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# THE DYNAMIC MECHANISM OF LOW-TEMPERATURE INSTABILITY OF PLASTIC DEFORMATION IN METALS

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**Abstract.** Under the assumption of structural softening of plastic deformation in metals, a solution of the system of equations for the ensemble of dislocations is obtained describing a running front for the dislocation density (soliton). Such dislocation charge is able to destroy deformation barriers and cause an athermal mechanism of the instability of the low-temperature plastic deformation.
**Keywords:** low-temperature deformation, kinetic instability, dynamic pile-up of dislocations

## 1. Introduction

Nowadays, the unstable character of plastic deformation is considered as a universal property of solid bodies, which is able to manifest itself in a wide range of temperatures [1]. In crystalline materials, it is caused by self-consistent collective movement of dislocations under the action of external and internal stresses [2,3]. At helium temperatures, the instability of plastic deformation (the serrated deformation) has been found in a large number of materials and it is typical at very low temperatures[4, 5]. Its character is determined by numerous parameters related to both the deformation conditions (deformation rate, temperature) and the properties of the material itself (lattice type, grain size, etc.).

Several hypotheses [6,7] have been proposed to explain the physical nature of the serrated deformation. Physically, this phenomenon has much to do with the softening of the material. A typical cause of the strength loss is the anomalous damping of dislocations (negative friction) as it was first established by Cottrell [8]. This effect is most pronounced in alloys, where the dependence of the damping force $F(v)$ acting on dislocations on their velocities $v$ is depicted by an *N*-shaped curve at elevated temperatures. At low temperatures, this instability $[F(v) < 0]$ can have a dynamical nature and be due, for example, to the dispersion of the elastic moduli at high dislocation velocities [9] or to the thermoplastic instability [10,11].

In addition to this mechanism of the plastic deformation instability at low temperatures, there can be instability associated with a "dry"-friction type of the strength loss of material (structural softening). In the last case, the moving dislocations destroy obstacles that impede them (e.g., precipitates [7]) and the barrier damping of dislocations is decreased. In the framework of this model, in the present paper a theoretical study of wave modes of plastic deformation due to softening of metals at low temperatures.

## 2. Dynamical equations of dislocation ensemble

Mathematically, the problem of unstable modes of plastic deformation can be formulated using a set of nonlinear equations describing the time evolution of the density of continuously distributed dislocations. These evolution equations follow from the conservation law of the

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Burgers vector of a system of dislocations. This vector is conserved during various reactions and multiplication of dislocations [9]. The Burgers vector conservation law can be written in the form of a differential consistency equation

$\partial\alpha_{ik}/\partial t + e_{ilm}\partial j_{mk}/\partial x_l = 0,$

which relates the tensors of the density $\alpha_{ik}$ and the flux $j_{ik}$ of dislocations [9], These tensors are expressed in terms of the scalar dislocation density $\rho_a(\mathbf{r},t)$ as

$\alpha_{ik} = \sum_a l_i b_k^a \rho_a, \quad j_{ik} = e_{ijm}\sum_a l_j b_k^a \rho_a v_m^a,$

where $\mathbf{v}_a(\mathbf{r},t)$ is the average velocity of dislocation glide and the index $a$ enumerates the possible directions of the Burgers vector $\mathbf{b}$ of a dislocation relative to the unit vector $\mathbf{l}$ tangential to the dislocation line. The set of evolution equations for the dislocation density $\rho_a(\mathbf{r},t)$ follows from the consistency equation and has the form

$$\frac{\partial\rho_a}{\partial t} + \mathrm{div}\rho_a\mathbf{v}_a = G_a(\rho_a), \qquad (1)$$

which reduces to the continuity equation, if the local interaction of dislocations is neglected $(G_a = 0)$. This can be done if the plastic deformation is localized in a narrow slip band and is dynamical in character. In this case (further considered here) the equation of motion for dislocations, from which the velocity $\mathbf{v}_a(\mathbf{r},t)$ can be found, has the form [9]

$$m^*\frac{dv_k^a}{dt} = e_{ijm}l_j b_k^a\{\sigma_{mk}^{\mathrm{ext}} - \sigma_{mk}^{\mathrm{f}}(C_j,\psi_j)\} - F_i(\mathbf{v}^a). \qquad (2)$$

In the right-hand side of equation (2), the first term is the Peach-Kohler force per unit of the dislocation length, with $\sigma_{mk}^{\mathrm{ext}}$ being the component of the external stress tensor in the slip plane of dislocations; $\sigma_{mk}^{\mathrm{f}}(C_j,\psi_j)$ is the "dry"-friction stress caused by various local obstacles, which are distributed in space with density $C_j$ and have stopping-force characteristics $\psi_j$; $m^*$ is the effective dislocation mass; and $F_i(\mathbf{v}^a)$ is the dissipative friction force.

Equations (1) and (2) are derived under the assumption that the radius of curvature $R_c$ of the dislocation lines is much larger than the average spacing $\bar{r}$ between dislocations. In this approximation, dislocations as a whole can be treated as straight lines, and, when studying the evolution of the entire ensemble of dislocations, one can divide this ensemble into smaller ensembles, each of which is a system of parallel dislocation lines. In this paper, we will investigate the dynamics of one of these ensembles.

Let us consider an ensemble of dislocations in a slip band of a width $L > \bar{r}$. The $x$ axis is taken to be along the slip direction of the given slip system of dislocations. Let the plastic flow involve a subsystem of positive edge dislocations $(\mathbf{b}\perp\mathbf{l})$ characterized by a density $\rho_+(\mathbf{r},t) = \rho(\mathbf{r},t)$ with its equilibrium value being equal to $\rho_+^0(\mathbf{r},t) = \rho_0$. Then the plastic flow in the chosen slip band is described by the set of equations

$$m^*\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right) = b\{\sigma_{\mathrm{ext}} - \sigma_{\mathrm{f}}(C_j,\psi_j)\} - F(v), \qquad (3)$$

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0. \qquad (4)$$

Dry-friction stress $\sigma_{\mathrm{f}}$ is usually considered as a fitting parameter. However, in the case of structural crystal softening, the quantities $C_j$ and $\psi_j$ can be a function of dislocation density $\rho$; therefore, one should generally assume that $\sigma_f = \sigma_f(\rho)$.

The set of equations (3) and (4) has a time-independent uniform solution:

$$\rho = \rho_0, \quad v = v_0, \tag{5}$$

where $v_0$ is determined from the requirement of the right-hand side of equation (3) being equal to zero,

$$F(v_0) = b\sigma_{ext} - b\sigma_f(\rho_0) = b\sigma_t, \tag{6}$$

while the density $\rho_0$ is determined from the boundary conditions.

Let us analyze the stability of the time-independent uniform solution (5) of the set of Eqns. (3) and (4) taking into account the above assumptions about $\sigma_{int}(\rho)$ and $\sigma_f(\rho)$. Linearizing the original equations and introducing dimensionless quantities

$$n(x,t) = \frac{\rho(x,t)}{\rho_0} - 1, \quad u(x,t) = \frac{v(x,t)}{v_0} - 1, \tag{7}$$

we arrive at the equations

$$\frac{\partial u}{\partial t'} + \frac{\partial u}{\partial x'} = +an - \gamma u, \tag{8}$$

$$\frac{\partial n}{dt'} + \frac{\partial n}{\partial x'} + \frac{\partial u}{\partial x'} = 0. \tag{9}$$

Here, we have also introduced the dimensionless variables $x' = x/\tau v_0$ and $t' = t/\tau$ and the parameters $\tau = m^* v_0/b\sigma_t$, $a = -\sigma'_f(\rho_0)\rho_0/\sigma_t$, and $\gamma = F'(v_0)v_0/b\sigma_t$. For wavelike solutions $n, u \sim \exp(\lambda t' - ikx')$, we obtain the dispersion relation

$$\lambda^2 + \lambda(\gamma - 2ik) - ik(\gamma + a) - k^2 = 0,$$

from which it follows that the unstable-mode branch is characterized by $\lambda_+ = \mu + i\omega$ given by

$$\mu = -\frac{\gamma}{2} + \frac{1}{2\sqrt{2}}\left[\gamma^2 + \sqrt{\gamma^4 + 16k^2a^2}\right]^{1/2}, \tag{10}$$

$$\omega = k + \frac{ak}{\gamma + 2\mu}. \tag{11}$$

Therefore, the solution becomes unstable $(\mathrm{Re}\lambda_+ = \mu > 0)$ in two cases,

$$\gamma < 0 \tag{12}$$

and

$$a > 0, \quad \gamma > 0. \tag{13}$$

In the case of inequality (12), we have an instability that is associated with negative friction $[F'(v) < 0]$ in the dislocation dynamics (we will refer to this instability as that of the first type), while in the case of inequality (13), an instability (of the second type) occurs because the barrier damping of the dislocations in the slip band becomes weaker as the dislocation density increases.

As follows from formula (10), the instabilities corresponding to inequalities (12) and (13) occur in the wavenumber range $0 < k^2 < \infty$. The fact that the spectrum of unstable modes contains zero-wavelength harmonics signifies that the set of equations (3) and (4) has no stationary nonhomogeneous solutions in the class of continuous functions. This will not be the case if the viscosity of the dislocation "gas" is taken into account. Physically, viscosity arises because moving dislocations pass into adjacent slip planes, thereby transferring their momentum into those planes. In gases, this is known to be the reason for internal friction between layers [12].

It can easily be shown that if the right-hand side of equation (3) contains the term $\eta v_{xx}$

responsible for viscosity ($\eta$ is the coefficient of dynamic viscosity), the spectrum of the unstable modes will be limited and, therefore, the problem will become regular. In this case, expressions (10) and (11) for the branch of unstable modes are redefined by the change in a parameter $\gamma$ ($\gamma \to \gamma + \nu k^2$, where $\nu = \eta/m^* v_0^2 \tau$ is the dimensionless coefficient of kinematic viscosity).

**3. Stationary structural softening waves**

Let us consider a model that exhibits instability associated with structural softening ($a > 0$). For the sake of definiteness, we assume that the dissipative damping of the dislocations is purely viscous; i.e., $F(v) = Bv$, where $B$ is the viscous damping constant. In this case, the parameter $\gamma$ involved in the condition for the instability of plastic deformation is equal to unity. We also assume that the dependence of the dry-friction stress on the dislocation density has the form $\sigma_\mathrm{f} \sim (K + \rho)^{-1}$, where $K$ is a constant (the Michaelis-Menten law [13]). In our problem, this law can be shown to be valid by using the following line of reasoning.

Let dislocations cut obstacles (e.g., precipitates) when moving through them so that these obstacles exhibit less resistance to each subsequent dislocation moving in the same slip plane and their stopping-force characteristics $\psi_j$ decrease. On the other hand, the deformation (cutting) of an obstacle by dislocations leads to an increase in the obstacle's surface energy; therefore, the obstacles tend to relax to their initial state with initial stopping-force characteristic $\psi_{0j}$. Taking these processes into account, one can write the kinetic equation for the variable $\psi_j$

$$\frac{\partial \psi_j}{\partial t} = \frac{\psi_{0j} - \psi_j}{\tau_\psi} - \kappa \psi_j \rho, \tag{14}$$

which, combined with equations (3) and (4), forms a complete set. Here, $\tau_\psi$ is the relaxation time to equilibrium value $\psi_{0j}$ and $\kappa(C_{0j})$ is a constant characterizing the interaction between the dislocations and stoppers. We assume that the recovery of the configuration of the obstacles occurs rather quickly $(\tau_\psi << \tau)$. Therefore, $\psi_j$ varies adiabatically and one can put $\partial \psi_j/\partial t = 0$ in equation (14); from which it follows that $\psi_j(\rho) = \psi_{0j}/(1 + \kappa\tau_\psi\rho)$. Further, putting $\sigma_\mathrm{f} \sim \psi_j$, we finally obtain $\sigma_\mathrm{f}(\rho) = \sigma_{0\mathrm{f}}/(1 + \kappa\tau_\psi\rho)$, where $\sigma_{0\mathrm{f}}$ is the friction stress in the absence of softening. It should be noted that the work-softening mechanism considered above is not the only possible one for which the density of structural defects $C_j$ is constant.

With the assumptions formulated above, the original equations take the form

$$m^*\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right) = \eta\frac{\partial^2 v}{\partial x^2} + b\sigma_\mathrm{ext}\left(-\frac{b\sigma_{0\mathrm{f}}}{1 + \kappa\tau_\psi\rho}\right) - Bv, \tag{15}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0. \tag{16}$$

From equation (15), the steady-state velocity can easily be determined to be $v_0 = b[\sigma_\mathrm{ext} - \sigma_{0\mathrm{f}}/(1 + \kappa\tau_\psi\rho)]/B$. This value can differ significantly from the velocity $v = b(\sigma_\mathrm{ext} - \sigma_{0\mathrm{f}})/B$ in the absence of the softening effect. For example, if the crystal softening is limited by friction stress, we have $\sigma_\mathrm{eff} = \sigma_\mathrm{ext} - \sigma_{0\mathrm{f}} << \sigma_\mathrm{ext}$. In this case, $v << v_0 \approx b\sigma_\mathrm{ext}/B(1 + p)$, where the parameter $p = (\kappa\tau_\psi\rho_0)^{-1}$ is less than or of order unity.

We will seek inhomogeneous wavelike solutions of equations (15) and (16) in the class of self-similar solutions by putting $v = v(\xi)$ and $\rho = \rho(\xi)$, where $\xi = x - ct$. Substituting these into the original equations, we obtain

$$\eta \frac{\partial^2 v}{\partial \xi^2} + R(v)\frac{dv}{d\xi} = -Q(v) = -\frac{\partial U}{\partial v}, \tag{17}$$

$$\rho = \rho_0 \frac{c - v_0}{c - v}, \quad (v < c), \tag{18}$$

where

$$R(v) = -m^*(v - c), \quad Q(v) = B(v - v_0)\left[\frac{a(1+p)v_0}{p(c - v) + c - v_0} - 1\right].$$

Equation (17) describes a nonlinear oscillator with a damping force $R(v)v'_\xi$. Therefore, the problem is reduced to investigating the motion of a particle in an effective potential well $U(v)$, which has the form of an integral of $Q(v)$. The function $Q(v)$ vanishes at singular points $v = v_0$ and $v = v_0[1 + (c/v_0 - 1 - a(1+p))/p] = v_c$, at which the potential $U(v)$ has extremal values. The maximum of $U$ is reached at $v = \min\{v_0, v_c\}$, while the minimum is at $v = \max\{v_0, v_c\}$. The equilibrium state at $v = v_c$, which arises in the system because of the change of variables, is always absolutely unstable, in contrast to the physically reasonable state at $v = v_0$. This means that the solutions of equation (17) that describe finite motion can exist only in the vicinity of the minimum of the potential $U - U(v_0)$ for $c < v_0(1+a)$. (It is easy to verify that $v_c < v_0$ if $c < v_0(1+a)$). Possible solutions that describe infinite motion must satisfy the condition $v(\pm\infty) = v_0$, which can be the case only if $c > v(1+a)$, i.e., when the motion is represented in the $(v, v'_\xi)$ phase plane by a separatrix going out and then entering the saddle point $(v_0, 0)$.

First, we consider possible solutions of equation (17) that describe finite motion, which is a self-oscillation of the variable $v$. This oscillation will be undamped if the work of the damping force is equal to zero in the average, which will be the case if $c \le c_0 = v_0$.

The maximum wave velocity $(c = c_0)$ corresponds to small-amplitude oscillations, $|v - v_0| < v_0$. Stationary waves of a relatively large amplitude propagate with a velocity $c$, which is only slightly less than $c_0$. In this case, the stationary wavelike solutions of equations (15) and (16) correspond to the limit cycle of equation (17) in the $(v, v'_\xi)$ phase plane. The wave velocity c is uniquely determined by the wave amplitude, which, in turn, depends on the given initial conditions.

Now, we consider solitary waves in the system described by equations (17) and (18). As indicated above, equation (17) describes the motion of a particle of mass $\eta$ in potential $U(v)$ in the presence of damping force $R(v)v'_\xi$, which depends on the velocity of propagation of the solitary wave $c$. The possible values of $c$ lie in the interval $c_{\min} < c < c_{\max}$. The lower limit $c_{\min} = v_0(1+a)$ is determined by the boundary conditions $v(\pm\infty) = v_0$, while the upper limit $c_{\max} = v_0[1 + a(1+p)]$ follows from the condition $v_c < c$, i.e., from the condition for the positivity of density $\rho$.

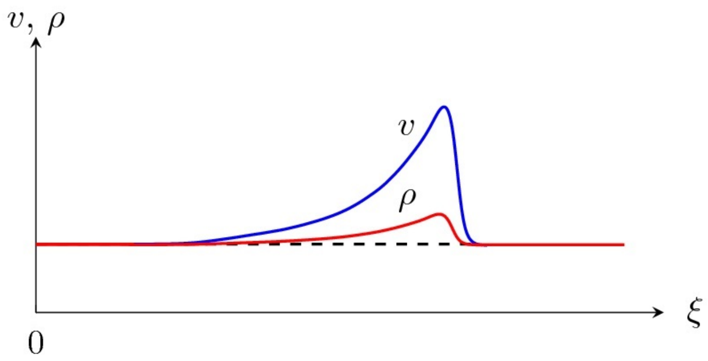


**Fig. 1.** The shape of solitary waves in the form of solitons for density $\rho(x,t)$ and velocity $v(x,t)$ of dislocations, normalized to $\rho_0$ and $v_0$, respectively

For a given potential $U(v)$, there is only one value of the velocity $c$ at which the energy loss due to damping in an infinite motion of the particle (corresponding to the motion along the separatrix in the phase plane) is strictly equal to zero. A numerical solution of equation (17) reveals that, in this case, the motion along a closed trajectory in the phase plane corresponds to a soliton-like solution (Fig. 1).

**4. Summary**

Thus, the model considered in the article describes the formation of a slip band, in the front of which there is a dislocation charge moving at a velocity close to the velocity of dislocations. Such dislocation charge is able to destroy dislocation barriers (precipitates, grain boundaries, etc.) and cause an athermal mechanism of the instability of low-temperature plastic deformation

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