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Estimation of breakdown pressure in laboratory experiments on hydraulic fracturing

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Abstract. Hydraulic fracturing is one of the most common methods used to study stress-strain state of hydrocarbon reservoir being developed. The process of fracture initiation and propagation is affected by initial stress state in the rock and tensile strength of the rock. Moreover, fracturing fluid filtered through the walls of the well into the well surrounding rock mass also affects the process of fracture propagation. The purpose of this work is to determine the best method for assessing the fracture breakdown pressure according to laboratory experiments. The experiments on hydraulic fracturing were carried out on a special laboratory setup that allows to create loading conditions on a model sample that are close to real conditions at a real field being developed. Some of the known methods based on analysis of the dependencies of borehole pressure on time were used for laboratory data processing.
Keywords: hydraulic fracturing; stress-strain state; breakdown pressure; backstress effect; laboratory experiments; data processing

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Introduction

Hydraulic fracturing is one of the most common methods used to study the stress-strain state of a hydrocarbon reservoir under development [1,2]. The hydraulic fracturing process involves a fracture initiation in the oil-saturated reservoir under development by pumping pressurized fluid into the reservoir through a specially drilled well. The resulting fracture is of interest to the researcher since it allows to estimate the values of the principal stresses acting in the undeformed formation [3].

Fracture initiation and propagation are influenced by stress distribution in the reservoir under development [4–6]. Therefore, there are various methods to determine the values of the main stresses acting in the formation using the analysis of borehole data. Some of the methods are based on the analysis of the pressure-time dependence function obtained from the well during hydraulic fracturing. For the first time, such an approach was described in the work [7].

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In addition to the initial stress-strain state of the medium, it is necessary to know the formation breakdown pressure to carry out hydraulic fracturing properly and for successful oil/gas field development [8]. This pressure can be defined as the pressure needed for tensile fracture initiation at a given depth, more precisely, at the depth of hydraulic fracturing. The paper considers one of the existing methods of theoretical assessment of formation breakdown pressure by analyzing pressure-time curves obtained during hydraulic fracturing. However, the main objective of the study was to evaluate the impact of some effects on the magnitude of formation breakdown pressure.

In particular, the authors of the work have shown that in addition to the existing stress distribution in the formation, so-called reverse stress (backstress) has a significant effect on fracture initiation. This is an additional stress of the fracturing fluid pumped into the formation and filling the porous medium of the near-well zone, which prevents fracture propagation [9–14].

A series of laboratory experiments on hydraulic fracturing was conducted within the framework of the current study. As a result, time-dependent pressure relationships for each experiment were obtained and further processed. Backstress and formation breakdown pressure were theoretically calculated for each of the experiments. The obtained theoretical values of the formation breakdown pressure were compared with the known values from the borehole data of the laboratory experiments.

Theoretical part

The study is generally aimed at understanding mechanical processes in a small zone near the wellbore, where hydraulic fracture initiation occurs. The well and fracture initiated after fluid injection as well as its propagation in the well surrounding rock masses are the main objects of the research. There are several approaches for estimating the hydraulic formation breakdown pressure [8,14,15].

The distribution of horizontal stresses in well surrounding rock masses was obtained from the modification of the classical Kirsch problem solution for stress concentration around a circular hole [16]. This distribution was consequently used to evaluate formation breakdown pressure for a formation. Fracture initiation occurs whenever minimum principal stress becomes tensile and its modulus exceeds the unconfined tensile strength (UTS) of the rock mass.

The problem of determining the formation breakdown pressure was solved under different boundary conditions. Therefore, it is possible to formulate four cases with different conditions.

Case 1. In the first case, the fracture occurs in a homogeneous nonporous medium, which is considered impermeable to the fracturing fluid. In this case, $\sigma_H = S_H$, $\sigma_h = S_h$ is true for effective stresses, since there is no pore pressure distribution in the formation. Compressive stresses are considered as positive hereafter, $S_H \geq S_h$.

The formula for estimating the formation breakdown pressure has the form:

$$P_w^* = 3S_h - S_H + UTS, \tag{1}$$

with fracturing fluid pressure P_w^*, in-situ horizontal stresses S_h and S_H , acting far from the well. P_w^* corresponds to pressure at well resulting into fracture initiation or formation breakdown pressure (FBP).

Case 2. In the second case, the fracture occurs in a homogeneous porous medium, which is considered impermeable to the fracturing fluid. Since the pore pressure distribution is now present in the medium, the Terzaghi representation [15] was used for effective stresses: $\sigma_H = S_H - P_0$, $\sigma_h = S_h - P_0$, $S_H \geq S_h$.

Then the formula for estimating the formation breakdown pressure has the form:

$$P_w^* = 3S_h - S_H - P_0 + UTS, \tag{2}$$

with pore pressure P_0.

Case 3. In the third case, the fracture occurs in a homogeneous porous medium, which is considered permeable to the fracturing fluid. In this case, in addition to the distribution of pore pressure in the environment, it is necessary to take into account the process of filtration of the fracturing fluid from the well into the formation $\alpha \frac{1-2\nu}{1-\nu}(P_w - P_0)$, with Biot's coefficient α [18] and Poisson 's ratio ν. Moreover, it is necessary to take into account the condition of the pre-saturation of the medium. The Terzaghi representation [17] was also used for effective stresses: $\sigma_H = S_H - P_0$, $\sigma_h = S_h - P_0$, $S_H \geq S_h$.

Then, the formula for estimating formation breakdown pressure takes the form:

$$P_w^* = \frac{3S_h - S_H + UTS - \alpha \dfrac{1-2\nu}{1-\nu} P_0}{2 - \alpha \dfrac{1-2\nu}{1-\nu}}. \quad (3)$$

Case 4. However, when using the Terzaghi representation, it is assumed that a fracture forms instantly as soon as the annular effective stress of the Terzaghi directly on the wall of the borehole exceeds the uniaxial tensile strength UTS. However, in this case, the influence of changing pore pressure on the well wall is not considered. Therefore, it was proposed by Schmidt [19] to rewrite the equations for effective stresses in the form: $\sigma_H = S_H - \beta P_0$, $\sigma_h = S_h - \beta P_0$, $S_H \geq S_h$, where β – pore pressure coefficient ($0 \leq \beta \leq 1$). In this case, it is also assumed that the fracture occurred in a homogeneous porous medium, which is considered permeable to the fracturing fluid. Here, as in the previous case, the process of filtration of the fracturing fluid from the well into the formation $\alpha \frac{1-2\nu}{1-\nu}(P_w - P_0)$ and the condition of pre-saturation of the medium are taken into account.

The formula for estimating the formation breakdown pressure has the form:

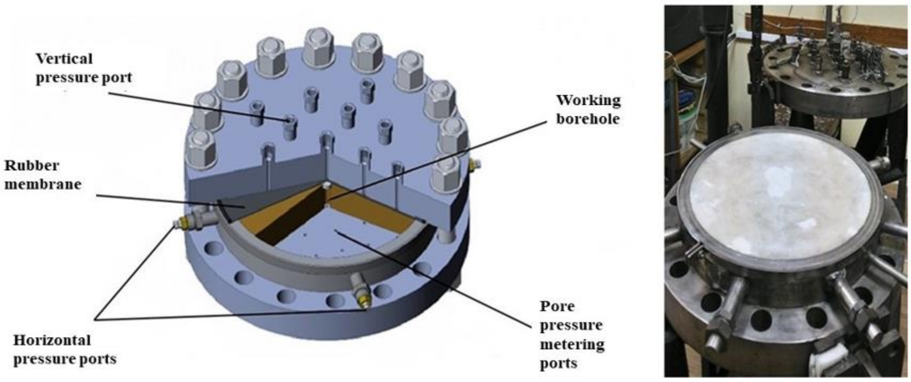
$$P_w^* = \frac{3S_h - S_H + UTS - \alpha \frac{1-2\nu}{1-\nu} P_0}{1 + \beta - \alpha \frac{1-2\nu}{1-\nu}}. \quad (4)$$

Experimental part

Laboratory experiment on hydraulic fracturing. A series of laboratory experiments were conducted to simulate the process of fracture formation in the area of the well in which the fluid is injected. A state-of-art setup was used (Fig. 1), which makes it possible to simulate the process of hydraulic fracturing. The design consists of two steel discs and a wide ring-wall between them. The lower disk and the ring-wall together form a working chamber, which has the following characteristic dimensions: diameter – 43 cm, height - 6.6 cm. The model sample of the formation is separated from the upper disk of the setup by a rubber membrane. There is a gap between the top lid of the setup and the membrane, which is filled with water under constant pressure to simulate lithostatic pressure in the reservoir model. Horizontal loading on the model sample is carried out by pumping gas or fluid into flat copper chambers located along the inner surfaces of the side walls of the working chamber. A more detailed description is provided in the work [20]. In this study, the setup is considered as a layered medium, i.e. a permeable layer, in which the fracture propagation occurs, surrounded by impermeable walls of the setup.

The laboratory experiment on hydraulic fracturing itself is carried out in several stages. Initially, a mixture of gypsum and cement is poured into the working chamber of the setup. After drying of the modeling material, a model sample is formed. The dimensions of the working chamber of the experimental setup determine the dimensions of the model sample. The resulting porous sample is then saturated with a solution of gypsum in water. After that, the setup is closed, loading of the model sample is done and the fracturing fluid, more precisely mineral oil, is injected into the preliminary prepared well at a constant flow rate. It is important to note that the horizontal stresses used in further calculations are the calculated horizontal stresses acting in the central region of the sample. They take into account both the applied horizontal loads and the friction between the sample and the metal covers.

The central well is a brass tube (diameter 16 mm), which is hermetically inserted into the lower cover of the setup. The upper end of the tube is closed with a screw plug. The tube has a vertical slot in the middle part, into which a thin brass gauze folded in half is inserted. The brass gauze serves as a seed for the fracture. The size of the gauze petals is 8 × 8 mm. The corners of the petals are cut off by about 2 mm. The tube has the ability to rotate freely around the vertical axis so that it is possible to orient the fracture perforation in a given direction.



(a) (b)

Fig. 1. A schematic illustration of the experimental setup (a), and an actual photo of the setup (b)

A series of laboratory experiments on hydraulic fracturing was carried out in the course of the study. As a result, two types of hydraulic fractures can be formed: vertical fractures (along the vertical well) and horizontal fractures (perpendicular to the well). In this series, experiments with vertical fractures were considered. An example of the obtained pressure-time curves for this type of fracture is presented below (Fig. 2).

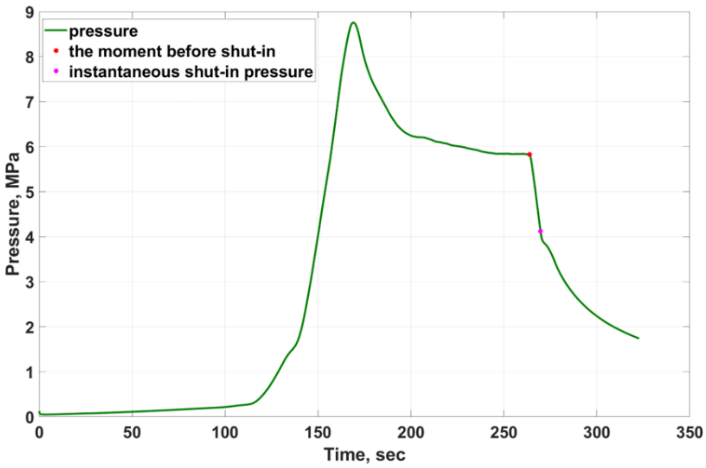


Fig. 2. An example of the dependence of pressure on time obtained in a laboratory experiment on the creation of a vertical fracture of a hydraulic fracturing

It is also important to note that prior to conducting laboratory experiments on hydraulic fracturing, the modelling materials were additionally studied. The modelling materials and fluids were selected in such a way that the propagation of a hydraulic fracture in a laboratory experiment was similar to its propagation in a real field. Similarity criteria were used to achieve the similarity of processes in laboratory and real models [20,21].

A mixture of gypsum and cement was found to be a suitable material for modelling the formation and mineral oil as the fracturing fluid. The numerical values of the material characteristics are presented in the table below (Table 1).

Table 1. Some properties of the modelling material and fluid used in the laboratory experiments

Fracturing fluid viscosity μ, Pa·s	0.12
Injection rate q, cm^3/s	0.17 – 0.37
Young modulus E, Pa	4×10^9
Poisson's coefficient ν	0.25
Model sample permeability k, m^2	2×10^{-15}
Model sample porosity φ	0.4

Determination of uniaxial tensile strength. One of the characteristics of the material used, which has been given special attention in this work, is the single-axis tensile strength. This parameter plays a major role in the framework of the conducted studies since the criterion for fracture formation was considered to be the moment when the local tensile stress in the stress concentrations area exceeds the ultimate tensile strength of the material in magnitude.

Therefore, an additional series of laboratory tests was initially carried out to determine the uniaxial tensile strength by applying the "Brazilian" method [22,23].

Samples from a model material with a diameter of 40 mm and a length of 42 mm were used in this series of experiments. The sample was placed between parallel steel plates during the manual press test. The tensile strength for such a series of experiments was calculated by the formula $\sigma_p = K\frac{P}{S}$, where K=2/π when loaded with plates, P is the breaking force, S is the product of the length of the sample by its diameter. The results of the tests by the "Brazilian" method are shown in Table 2.

Table 2. The results of the tests by the "Brazilian" method

Sample number	Sample diameter d, mm	Sample height h, mm	Breaking force P, kN	Tensile strength UTS, MPa
1	40	42.2	1.7	0.6
2	40	42.2	2.6	1.0
3	40	42.2	2.0	0.8
4	40	42.2	2.1	0.8
5	40	42.2	1.8	0.7
6	40	42.2	2.3	0.9

Based on the calculations obtained, the value of the tensile strength can be considered equal to 0.8 MPa.

According to some sources [24,25], the tensile strength can be determined by knowing the re-opening pressure of the hydraulic fracture. Another series of laboratory experiments was carried out to clarify the obtained tensile strength. In this case, all experiments were carried out in two stages. The first stage was a standard laboratory experiment on hydraulic fracturing, which is described above. The second stage involved re-conducting hydraulic fracturing to expand the existing fracture.

However, after processing the pressure drop curves, it was detected that the uniaxial tensile strength of the material varies in the framework of laboratory experiments. According to the sources [24–26], it is no longer necessary to overcome the tensile strength of the medium in the course of repeated fracturing, because the fracture already exists. Therefore, repeated fracturing is considered to be another applicable method for determining the tensile strength by subtracting the breakdown pressure of the reopened fracture from the breakdown pressure of the primary fracture. Two such two-stage experiments were conducted. The loading parameters of the samples and the results of the tensile strength calculations are shown in Table 3.

Table 3. Results of defining the tensile strength using two-stage laboratory experiments on hydraulic fracturing

Expansion of a vertical fracture	stage 1	experimental formation breakdown pressure	8.80	MPa
	stage 2	experimental formation reopening pressure	6.40	MPa
		Tensile strength	*2.40*	*MPa*
Expansion of a vertical fracture	stage 1	experimental formation breakdown pressure	7.69	MPa
	stage 2	experimental formation reopening pressure	5.96	MPa
		Tensile strength	*1.73*	*MPa*

The corrected tensile strength value is 2.07 MPa. The corrected tensile strength value is used in further calculations for the formation breakdown pressure because it is considered more correct under the conditions of hydraulic fracturing.

Results

Experimental data processing. In the course of this study, a series of six standard one-stage laboratory experiments on the formation of a vertical hydraulic fracture was carried out. The formation breakdown pressure was estimated for all four cases described, taking into account the characteristic parameters of the experiments (Tables 1, 2). The values of the formation breakdown pressure were obtained after analyzing the curves of pressure versus time, which were recorded using sensors during laboratory experiments on hydraulic fracturing. The pattern of the obtained experimental pressure-time curves corresponds to the classical characteristic pattern of hydraulic fracturing pressure curves, which was described in [27].

These curves have a characteristic pattern. Provided that the fluid is pumped at a constant rate, there is a linear increase in pressure over time in the beginning, which is violated at the first appearance of leaks, which is sometimes considered the moment of fracture initiation. The point of maximum pressure on the graph corresponds to the moment of fracture formation. Thereafter, there is a pressure drop to a value called fracture propagation pressure. Further, there is a rapid pressure drop after stopping fluid injection into the well to the fracture closure pressure. Thus, experimental formation breakdown pressures were determined from these pressure curves. The results are shown in Table 4: each experiment was carried out with its own vertical and horizontal loads forming far-field stresses S_h and S_H (equal to each other) and vertical stress S_V stated in the table.

Table 4. A comparison table of the values of the hydraulic formation breakdown pressure obtained experimentally and calculated theoretically

Experiment number	Q, cm^3/s	Horizontal stress ($S_h = S_H$), MPa	Vertical stress S_V, MPa	*FBP*, MPa (experimental)
1	0.17	2.01	6.6	8.76
2	0.37	2.01	7.1	12.55
3	0.20	1.32	4.8	7.07
4	0.20	1.12	4.6	10.96
5	0.20	1.29	4.7	7.74
6	0.20	1.50	5.1	13.46

The formation breakdown pressure was estimated under different assumptions (four cases), taking into account the characteristic parameters of each of the experiments (Table 4). All formulas (1) – (4) given in the theory are valid in the case of initiation of a vertical fracture in an infinite homogeneous formation. Ideally, it is assumed that the fracture formation process occurs on a vertical well with an open borehole.

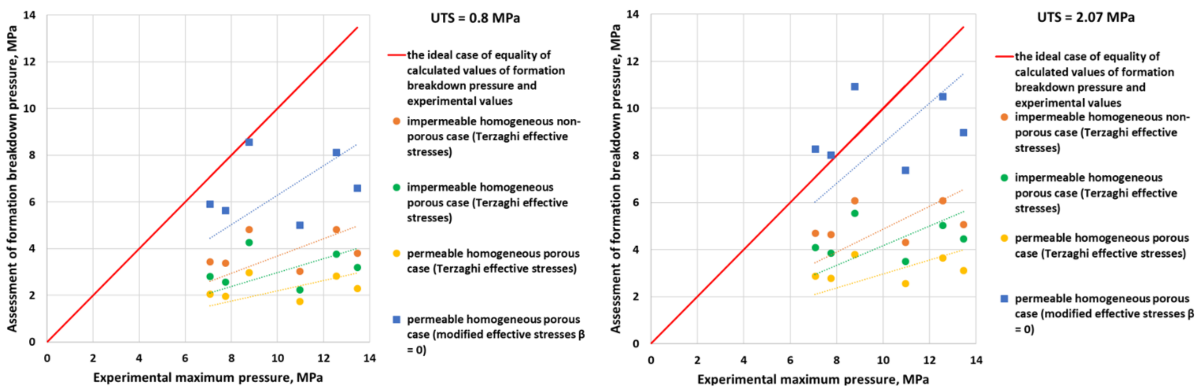
The experiments presented in this paper differ from the ideal theory in the finite size of the sample along the radius, the restriction of the sample from above and below by metal plates, and the fact that the well is cased. However, the cased well does not contradict ideal conditions, because perforations are made in it in the direction of fracture development, which seamlessly connects the well with the sample. It is important to note that the horizontal stresses given in the work represent the calculated horizontal stresses acting in the central region of the sample. They take into account both the applied horizontal loads and the friction between the sample and the metal covers. Therefore, the use of formulas (1) – (4) in the processing of experimental data can be considered sufficiently conditioned and correct.

The results of theoretical estimates of formation breakdown pressure are presented in Table 5. Theoretical estimates of formation breakdown pressure were made for each experiment.

Table 5. Experimental values of formation breakdown pressure and theoretical estimates were made for different boundary conditions and taking into account different values of uniaxial tensile strength

Experiment number	Experimental *FBP*, MPa	Theoretical *FBP*, MPa							
		UTS = 0.8 MPa				*UTS* = 2.07 MPa			
		Case							
		1	2	3	4 (β = 0)	1	2	3	4 (β = 0)
1	8.76	4.82	4.27	2.98	8.56	6.09	5.54	3.80	10.94
2	12.55	4.82	3.77	2.82	8.12	6.09	5.04	3.65	10.50
3	7.07	3.44	2.82	2.05	5.90	4.71	4.09	2.88	8.28
4	10.96	3.04	2.24	1.74	5.00	4.31	3.51	2.57	7.38
5	7.74	1.29	2.58	1.96	5.64	4.65	3.85	2.79	8.02
6	13.46	1.50	3.20	2.30	6.60	5.07	4.47	3.12	8.98

As can be seen from the data obtained (Fig. 3), the best results are obtained by the approach to calculating the formation breakdown pressure (with modified expressions for effective stresses) proposed by Schmidt. It is also obvious that the use of the adjusted uniaxial tensile strength contributes to a significant reduction in the difference between the theoretically calculated values of the formation breakdown pressure and the experimental ones.



(a) (b)

Fig. 3. Comparison of the results of calculations of the formation breakdown pressure under different boundary conditions and different values of uniaxial tensile strength *UTS*: (a) *UTS* = 0.8 MPa; (b) *UTS* = 2.07 MPa

Analysis of calculated values of formation breakdown pressure. The obtained experimental values of the formation breakdown pressure allow us to evaluate the influence of various parameters on the theoretical assessment of the formation breakdown pressure. In some cases, the following normalized:

$$FBP_n = \frac{FBP_{\text{theor}}}{FBP_{\text{exp}}}, \quad (5)$$

theoretically evaluated formation breakdown pressure FBP_{theor} is normalized by the 'true' corresponding experimental measurement FBP_{exp}. This particular indicator was chosen due to both parameters being positive in all experiments and a general tendency of FBP_{exp} exceeding FBP_{theor}. It is clear from formula (5) that the best match between theoretical and experimental formation breakdown pressures is achieved if $FBP_n = 1$.

Various factors affecting *FBP* evaluation can be analyzed using the reported experimental results, especially the mechanical properties of the studied medium. The state of the medium is characterized by stresses S_h and S_H, as well as by pore pressure P_0, in particular, the pore pressure coefficient β; other notable properties of the medium are uniaxial tensile strength UTS and parameters – Biot's coefficient α, Poisson's ratio ν.

Let's consider the influence of the parameter β on the formation breakdown pressure. Equation (4) assumes that an increase in this parameter leads to a corresponding nonlinear decrease in the formation breakdown pressure: at the maximum possible value $\beta = 1$ equation (4) becomes the same as equation (3), and we expect that the estimate obtained for such a case will be the same as from equation (1), where the process of radial filtration of fluid from the well into the medium and the pressure drop in the medium are not considered. Figure 4 confirms these expectations as it presents different *FBP* estimates, where all parameters, except for β, remain constant.

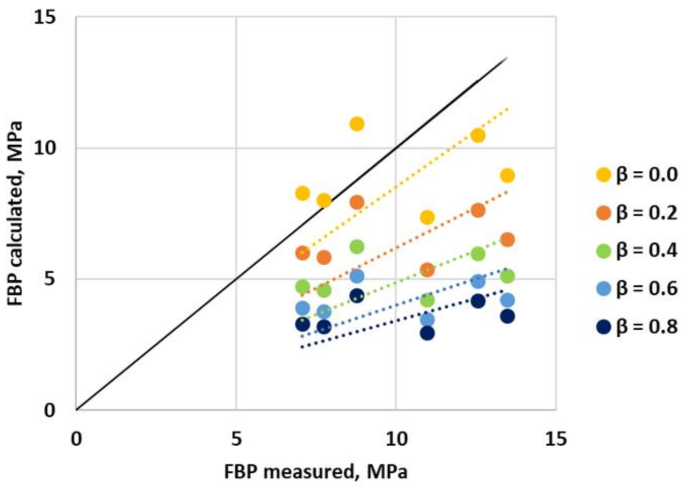


Fig. 4. Calculated *FBP* vs. measured *FBP* for a set of β parametrs – other properties and state of the medium remain unchanged. Theoretical estimations were in the conditions of the case 4

Parameter β was claimed to depend on rock porosity – it was suggested as close to unity for highly porous rocks and vanish for rocks with vanishing porosity. To our understanding, this parameter can be altered due to the redistribution of fluid in the well surrounding rock masses: filtration of hydraulic fracturing fluid into the environment surrounding the well reduces the effective porosity and permeability after replacing the reservoir fluid with a fracturing fluid with a relatively lower viscosity. A circular layer around the well can be formed around the well during the leak-off preceding fracture initiation. This layer can be quite thin – an analysis of its size and evolution during fluid filtration deserves a distinct study – but since tensile failure occurs near the wellbore, whenever this layer has a non-zero size, the effective stresses determined by the parameter β have to be used in the tensile failure law. This leads to an increase in effective stresses and a corresponding increase in the pressure in the well required to initiate tensile failure.

Figure 5 demonstrates the dependency of the formation breakdown pressure on Biot's coefficient. Figure 5 shows calculated *FBP* (not normalized) is plotted versus measured value. Equation (4) is used for the measurement of the *FBP* values for a set of Biot's coefficients between 0.0 and 1.0 (the natural range of Biot's coefficients for saturated media). Poisson's ratio remains unchanged – it stays equal to 0.25. It can be seen that an increase in Biot's coefficient leads to the growth of the slope of the calculated vs measured *FBP* curve. As it only exceeds 1 for the greatest Biot's coefficient, a certain preliminary conclusion can be drawn: the effect of the process of radial fluid filtration from the well into the medium grows for high Biot's coefficients. Keeping in mind that Biot's coefficient remains in a positive correlation with the permeability of rocks [26] one can conclude that error in *FBP* evaluation due to this effect is larger for highly permeable rocks. This conclusion, however, is not as clear as it could seem.

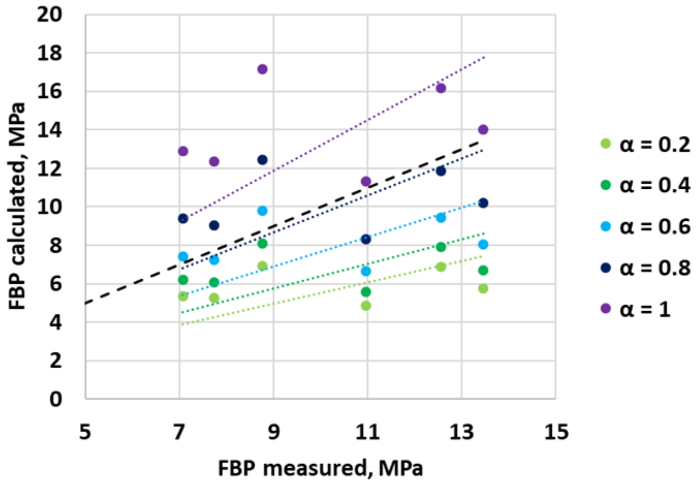


Fig. 5. Calculated *FBP* vs. measured *FBP* for a set of Biot's coefficients – other properties and state of the medium remain unchanged. Theoretical estimations were in the conditions of the case 4

Figure 6 represents normalized formation breakdown pressure *FBPn* as a function of measured *FBP* as well. The usage of *FBPn* instead of *FBP* makes it possible to see the rate at which Poisson's ratio alteration affects the discrepancy between measured and true values. Biot's coefficient remains equal to 0.7 in this case.

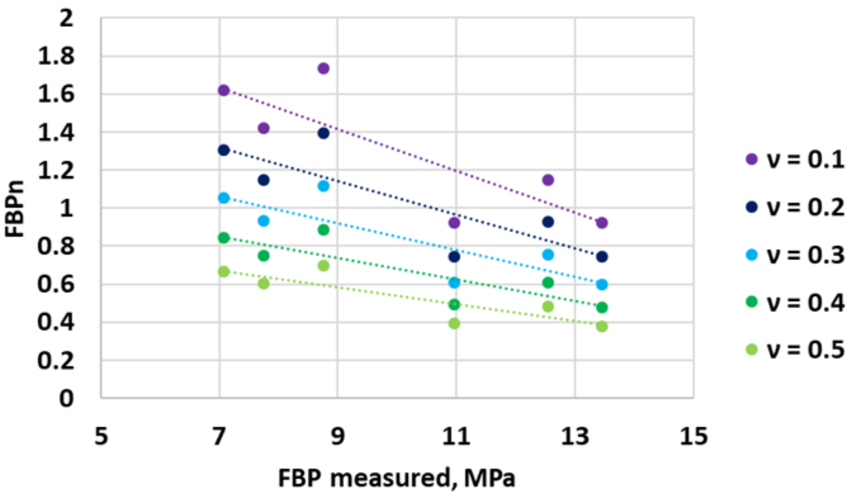


Fig. 6. Calculated *FBP* vs. measured *FBP* for a set of Poisson's rations – other properties and state of the medium remain unchanged. Theoretical estimations were in the conditions of the case 4

An increase in Posson's ratio leads to a decrease in the average *FBPn* as well as alters the slope of the *FBPn* vs. *FBP* measured curve. Under ideal conditions, *FBPn* would not depend on measured *FBP* and stayed at a constant value of 1. Interestingly enough, while larger Poisson's ratios lead to larger discrepancies between measured and calculated values, the slope decreases, so the dependency on measured *FBP* weakens.

Figures 5 and 6 only deal with one parameter affecting the petroelastic coefficient each. These findings can be summed up in a form of Fig. 8 presenting the series of curves where averaged *FBPns* – or, in other words, average slopes of calculated *FBP* vs. measured *FBP* dependencies – are plotted as functions of Biot's coefficient and Poisson's ratio. Biot's coefficient is on the horizontal axis, while Poisson's ratio remains constant along each line of the corresponding color according to the legend. Both mechanical properties are taken from naturally possible ranges: Biot's coefficient α remains between 0.0 and 1.0; Poisson's ratio v is between 0.0 and 0.5.

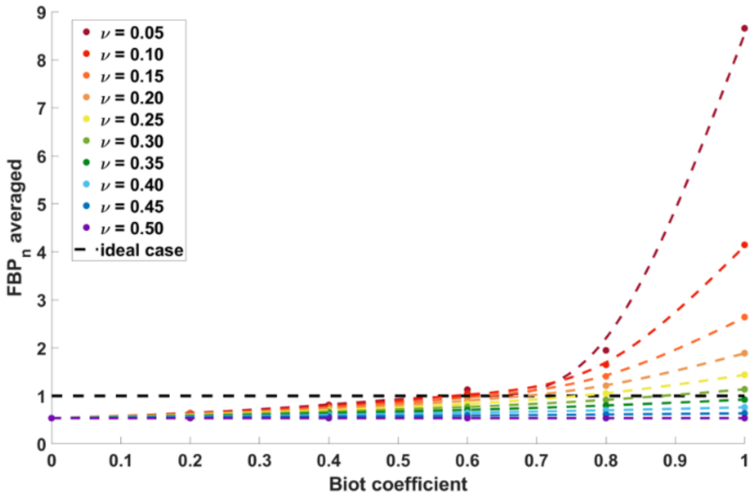


Fig. 7. Averaged FBP_n as function of the medium's elastic properties

The analysis of the influence of the mechanical properties of the medium on the discrepancy between the experimentally measured and theoretically calculated values of the formation breakdown pressure is presented. To sum up, both the state of the environment and its properties have a certain impact on the *FBP* assessment: generally, the higher the permeability and compressive stresses are, the lower discrepancy between the measured and calculated formation breakdown pressure. The Bio coefficient and Poisson's ratio affect the calculated *FBP* in a non-linear way: the calculated *FBP* increases with larger Bio coefficients and smaller Poisson's coefficients (Fig. 7).

Discussion

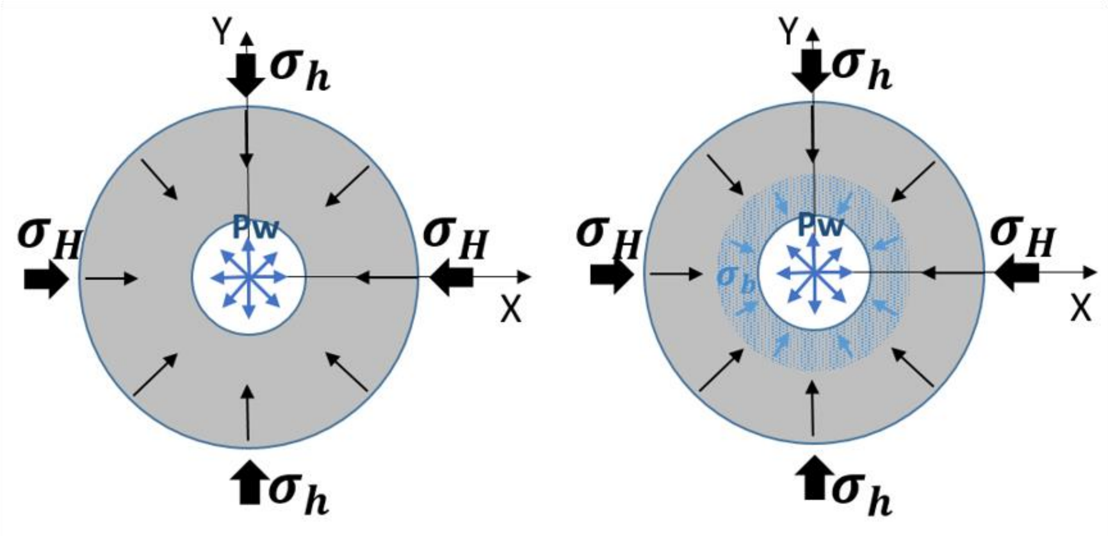
In the discussion, we propose to consider such an effect as backstress and its possible impact on the theoretical assessment of formation breakdown pressure. The backstress effect has previously been investigated in the context of hydraulic fracture closure pressure [13]. Recent works in which the backstress effect is discussed in the most complete and detailed manner belong to the authorship of Baykin and Golovin [9–12]. When considering the fracture closure pressure, backstress was introduced as additional stress caused by the fracturing fluid filtered out of the fracture.

We propose to consider this effect concerning the process of fracture formation. The material of the near-well area is strengthened when fracturing fluid saturates the pores. This process can be mathematically described by creating additional pressure of the fluid on the walls of the well, which in turn leads to an increase in the hydraulic formation breakdown pressure (Fig. 8).

In the context of the task under consideration, namely, the hydraulic fracture initiation and propagation in well surrounding rock masses with fracturing fluid injection at a constant rate, the authors of these papers propose to calculate backstress in well surrounding rock masses using the following equation:

$$\sigma_b = 2\eta(FBP - P_0), \quad \eta = \frac{\alpha(1-2\nu)}{2(1-\nu)}, \tag{6}$$

where η is a poroelastic coefficient, α is Biot's coefficient, and ν is Poisson's ratio. This coefficient deserves extra attention. There are reported studies [9–12,14,19], for backstress being given by equation (6). This equation is proposed for the case of one permeable layer representing a hydrocarbon reservoir being between two impermeable layers preventing fracturing fluid from filtrating into upper and lower layers [13]. In this paper, the application of such an assumption to calculations is considered correct, since the laboratory experimental setup can be represented as a layered medium, where the model material acts as a permeable layer and the steel upper and lower covers act as impermeable layers of the medium surrounding the permeable one.



(a) (b)

Fig. 8. Schematic representation of the stress distribution in the reservoir with the well into which the fluid is injected

The increase in formation breakdown pressure is since fracture formation criterion alters due to hydraulic fracture initiation in a medium. Thus, taking into account the additional compressive stress acting externally on the borehole walls, which will be referred to as backstress σ_b, it is possible to rewrite the expression for the formation breakdown pressure *FBP* as follows:

$$FBP = 3S_h - S_H + UTS + \sigma_b. \tag{7}$$

It is interesting to note that after substituting (6) into equation (7), the expression for the formation breakdown pressure will have the form:

$$FBP = \frac{3S_h - S_H + UTS - \alpha\frac{1-2\nu}{1-\nu}P_0}{1-\alpha\frac{1-2\nu}{1-\nu}}. \tag{8}$$

The form of equation (8) coincides with equation (4), provided that β = 0. Thus, the backstress effect can replace the total impact of the process of filtration of the fracturing fluid from the well into the formation $\alpha \frac{1-2\nu}{1-\nu}(P_w - P_0)$ and the condition of saturation of the medium. Figure 9 shows a comparison of the results of the formation breakdown pressure estimation calculated by formulas (4) and (8).

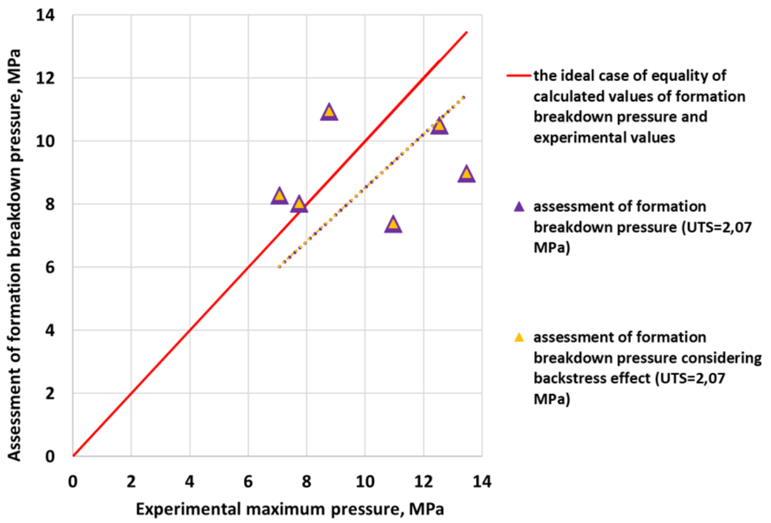


Fig. 9. Comparison of experimental and theoretical values of formation breakdown pressure during hydraulic fracturing

Conclusion

The current study deals with laboratory modeling of hydraulic fracture initiation and propagation. While there are evident problems with rescaling laboratory data to field scale, there is still a positive side: properties of the studied material and its state are known for sure if the controlled experiment is carried out in a laboratory. This provides an opportunity to study interrelationships between various parameters controlling fracture initiation and growth. The current study was aimed at understanding formation breakdown pressure – a parameter essential for hydraulic fracturing procedures.

The data from six laboratory experiments on hydraulic fracturing were analyzed in the work. In the course of all experiments, pressure dependences on time in the well during hydraulic fracturing were obtained, according to which experimental values of formation breakdown pressures were determined. These values were compared with theoretically calculated values according to four theories. According to the first theory, the formation breakdown pressure was calculated without taking into account fluid filtration. The second theory, when calculating the formation pressure, took into account the saturation condition of the medium impervious to the fracturing fluid from the well. According to the third theory, both the saturation of the medium and the filtration of fluid from the well were taken into account. The fourth theory is some improvement of the third one, which takes into account the gradual penetration of the fracturing fluid into the medium through the pore pressure coefficient (beta). The best agreement with the experimental data was shown by the latest theory at β = 0. In general, β tends to zero with the vanishing porosity of the medium. On the one hand, the porosity of the material is quite large (about 40 %), but when filling the impacted zone with the fracturing fluid, the porosity, in fact, becomes disappearing for the fluid saturating the sample. Consequently, the use of the pore pressure coefficient β, tending to 0, becomes a fairly reasonable step. In addition, an interesting fact was demonstrated in the discussion. There was a proposal to investigate the backstress effect at the moment of fracture formation. Previously, backstress was only used in the context of the connection of minimum stress and fracture closure pressure. It was proposed to replace the condition of saturation of the medium and the condition of penetration of the fracturing fluid from the well with backstress, considering it in the equation for the formation breakdown pressure for the simplest case (unsaturated and impermeable medium). It turned out that in this case, the equation for the formation breakdown pressure completely coincided with the equation that takes into account both the saturation of the medium and the filtration of the fracturing fluid at $\beta = 0$. Thus, the authors summarize that under experimental conditions, the best coincidence of the calculated formation breakdown pressures with the experimental values was shown by the backstress theory.

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