years. Taking into account the Bauschinger effect is important for fatigue failure problems, as well as for the analysis of damage accumulation in the case of isotropic and kinematic hardening, to which the works are devoted [14–17].

In addition, the complexity of the study of the Bauschinger effect is due to the need to take into account the nonlinearity of the deformation process. Some issues of plastic deformation under the nonlinear hardening law require the use of numerical research methods, some aspects of the analytical description of the plastic behavior of the material are considered in [18]. Also, the problem of solving problems of nonlinear plasticity was considered in [19–21]. The deformation hardening that occurs in this case is important for assessing the ultimate deformations, which is noted in [22].

Taking into account the Bauschinger effect is also used in problems of elastic-plastic deformation in the calculation of residual stresses. The effect of the Bauschinger effect on the picture of the residual stress-strain state is analyzed in [23,24].

In this article, we will consider an important issue for the analytical description of the Bauschinger effect of obtaining an expression of the stress – strain dependence during unloading and subsequent loading by the reverse sign force. To do this, we will use the generalized Masing principle [1].

Generalized Masing principle

The results of theoretical studies made it possible for Masing to suggest that the curve of repeated alternating loading coincides with the corresponding curve at the first loading, but constructed in axes with a doubled scale and reverse direction. This assumption will be called the Masing principle [1]. If, at the first loading, the stresses and the corresponding strains are connected by the equation:

$$\sigma^{(1)} = \Phi(e), \qquad (1)$$

then according to the Masing principle, the stress differences $\bar{\sigma} = \sigma' - \sigma^{(2)}$ and the strain differences $\bar{e} = e' - e$ satisfy the equation (Fig. 1) [1]:

$$\bar{\sigma} = 2\Phi\left(\frac{\bar{e}}{2}\right), \tag{2}$$

or

$$\sigma^{(2)} = \sigma' - 2\Phi\left(\frac{e' - e}{2}\right), \tag{3}$$

where $\sigma^{(1)}$ – equation relating stresses and Hencky strains at the first loading in the coordinate system (σ, e); $\bar{\sigma}$ – equation describing the curve of repeated alternating loading in the coordinate system $(\bar{\sigma}, \bar{e})$; $\sigma^{(2)}$ – equation relating stresses and strains under repeated alternating loading in the coordinate system (σ, e); σ' и e′ – maximum stresses and strains obtained during the first loading.

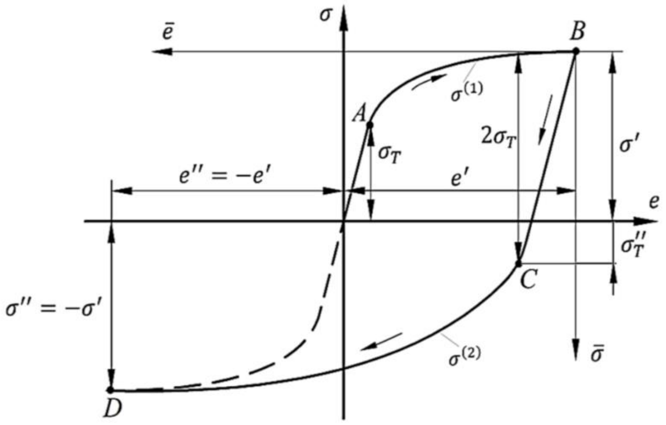

Fig. 1. Diagram of repeated alternating deformation of Masing [1]

If the diagram of the behavior of the material under the first loading (1) is known, then Eq. (3) relates the stresses $\sigma^{(2)}$ and the corresponding deformations e under repeated alternating loading. It should be noted that if the maximum stresses σ' and strains e' were reached during the first loading, then with repeated loading in the opposite direction, the yield strength is determined by the equation (Fig. 1) [1]:

$$\sigma_T'' = 2\sigma_T - \sigma'. \tag{4}$$

Thus, if, after stretching to a plastic state, the sample is unloaded and then compressed, then the yield strength decreases, and, as follows from Eq. (4), the sum of the absolute values of the maximum tensile stress and the new yield strength during compression is equal to twice the yield strength of the undeformed material. In this case, the Bauschinger effect is usually called ideal [25] and a classical hysteresis loop is observed.

Let us consider the important question for the analytical description of the Bauschinger effect of obtaining an expression of the stress – strain dependence during unloading and subsequent loading by the force of the reverse sign. The difficulties encountered in this case are due to the fact that, unlike the primary deformation curve, when the stress $\sigma^{(1)}$ is a function of only e, with repeated loading, the stress $\bar{\sigma}$ is a function not only of $\bar{e}$, but also, as experiments have shown, depends on the parameter e' – the maximum strain obtained during the first loading.

As one of the possible generalizations of the Masing principle, the assumption is introduced that the deformation curve under alternating loading coincides with the curve of the previous loading $\sigma^{(1)} = \Phi(e)$, but with a change in scale α_e times along the strain axis and α_σ times along the stress axis, and in the general case α_e and α_σ can be functions of the preceding deformation at $e' \geq e_T$: $\alpha_e = \alpha_e(e')$, $\alpha_\sigma = \alpha_\sigma(e')$ [1]. In this case (Fig. 1):

$$\bar{\sigma} = \alpha_\sigma \Phi\left(\frac{\bar{e}}{\alpha_e}\right), \tag{5}$$

or

$$\sigma^{(2)} = \sigma' - \alpha_\sigma \Phi\left(\frac{e' - e}{\alpha_e}\right). \tag{6}$$

The functions $\alpha_e(e')$ and $\alpha_\sigma(e')$ are called Masing functions and they must satisfy certain conditions. If the stretching and compression curves of the source material coincide with each other, then for $e' = e_T$ there should be [1]:

$$\alpha_e(e_T) = 2,\ \alpha_\sigma(e_T) = 2. \tag{7}$$

If we take into account the deformation anisotropy of elastic constants, then before the appearance of plastic strains during repeated alternating loading:

$$\bar{\sigma} = \bar{E}(e')\,\bar{e},$$

where $\bar{E}(e')$ – variable modulus of elasticity, depending on the previous strain. In this case, according to [1]:

$$\bar{\sigma} = \alpha_\sigma E \frac{\bar{e}}{\alpha_e}, \qquad \frac{\alpha_\sigma}{\alpha_e} = \frac{\bar{E}(e')}{E}. \tag{8}$$

Therefore, if the function of changing the Young's variable modulus $\bar{E}(e')$ is known, then from experiments on alternating loading it remains to determine only one function depending on the parameter e' – $\alpha_e(e')$. If the deformation anisotropy of elastic constants is not taken into account, then $\bar{E} = E$. In this case, according to [1] it follows from Eq. (8) that:

$$\alpha_\sigma(e') = \alpha_e(e') = \alpha(e'). \tag{9}$$

As experimental data [26] show, the changes in the Young's modulus under alternating loading are insignificant and, as calculations show, these changes are commensurate with the errors in determining elastic constants, which is introduced by the hypothesis of incompressibility of the material. In the future, when considering alternating loading, we will not take into account the anisotropy of elastic constants.

Definition of Masing functions

The Bauschinger effect is defined as a decrease in the yield strength of a material under compression as a result of previous tensile strain. The majority of researchers agree that the degree of preliminary strain has a significant impact on the Bauschinger effect [27,28]. To quantify the Bauschinger effect, we take the ratio of the yield strength $\sigma''_T(e')$ observed during compression to the maximum stress in the previous loading σ' (Fig. 1).

We call it the coefficient of the Bauschinger effect and denote β_T:

$$\beta_T = \beta_T(e') = \frac{\sigma''_T(e')}{\sigma'(e')}, \tag{10}$$

where e' – the maximum strain obtained during the first loading; $\sigma'(e')$ – the maximum stress corresponding to this strain.

In accordance with the generalized Masing principle, by analogy with Eq. (4), we can write $\sigma''_T(e') = \alpha(e')\sigma_T - \sigma'(e')$, or taking into account Eq. (10):

$$\alpha = \alpha(e') = \frac{\sigma'(e')}{\sigma_T}\big(1 + \beta_T(e')\big). \quad (11)$$

Determination of the Masing function based on experimental data

For the calculations, we take the experimental results presented in [29]. In this work, the results of tests on steels of grades St3, SHL-4 (10HCND), 09G2 and on alloy steels having $\sigma_{0.2} = 600 \div 1200$ MPa are presented. The research was carried out by the method of "probing". The sample was loaded to a certain specified strain, determined by the residual strain $\varepsilon_{\text{long}}^{\text{res}}$ and the stress σ^D, then unloading and loading of the opposite sign was carried out to a residual strain of 0.2 %, determined by the stress $\sigma_{0.2}^{\text{C}}$. Then the direct loading was repeated until the next level of the specified strain was reached, "probing" was carried out again, etc. The Bauschinger effect in this case was characterized by the ratio $\sigma_{0.2}^{\text{C}}/\sigma^D$ and we will denote it $\beta_{0.2}^D$.

To display the test results on the deformation diagram (σ_i, e_i) we will use the hypothesis of a single curve, put forward by Ludwik and described in a scientific work [25], and recalculate relative strains into logarithmic, and conditional stresses into true stresses according to the formulas: $\varepsilon^D = \left(\varepsilon_{long}^{res} + \frac{\sigma^D}{E}\right)$, $e'_i = ln(1 + \varepsilon^D)$, $\sigma'_i = \sigma^D(1 + \varepsilon^D)$, $\sigma''_{i0.2} = \sigma_{0.2}^{\text{C}}\left(1 - \left(0.002 + \frac{\sigma_{0.2}^{\text{C}}}{E}\right)\right)$.

Calculations have been carried out for 45HGMA steel, which has $\sigma_{0.2} = 800$ MPa, which corresponds to the data given in [29]. The calculation results are presented in Table 1.

Table 1. Steel $\sigma_{0,2} = 800$ MPa (45HGMA - GOST 4543-2016)

$\varepsilon_{\text{long}}^{\text{res}}$	ε^D	e'_i	σ^D, MPa	σ'_i, MPa	$\sigma_{0,2}^{\text{C}}$, MPa	$\sigma''_{i0,2}$, MPa	$\beta_{0,2}^D = \frac{\sigma_{0,2}^C}{\sigma^D}$	$\beta_{0,2} = \frac{\sigma''_{i0,2}}{\sigma'_i}$
0.0022	0.0061	0.0061	825	830.1	675	671.5	0.82	0.81
0.0072	0.0113	0.0112	863	872.8	488	485.9	0.57	0.56
0.0140	0.0182	0.0181	888	904.2	450	448.1	0.51	0.50
0.0216	0.0259	0.0256	900	923.3	425	423.3	0.47	0.46
0.0288	0.0332	0.0327	925	955.7	438	436.2	0.47	0.46
0.0358	0.0402	0.0394	925	962.2	400	398.4	0.43	0.41
0.0432	0.0477	0.0466	938	982.7	413	411.4	0.44	0.42
0.0504	0.0549	0.0534	938	989.5	425	423.3	0.45	0.43
0.0572	0.0617	0.0598	938	995.8	425	423.3	0.45	0.43
0.0644	0.0689	0.0666	938	1002.6	388	386.5	0.41	0.39
0.0716	0.0761	0.0733	938	1009.4	388	386.5	0.41	0.38
0.0800	0.0845	0.0811	938	1017.2	425	423.3	0.45	0.42
0.0858	0.0902	0.0864	925	1008.4	400	398.4	0.43	0.40

To describe the deformation diagram of the material, we apply a linear-power approximation [30] in the form:

$$\sigma_i = \Phi(e_i) = \begin{cases} Ee_i, if\ e_i \le e_{iT} \\ A(e_i - e_{0i})^n, if\ e_i > e_{iT} \end{cases}, \quad (12)$$

where σ_i – intensity of stresses; e_i – intensity of Hencky strain; e_{iT} – the value of the strain intensity corresponding to the transition point of the linear dependence to the power-law (yield strength); $e_{0\text{i}}$ – the magnitude of the displacement of the power function along the strain axis; A и n – power function parameters.

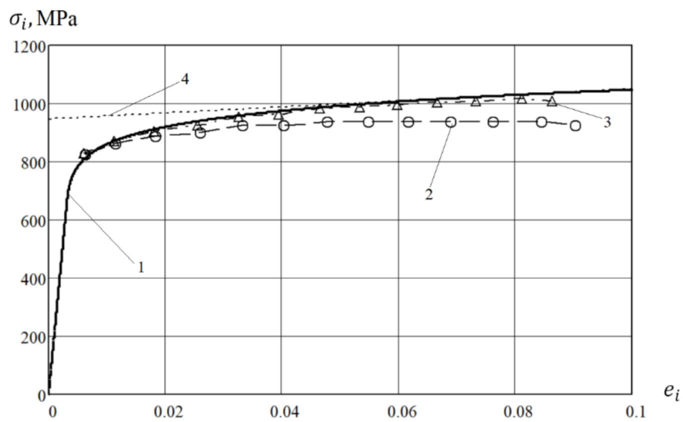

Fig. 2. Approximation of the deformation diagram of 45XGMA steel (E=210000 MPa, e_{iT} = 0.00367, σ_{iT} =771.14 MPa, A =1301.29 MPa, n =0.0933) and experimental data for steel $\sigma_{0,2}$=800 MPa (Table 1): 1 – diagram constructed according to equation (12); 2 – diagram of conditional stresses σ^D; 3 – diagram of true stresses σ_i'; 4 – tangent, characterizing the property of the deformation diagram of the III kind [30]

Figure 2 shows the results of the approximation of the 45HGMA steel deformation diagram and experimental data. As you can see, the results match quite well. In the case of approximation of the deformation diagram by Eqs. (5), (6), and (12), taking into account Eq. (9), will be written as:

$$\bar{\sigma}_i = \alpha(e_i')\Phi\left(\frac{\bar{e}_i}{\alpha(e_i')}\right) = \begin{cases} \alpha(e_i')E\left(\frac{\bar{e}_i}{\alpha(e_i')}\right), if\ 0 \le \bar{e}_i \le \alpha(e_i')e_{iT} \\ \alpha(e_i')\,A\left(\frac{\bar{e}_i}{\alpha(e_i')} - e_{0i}\right)^n, if\ \ \bar{e}_i > \alpha(e_i')e_{iT} \end{cases}, \tag{13}$$

and

$$\sigma_i^{(2)} = \sigma_i'(e_i') - \alpha(e_i')\Phi\left(\frac{(e_i' - e_i)}{\alpha(e_i')}\right) =$$
$$= \begin{cases} \sigma_i'(e_i') - \alpha(e_i')E\left(\frac{(e_i' - e_i)}{\alpha(e_i')}\right), if\ e_i' \ge e_i \ge e_i' - \alpha(e_i')e_{iT} \\ \sigma_i'(e_i') - \alpha(e_i')\,A\left(\frac{(e_i' - e_i)}{\alpha(e_i')} - e_{0i}\right)^n, if\ e_i < e_i' - \alpha(e_i')e_{iT} \end{cases}, \tag{14}$$

where σ_i' и e_i' – accordingly, the intensity of stresses and the intensity of strains obtained during the first loading.

To determine the Masing function $\alpha(e')$, we use the Eq. (11), replacing, in accordance with the hypothesis of a single curve e' and $\sigma'(e')$ on e_i' and $\sigma_i'(e_i')$:

$$\alpha(e_i') = \frac{\sigma_i'(e_i')}{\sigma_T}\left(1 + \beta_T(e_i')\right). \tag{15}$$

According to the available experimental data, we obtained the Bauschinger effect coefficient (Table 1) $\beta_{0.2}(e_i') = \sigma_{i0.2}''/\sigma_i'$. To analytically describe the Bauschinger effect using Masing functions (15), we need to determine the Bauschinger effect coefficient $\beta_T(e_i') = \sigma_{iT}''/\sigma_i'$.

If we determine the difference between $\sigma_{i0.2}''$ and σ_{iT}'', then we can obtain an equation connecting β_T and $\beta_{0.2}$:

$$\beta_T(e_i') = \frac{\sigma_{iT}''}{\sigma_i'} = \frac{\sigma_{i0,2}''}{\sigma_i'} - \frac{(\sigma_{i0,2}'' - \sigma_{iT}'')}{\sigma_i'} = \beta_{0,2}(e_i') - \frac{(\sigma_{i0,2}'' - \sigma_{iT}'')}{\sigma_i'}. \quad (16)$$

Considering that $(\sigma_{i0,2}'' - \sigma_{iT}'') = (\bar{\sigma}_{i0,2} - \bar{\sigma}_{iT})$ we define $\bar{\sigma}_{iT}$ and $\bar{\sigma}_{i0,2}$ when unloading and subsequent loading of the opposite sign in accordance with Eq. (13):

$$\bar{\sigma}_{iT} = \alpha(e_i')\Phi\left(\frac{\bar{e}_{iT}}{\alpha(e_i')}\right) = \alpha(e_i')E\left(\frac{\bar{e}_{iT}}{\alpha(e_i')}\right) = E\bar{e}_{iT}, \quad (17)$$

$$\bar{\sigma}_{i0,2} = \alpha(e_i')\Phi\left(\frac{\bar{e}_{i0,2}}{\alpha(e_i')}\right) = \alpha(e_i')\, A\left(\frac{\bar{e}_{i0,2}}{\alpha(e_i')} - e_{0i}\right)^n, \quad (18)$$

where $\bar{e}_{iT}$ и $\bar{e}_{i0.2}$ – strains corresponding to the stress $\bar{\sigma}_{iT}$ and $\bar{\sigma}_{i0.2}$ when describing the deformation diagram by the function (12).

Thus, it is possible to write

$$(\sigma_{i0.2}'' - \sigma_{iT}'') = \alpha(e_i')\, A\left(\frac{\bar{e}_{i0.2}}{\alpha(e_i')} - e_{0i}\right)^n - E\bar{e}_{iT}. \quad (19)$$

Moreover, $\bar{e}_{iT} = \alpha(e_i')e_{iT}$, and $\bar{e}_{i0,2}$ is determined numerically from the equation:

$$\alpha(e_i')\, A\left(\frac{\bar{e}_{i0.2}}{\alpha(e_i')} - e_{0i}\right)^n = E(\bar{e}_{i0.2} - 0.002). \quad (20)$$

It follows from Eqs. (16), (19), and (20) that the function $\beta_T(e_i')$ depends on $\alpha(e_i')$. Since initially the value of the Masing function is not known, we take $\alpha(e_i') = 2$ for calculations in the first approximation, as for the ideal Bauschinger effect. Next, using Eqs. (16) and (18), we determine the Masing function $\alpha(e_i')$ by Eq. (15).

As calculations show, in the first approximation, the values of $\sigma_{i0,2}^{(2)}$ calculated by Eq. (14) differ significantly from the values of $\sigma_{i0.2}''$ obtained experimentally (Table 1). In this case, the average arithmetic error of determining $\sigma_{i0.2}^{(2)}$ is 9.5 %, and the maximum is 20.7 %. To improve the result, a second approximation is carried out. In this case, the Masing function $\alpha(e_i')$ obtained in the first approximation is substituted into Eqs. (19), (20) and using Eqs. (15) and (16), the Masing function $\alpha(e_i')$ is refined in the second approximation.

Figure 3 shows the functions $\beta_T = \beta_T(e_i')$ and $\beta_{0.2} = \beta_{0.2}(e_i')$ obtained in the second approximation by processing experimental data. Since the Bauschinger effect begins to manifest itself only after the e_{iT} yield stress deformations or the $e_{i0,2}$ conditional yield stress deformations are reached during the first loading, the graphs of the function $\beta_T = \beta_T(e_i')$ *and* $\beta_{0.2} = \beta_{0.2}(e_i')$ were constructed in such a way that $\beta_T = \beta_T(e_{iT}) = 1$ and $\beta_{0.2} = \beta_{0.2}(e_{i0.2}) = 1$. In addition, when constructing the curve $\beta_T(e_i')$, the experimental points $\beta_{0.2}(e_i')$ shifted not only along the ordinate axis down by an amount $(\bar{\sigma}_{i0.2} - \bar{\sigma}_{iT})$, but also along the abscissa axis to the left by an amount $(\bar{e}_{i0.2} - \bar{e}_{iT})$.

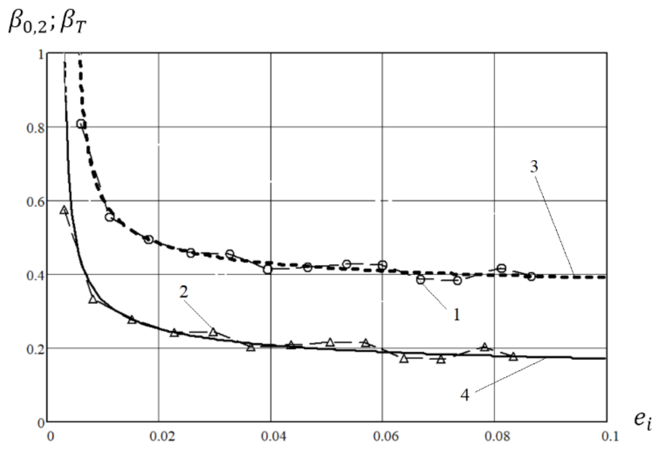

Fig. 3. Curves of changes in the coefficients of the Bauschinger effect $\beta_{0.2} = \beta_{0.2}(e_i')$ and $\beta_T = \beta_T(e_i')$ obtained on the basis of processing experimental data (Table 1) and approximation by equations of the form (21): 1 – experimental values $\beta_{0,2}$ (Table 1); 2 – β_T values obtained by processing experimental data; 3 – function $\beta_{0.2} = \beta_{0.2}(e_i')$; 4 – function $\beta_T = \beta_T(e_i')$

To approximate the curves obtained experimentally, an equation of the form $\beta(e_i') = a/(e_i' + b)^c + d$ was used. Provided that the curve passes through a given point ($e_i'0$; β0), the equation will take the form:

$$\beta(e_i') = \frac{a}{(e_i' + b)^c} - \frac{a}{(e_i'0 + b)^c} + \beta 0. \tag{21}$$

In our case $\beta 0 = 1$ and $e_i'0 = e_{i0.2}$ when determining $\beta_{0.2}(e_i')$ and $e_i'0 = e_{iT}$ when determining $\beta_{\mathrm{T}}(e_i')$. The coefficients *a, b* and *c* were determined from the given points by the least squares method.

After determining the Bauschinger coefficient $\beta_T = \beta_T(e_i')$, by the Eq. (15) we determine the Masing function $\alpha = \alpha(e_i')$. Figure 4 shows graphs of these functions, constructed from experimental data and according to the Eq. (15).

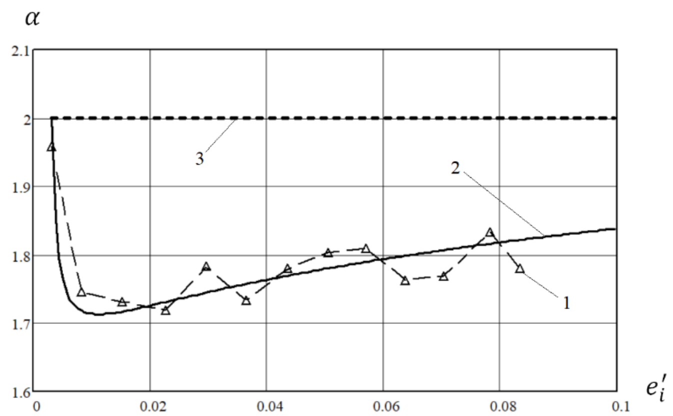

Fig. 4. The Masing function $\alpha = \alpha(e_i')$, determined by experimental data and by Eq. (15): 1 – the values of α obtained by processing experimental data; 2 – the function $\alpha = \alpha(e_i')$, obtained by Eq. (15); 3 – the values of the Masing function for an ideal Bauschinger effect, $\alpha = \alpha(e_i') = 2 = \mathrm{const}$

Figure 5 shows diagrams of deformation of 45HGMA steel under reverse loading. The loading diagram was constructed using Eq. (12). Diagrams of unloading and further loading of the opposite sign were constructed on the basis of the generalized Masing principle according to Eq. (14) using the resulting Masing function shown in Fig. 4. As can be seen from the comparison of the calculation results with experimental data on the definition of $\sigma''_{i0.2}$, good convergence is obtained. The average arithmetic error of determining $\sigma^{(2)}_{i0.2}$ is 4.8 %, and the maximum is 15.1 %. Moreover, the greatest error is observed for the initial loading stage at $e'_i < 0.01$, which corresponds to the descending branch of the graph $\alpha = \alpha(e'_i)$ (Fig. 4). In addition, a comparison of the results obtained with the calculated data for the ideal Bauschinger effect shows (Fig. 4) that the application of the generalized Masing principle allows get more accurate results.

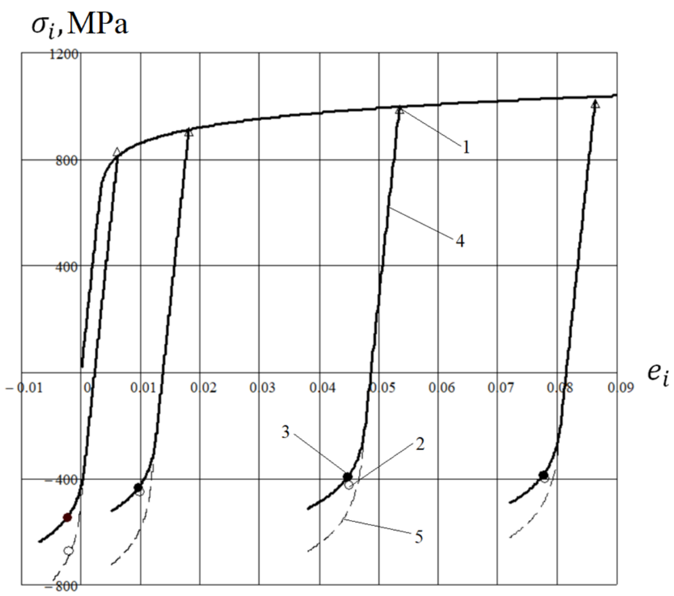

Fig. 5. Diagrams of deformation of 45HGMA steel under reverse loading: 1 – experimental data of the initial loading level σ'_i; 2 – experimental data on the definition of $\sigma''_{i0,2}$ under reverse loading; 3 – calculated data on the definition of $\sigma^{(2)}_{i0.2}$ under reverse loading; 4 – calculated diagrams of forward and reverse loading; 5 – calculated diagrams of reverse loading for the ideal Bauschinger effect

Conclusion

The possibility of an analytical description of the Bauschinger effect using experimental data allows us to obtain good convergence of the results and conclude that the generalized Masing principle can be applied in the study of various alternating loading processes.

In general, the question of the behavior of the material in the plastic region during repeated compression requires separate experimental studies, but to solve problems when repeated plastic strains are insignificant, it is possible to successfully apply the generalized Masing principle to describe the Bauschinger effect. Such tasks include, for example, calculations related to the determination of the stress-strain state and residual stresses during autofrettage of thick-walled pipes and high-pressure vessels.

References

1. Moskvitin VV. *Plasticity under variable loads*. Moscow: Publishing House of Moscow State University; 1965. (In-Russian)
2. Pisarenko GS, Mozharovsky NS. *Equations and boundary value problems of the theory of plasticity and creep. Reference manual*. Kiev: Nauk. Dumka; 1981. (In-Russian)
3. Shinkin V. Bauschinger Effect, Masing Principle and Hencky Theorem for Bending Moment at Repeated Alternating-Sign Bending of Steel Sheet. *Materials Science Forum*. 2022;1052: 346-351.
4. Miryala M, Chauhan H, Murakami M, Saitoh T, Segawa K, Kamada K, Skelton R, Maier H, Christ H. The Bauschinger effect, Masing model and the Ramberg-Osgood relation for cyclic deformation in metals. *Materials Science and Engineering: A*. 1997;238(2): 377-390.
5. Shchedrin AV, Ul'yanov VV, Skoromnov VM, Bekaev FF. Bauschinger effect in complex machining methods. *Russian Engineering Research*. 2008;28(8): 797-799.
6. Motova YA, Nikitina NY. Experimental investigation by ultrasound of engineering materials behavior under the cyclic loading. *Materials Physics and Mechanics*. 2016;28(1-2): 43-47.
7. Kravchenko GN. Influence of Cyclic Loading on the Effectiveness of Repeated Shot Hardening. *Russian Engineering Research*. 2022;42(4): 365-369.
8. Mahato JK, De PS, Sarkar A. Effect of prestrain and stress rate on Bauschinger effect of monotonically and cyclically deformed ofhc copper. In: *XVII International Colloquium on Mechanical Fatigue of Metals (ICMFM17), Verbania, 25–27 June 2014*. 2014. p.368-375.
9. Adigamov PR, Andreev PA, Rogacheva SO. Manifestation of the Bauschinger effect in alternating deformation. *Izvestiya. Ferrous Metallurgy*. 2022;65(7): 455-466. (In Russian)
10. Savchenko VG, Babeshko ME. Technique of Allowing for Plastic Strains Under Unloading in Thermoplasticity Problems for Axisymmetric Bodies. *International Applied Mechanics*. 2019;55(4): 416-425.
11. Castro E Sousa AD, Hartloper AR, Lignos DG. Cyclic Metal Plasticity Model Parameters with Limited Information: Constrained Optimization Approach. *Journal of Engineering Mechanics*. 2021;147(7): 04021035.
12. Hwan AD, Hwan DV, Samsonova VI. Hardening of thin-walled cylindrical racks by plastic deformation. *Forging and stamping production. Processing of materials under pressure*. 2019;1: 14-17.
13. Makarov PV. The Model of Dynamic Stress Relaxation of Elastoplastic Materials. *Russian Physics Journal*. 2021;63(11): 1876-1884.
14. Andrianov IK. Minimization of dies volume using uniform distribution of hollows in the conditions of multi-cycle fatigue dependence on temperature. *CIS Iron and Steel Review*. 2022;23: 33-38.
15. Kroon MA. Rubin MB. Simple Scalar Directional Hardening Model for The Bauschinger Effect Compared With a Tensorial Model. *Journal of Mechanics of Materials and Structures*. 2020;15(4): 510-539.
16. Fedorenkov DI, Kosov DA, Tumanov AV. Methodology for determining constants and parameters of the damage accumulation model with isotropic and kinematic hardening. *Physical Mesomechanics*. 2022;25(6): 63-74.
17. Temis YM. Mathematical simulation of low cycle fatigue of high-loaded engine parts. *Propulsion and Power Research*. 2018;7(4): 277-287.
18. Shinkin VN. Analytical description of metal plasticity at shift. *CIS Iron and Steel Review*. 2020;19: 66-70.
19. Zemlyanushnova NY, Porokhin AA, Zemlyanushnov NA. Stress–strain state of the valve spring in an auto engine during plastic hardening. *Russian Engineering Research*. 2016;36(7): 535-540.
20. Shen J, Wei X, Kraposhin VS, Vekshin BS. High cold plastic deformation of die steel 4Kh5VF1S subjected to hardening and tempering. *Metal Science and Heat Treatment*. 2012;53(9-10): 503-504.
21. Domichev K, Petrov A, Steblyanko P. Mathematical model shear lines for plastic deformation. *The Scientific Heritage*. 2021;61: 54-56.
22. Zheng M, Li Z. Strain-hardening effect on critical strain assessment of pipe plastic bending at buckling. *Materials Physics and Mechanics*. 2020;44(1): 48-60.
23. Abashkin EE, Burenin AA, Zhilin SG. Modeling of residual stress distribution in a welded joint. *Physics and Mechanics of Materials*. 2019;42(5): 671-689.
24. Aleksandrov S, Lyamina E, Yeau-Ren NG. Influence of the Bauschinger effect on the distribution of residual stresses in a thin hollow hyperbolic disc under external pressure. *Materials Science Forum*. 2019;972: 105-110.
25. Malinin NN. *Applied theory of plasticity and creep*. Moscow: Mashinostroenie; 1975. (In-Russian)

26. Perl M, Perry J. An Experimental-Numerical Determination of the Three-Dimensional Autofrettage Residual Stress Field Incorporating Bauschinger Effects. *Journal of Pressure Vessel Technology*. 2006;128(2): 173-178.
27. Adigamov RR, Andreev VA, Rogachev SO, Fedotov ES, Khadeev GE, Yusupov VS, Adigamov PR. Bauschinger effect during alternating deformation. *Izvestiya. Ferrous Metallurgy*. 2022;65(7):455-466. (In Russian)
28. Pykhtunova SV. On the issue of the Baushinger effect. *Quality in Materials Processing*. 2015;1(3): 75-77. (In-Russian)
29. Nigmatullin VI. Experimental investigation of the effect of preliminary plastic deformation on the behavior of structural steels under reverse loading. *Transactions of the Krylov State Research Centre*. 2011;60: 119-132. (In Russian)
30. Andrianov IK, Feoktistov SI. *Fundamentals of constructing deformation diagrams taking into account the compressibility of the material and the Bauschinger effect*. Komsomolsk-on-Amur: KnAGU; 2022. (In-Russian)

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