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AUTOWAVE MECHANISM OF LOCALIZATION OF LOW-TEMPERATURE PLASTIC DEFORMATION

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Abstract. The mechanism of localization of low-temperature plastic deformation is investigated within the framework of the thermal activation model. The localization mechanism is considered as a result of the autowave character of the stationary solution of a system of equations that describes both the processes of thermal conductivity and plastic deformation. It is established that the stationary solution of the initial problem is given by the wave fronts of switching waves of temperature and plastic deformation. It is shown that the considered autowave mechanism determines the process of localization of high-temperature areas of plastic deformation in the cross-section of the sample in the form of either a neck or separate deformation bands.

Keywords: Deformation localization, thermally activated plastic deformation, low temperatures, autowaves

1. Introduction

An important problem of modern materials science is the instability and localization of plastic deformation of mono- and polycrystals in the low temperature region [1]. This phenomenon consists of the serrated deformation and deformation stratification of crystals into local zones of intense shear formation at temperatures of the order of 0.5-50 K, i.e. bands of deformation inside an almost undeformed crystal. Localization of deformation was found in many metals. However, the nature of this phenomenon remains largely unclear [2,3].

Since the localization of deformation is associated with an increase in the plastic flow velocity in a certain slip band, this increase is usually explained by a sharp increase either in the dislocation density in the localized zone, or in the velocity of dislocations (depending on temperature). Physically, this process is related to the excitation of the corresponding wave fronts, similar to the appearance of the Luders bands at elevated temperatures [4]. On the other hand, there is a point of view that the appearance of the deformation bands at very low temperatures is due solely to thermal effects. In this regard, this paper proposes a model for the formation of a localized slip band in the framework of the thermoactivation model of plastic deformation.

2. Autowave model

We investigate a model, which is often used to establish a criterion of plastic deformation instability at thermal activation sliding of dislocations, for the occurrence of autowave solutions [5,6]. Then for thin enough metal samples ($R << L$, where R is the radius of the cylindrical sample, L is the length of the sample) the processes of deformation and thermal conductivity (non-uniform along the cylinder axis) can be described by the following system

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of equations[7,8]

$$C\frac{\partial T}{\partial t}-\kappa\frac{\partial^2 T}{\partial x^2}=-\frac{2h}{R}(T-T_0)+\sigma\dot{\varepsilon}. \tag{1}$$

$$\frac{\partial\varepsilon(t)}{\partial t}=\dot{\varepsilon}, \qquad \dot{\varepsilon}=\nu\exp[-W/k_BT], \tag{2}$$

Here the equation (1) is the thermal conductivity equation, where T is the temperature of the metal sample, κ is the thermal conductivity coefficient, T_0 is the ambient temperature, h is the heat transfer coefficient, σ is deforming stress, C is the heat capacity of the sample, which is taken to be a constant value for simplicity as in [5] .

The thermal activation mode of plastic deformation is characterized by the Eq. (2), where $\dot{\varepsilon}$ is the local rate of plastic deformation in the deformation zones, ε is the value of plastic deformation, W is the activation energy, ν is a pre-exponential multiplier, k_B is the Boltzmann constant.

The system (1), (2) belongs to the class of autowave systems if the right side of the Eq. (1)

$$F(T)=\sigma\nu\exp[-W/k_BT]-\frac{2h}{R}(T-T_0) \tag{3}$$

has a descending section in a certain temperature interval and the deforming stress σ must intersect the abscissa axis at least in three points [9]. From the condition $F(T)=0$ we get the dependence $\sigma=\sigma(T)$, which is characteristic for the stationary case:

$$\sigma=\frac{2h}{Ru}(T-T_0)\exp(W/k_BT). \tag{4}$$

This dependency is N -like shape (see Fig. 1) under condition $\alpha=W/k_BT>4$ [10] and this implies that the autowave mode of plastic deformation (for typical values of the parameters [5]) is possible only at a sample temperature below $T<W/4k_B\approx 15K$.

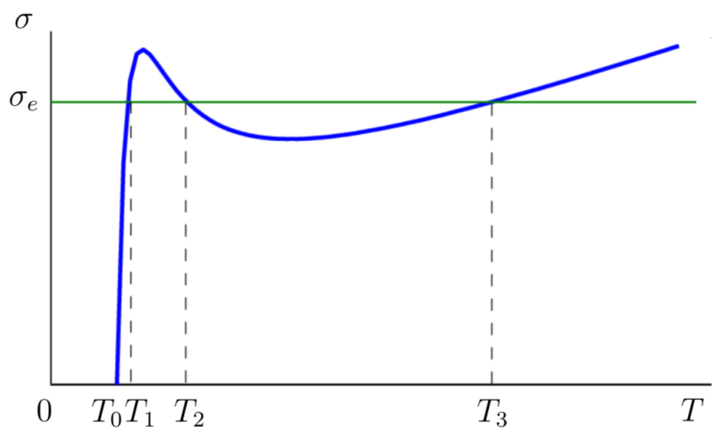

Fig. 1. Dependency $\sigma=\sigma(T)$ when the parameter $\alpha=W/k_BT>4$

The line $\sigma=\sigma_e$ can intersect the $\sigma(T)$ dependency in different ways. Let us consider the case when the intersection occurs at three points (Fig. 1).

For further analysis, we write the system (1), (2) as a single equation entering a dimensionless temperature $\theta=T/T_0$

$$\frac{d\theta}{dt} = \eta\frac{\partial^2\theta}{\partial x^2} + f(\theta), \tag{5}$$

where

$$f(\theta) = \mu\sigma\exp\left[-\frac{\alpha}{\theta}\right] - \beta(\theta - 1), \tag{6}$$

$\mu = \nu/CT_0$, $\beta = 2h/CR$, $\eta = \kappa/C$.

The graph of the dependency $f(\theta)$ at a given external stress level $\sigma = \sigma_e$ is shown in Fig. 2. In this case, the system (5), (6) has three equilibrium states: two stable states corresponding to temperatures $\theta = \theta_1$ and $\theta = \theta_3$ and unstable one corresponding to $\theta = \theta_2$.

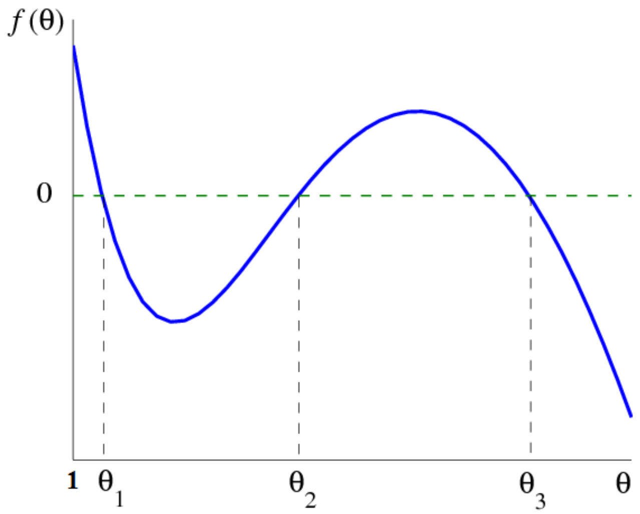

Fig. 2. Dependence of the right side of the heat equation (5) on the normalized temperature $\theta = T/T_0$

To find stationary running solutions of the system (5) let us go to the self-similar variable $\xi = x - ct$ assuming $\theta = \theta(\xi)$. Substituting a solution of the assumed type in the original system, we obtain

$$-c\frac{d\theta}{d\xi} = +\eta\frac{d^2\theta}{d\xi^2} + f(\theta). \tag{7}$$

We choose the following boundary conditions for the variable $\theta(x,t)$: $\theta(-\infty,0) = \theta_3$, $\theta(\infty,0) = \theta_1$.

We will study the system (7) on the phase plane (θ, W), assuming $W = d\theta/d\xi$. We have

$$\eta\frac{dW}{d\xi} = -cW - f(\theta), \tag{8}$$

$$\frac{d\theta}{d\xi} = W. \tag{9}$$

This system has three fixed points on the plane (θ, W) $(\theta_1,0), (\theta_2,0)$ and $(\theta_3,0)$.

The point $(\theta_2,0)$ is the focus, and the singular points $(\theta_1,0)$ and $(\theta_3,0)$ are saddle points, through which two trajectories pass (Fig. 3). The only stable stationary solution is described by a separatrix going from saddle to saddle, which corresponds to a certain value of the switching wave speed c.

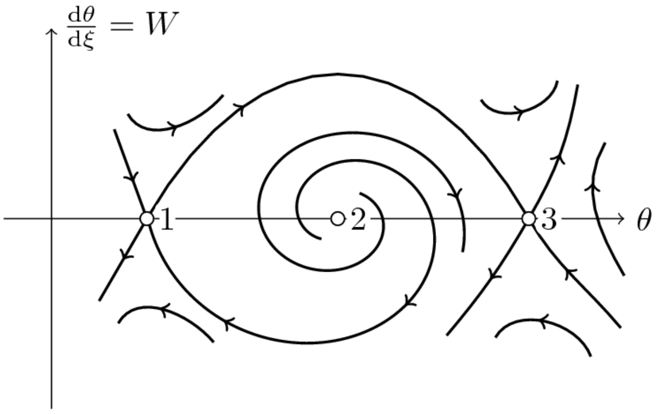

Fig. 3. Phase plane of the variables $\theta - W$

The system (8), (9) can be reduced to the boundary problem

$$\eta W \frac{dW}{d\theta} - cW - f(\theta) = 0, \quad (10)$$

with the boundary conditions $W(\theta_1) = W(\theta_3) = 0$.

To get analytical results we approximate $f(\theta)$ by cubic trinomial $f(\theta) = -\gamma(\theta - \theta_1)(\theta - \theta_2)(\theta - \theta_3)$ and, assuming $W = \delta(\theta - \theta_1)(\theta - \theta_3)$, obtain

$$c = \sqrt{\frac{\eta\gamma}{2}}(\theta_1 + \theta_3 - 2\theta_2). \quad (11)$$

By integrating the function $W = d\theta/d\xi = \delta(\theta - \theta_1)(\theta - \theta_3)$ we find the profile of the wave solution

$$\theta(x,t) = \theta_1 + (\theta_3 - \theta_1)\left[1 + C_0 \exp\left(\frac{x - ct}{\Lambda}\right)\right]^{-1}, \quad (12)$$

where $\Lambda = \sqrt{8\eta/\gamma}(\theta_3 - \theta_1)^{-1}$ is characteristic width of the wave front, C_0 is an integration constant.

The obtained solution is a temperature wave of heating $\theta = \theta(x - ct)$ from the state θ_1 to the state θ_3 running to the right for $c > 0$ to which the wave of plastic deformation softening $\dot{\varepsilon}(x\text{-}ct)$ corresponds. The wave speed sign is determined by the condition (11), i.e. $c > 0$ is implemented at $\theta_1 + \theta_3 > 2\theta_2$ under appropriate level of deforming stresses σ_e. At some critical stress $\sigma_e = \sigma_c$, when the condition $\theta_1 + \theta_3 = 2\theta_2$ holds, the solution is a standing wave.

It should be noted that the original system is invariant with respect to a change of sign x. Therefore, the solution of the equation (5) could be looking for the self-similar variable $\xi = x + ct$ with boundary conditions of the form: $\theta(\infty,0) = \theta_3$, $\theta(-\infty,0) = \theta_1$. In this case, the general solution of the system takes the form similar to (13), but the corresponding switching wave propagates in the opposite direction at the same speed $c > 0$:

$$\theta(x,t) = \theta_1 + (\theta_3 - \theta_1)\left[1 + C_0 \exp\left(\frac{x + ct}{\Lambda}\right)\right]^{-1}. \quad (13)$$

In physically realistic conditions for the appearance of wave solutions, for example, when a slip band with corresponding heating to the temperature θ_3 is formed in cross section of a sample, the actual form of the running wave fronts is shown in Fig. 4 and it corresponds to the temperature zone of localization in a certain section of the crystal, which expands with the speed $2c$.

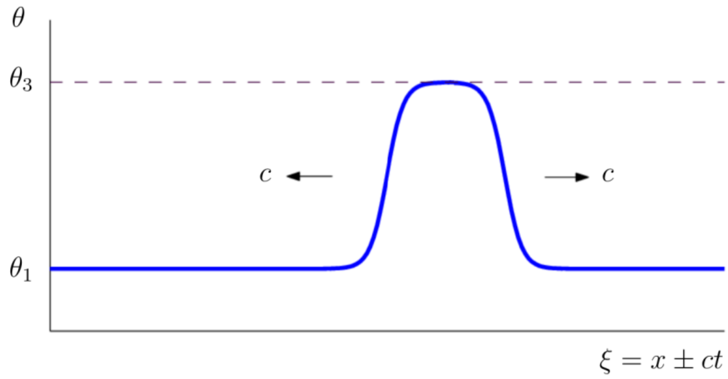

Fig. 4. Localized heating zone of a sample formed in its cross-section

3. Conclusion

The original system of equations (1), (2) was considered above under the constant load condition $(\sigma = \sigma_e = const)$. In reality, this condition is got on a special machine that maintains the tensile speed of a metal sample constant and is described by the equation

$$\frac{\partial\sigma}{\partial t} = G^*[\dot{\varepsilon}_0 - \frac{1}{L}\int_0^L \dot{\varepsilon}(x,t)dx], \tag{14}$$

where $G^* = Kh_0/S$ is the effective modulus of elasticity, K is machine-sample system stiffness, h_0 and S are height and cross-section of the sample, $\dot{\varepsilon}_0$ is the specified rate of plastic deformation.

Solutions (12), (13) have a small width of the temperature wave fronts and, correspondingly, the deformation rate compared to with the sample length ($\Lambda << L$). Then the average plastic deformation rate over the whole length $(0, L)$ can be expressed approximately as

$$\bar{\dot{\varepsilon}}(t) = \frac{1}{L}\int_0^L [\dot{\varepsilon}(x-ct) + \dot{\varepsilon}(x+ct)]dx \approx [\dot{\varepsilon}(\theta_3)l_r + \dot{\varepsilon}(\theta_1)(L - l_r)]/L, \tag{15}$$

where the softening zone width l_r moves at the speed dl_r/dt. In a stationary case ($c = 0$, $\bar{\dot{\varepsilon}}(t) = \dot{\varepsilon}_0$) the width of plastic deformation zone l_r is determined easily from (15):

$$l_r = L\frac{\dot{\varepsilon}_0 - \dot{\varepsilon}(\theta_1)}{\dot{\varepsilon}(\theta_3) - \dot{\varepsilon}(\theta_1)}. \tag{16}$$

Let us consider the case when $\sigma \approx \sigma_c$. In this case, the share of randomly generated propagating thermal pulses (formed on the boundary or inside of the material) is small enough and does not lead to a noticeable change of the integral in (14). For this reason, no macroscopic changes of the load σ occurs. However, if σ exceeds σ_c slightly, then the softening effect takes place, as the deformation zone is continuously expanded and the value of the integral in (14) increases. According to (14) this reduces the load σ to a stable value σ_c.

There may be a situation when this tuning occurs with some delay and then, at $\sigma \approx \sigma_c$, it is possible a damped pulsating mode of the zone of plastic deformation softening and changing of the load σ.

The transverse plastic zone l_r with elevated temperature, which is formed under these conditions, allows multiple characters. Instead of one large plastic deformation zone, there may be several smaller zones with the same total "width". The location and number of such zones are determined by the initial conditions.

Thus, since, in general, the width of the plastic deformation zone l_r is the value, which is adjusted to the specified conditions of crystal deformation, then the described autowave mechanism determines also the process of localization of high-temperature domains of plastic deformation in the cross-section of the sample in the form of either a neck or separate deformation bands.

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