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Nuclear geometry: from potassium to titanium

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Abstract. Nuclear geometry has been developed by analogy with the fullerene geometry. On the basis of this geometric approach, it was possible to design the structure of potassium, calcium, scandium, and titanium isomers as well as their isotopes, which can be obtained by means of nuclear synthesis. The nuclei can be classed into two groups: basic nuclei having equal numbers of protons and neutrons and isotopes having one, two, and more neutrons. The latter ensure their mechanical stability with respect to shear stresses, sending their electron to the coat of mail created by the basic nuclei.

Keywords: calcium, graph representation, isomer, isotope, nuclear electron, nuclear geometry, nuclear reaction, potassium, scandium, titanium

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1. Introduction

There are various models of nuclear structure, but [1] "all the nuclear models play the role of more or less probable working hypotheses. The nuclear models are approximate representations used for the description of some properties of nuclei. It is assumed that a nucleus is identical to a physical system, which properties are well studied". Although these models often contradict each other, usually they describe different features of a nucleus and therefore supplement each other. Any model is based on experimental facts and allows explaining some properties which are of interest. At the same time "the consistent explanation of the most important properties of atomic nuclei on the *firm basis of general physical principles* is one of the unsolved fundamental problems of nuclear physics" [1].

The modern existing nuclear models were developed in the framework of the proton-neutron conception. Their general feature is that they are shapeless, i.e. they describe quantum states but not the geometry of nuclei. How to find the firm basis for constructing a consistent nuclear theory? Let us return to Aristotle (384-322 BC). He assumed that all essential originates and consists of two principia: matter and form; the form being the leading principle. The matter in itself is the passive principle of nature; it is the possibility for the appearance of a real thing. In order for the thing to become reality; it must receive the form, which transforms the possibility into reality [2, p. 27].

Up to now in nuclear physics, we have only the matter (protons, neutrons, electrons, etc.), but no space form. In other words, the leading principle is absent in nuclear physics elsewhere. To my mind, the firm ground for constructing a new consistent theory of nuclear physics is nuclear geometry, i.e. 'Aristotle's form' of nuclei. The nuclear geometry has been created and developed by analogy with fullerene geometry in [3-7]. It should be emphasized that such a geometric approach explained not only the generation of elements from hydrogen to argon but also that of *their isotopes and isomers* in the framework of one and the same unified approach. Moreover, the geometric models of nuclei have given us an insight into the problem, of why the nuclei have a definite number of stable isotopes and isotopes having a large half-decay period.

2. Parallels between tetrahedral fullerene C_4 and helium $_2He^4$

Tetrahedral fullerene C_4 can be inscribed into a sphere (Fig. 1a). The atoms and shared electron pairs, forming covalent bonds, are located on one and the same sphere [3]. It should be emphasized that the covalent bond can be represented not only as a straight line but as a small arc on a geodesic line (great circle). Being less than a semicircle, it is the least path between the ends of the arc [8].

The covalent bonds are created by shared electron pairs [9]. According to the theory by Sidgwick and Powell [9,10], each shared electron pair can be considered as a point charge; all the charges repel each other and arrange themselves into such a configuration, which ensures their maximal removal from each other. Tetrahedral fullerene C_4 has four atoms and six point charges. All the charges are located on the great circles, which pass through any two atoms connected by a respective electron pair. It is interesting to note that six point charges form a regular octahedron inscribed into the same sphere (Figs. 1b and 1c).

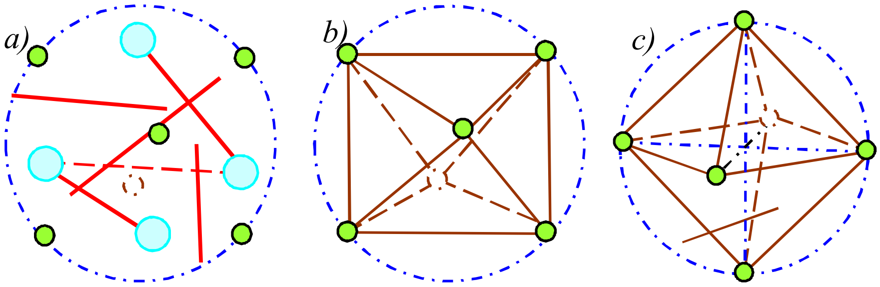


Fig. 1. Tetrahedral fullerene C_4; large turquoise spheres are atoms and small green spheres are shared electron pairs. Here: (a) position of the shared electron pairs on geodesic lines; (b) point charge octahedron corresponding to (a); (c) usual form of a regular octahedron

Consider a nucleus of helium $_2He^4$. The name derives from the Greek ηλιος for "sun". The element was discovered by spectroscopy during a solar eclipse in the sun's chromosphere by the French astronomer Pierre-Jules-Cesar Janssen in 1868. It was independently discovered and named helium by the English astronomer Joseph Norman Lockyer. It was thought to be only a solar constituent until it was later found to be identical to the helium in the uranium ore cleveite by the Scottish chemist William Ramsay in 1895. Ramsay originally called his gas krypton, until it was identified as helium. The Swedish chemists Per Theodore Cleve and Nils Abraham Langet independently found helium in cleveite at about the same time [11].

According to the proton-neutron conception [1], helium 4 has 2 protons and 2 neutrons. Similar to tetrahedral fullerene C_4, they can form a tetrahedron; however such tetrahedron is asymmetric from the physical and geometric standpoint. The structure does not look beautiful, so it cannot be a veritable from an aesthetic point of view. In order to conserve the symmetry of proton-neutron tetrahedron, one is compelled to accept for a fact that

- Each neutron in a nucleus decomposes into a proton and negatively charged particles.
- All the apices of the tetrahedron are equivalent and therefore they must be protons.

- For helium 4, the number of negatively charged particles is equal to the number of the tetrahedron edges, so they must have a charge ⅓ that of an electron.
- The negatively charged particle will be named a tertion (Latin *tertia* − one third).
- Interaction of the tertions creates hidden symmetry of the electronic pattern (tertion net).
- The electronic symmetry does not coincide with that of the protonic cell but determines it.

It should be emphasized that the last postulate is advanced by analogy with the experimental result for fullerenes [12]. The example is shown in Fig. 1.

The question arises as to whether postulate 1 is formulated on legal grounds. It is known that a free neutron is unstable and decays according to the scheme $n \to p + e^- + \tilde{\nu}_e$ into proton, electron, and antineutrino (β−decay), the average lifetime being equal approximately to 15.3 min [13]. It seems very illogical that the neutron becomes stable inside a nucleus in the strong electric field created by surrounding protons. Apparently "the absolutely new assumption according to which each nuclear electron is connected with one of the nuclear protons forming a neutron" (George Gamov, 1932) needs to be reformulated. To my mind, neutron electrons are similar to valence electrons in molecules and solids, having a possibility to be removed from their parent neutrons in order to create "covalent" bonds. At that, the neutron electron can form various orbitals. Moreover, both subsystems, nuclear protons, and nuclear electrons produce their own patterns of different symmetry.

On the basis of these postulates, it is possible to design the structure of other nuclei [3-7]; for helium, which are shown in Fig. 2.

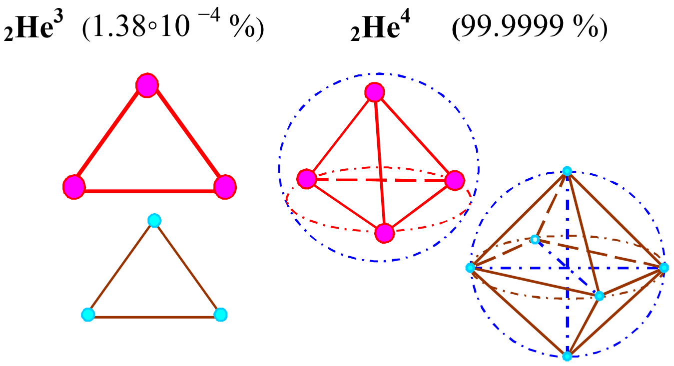


Fig. 2. Proton cells (red) and tertion nets (brown) of helium and its isotope

3. Parallels between lithium, beryllium, and body-centered crystals

The name lithium derives from the Latin *lithos* for "stone" because lithium was thought to exist only in minerals at that time. It was discovered by the Swedish mineralogist Johan August Arfwedson in 1818 in the mineral petalite $LiAl(Si_2O_5)_2$. Lithium was isolated in 1855 by the German chemists Robert Wilhelm Bunsen and Augustus Matthiessen [11]. There are two stable isotopes of lithium: $_3Li^6$ (7.5 %) and $_3Li^7$ (92.5 %) [14].

The name beryllium derives from the Greek word βηρυλλος for a gemstone "beryl" (3BeO_Al_2O_3_$6SiO_2$), in which it is found. It was discovered by the French chemist and pharmacist Nicholas-Louis Vauquelin in beryl and emerald in 1797. The element was first separated in 1828 by the French chemist Antoine-Alexandre-Brutus Bussy and independently by the German chemist Friedrich Wöhler. Because the salts of beryllium have a sweet taste, the element was also known as glucinium from the Greek γλυκυς for "sweet", until IUPAC selected the name beryllium in 1949 [11]. There is only one stable isotope of beryllium: $_4Be^9$ (100 %) [14].

On the basis of the postulates considered above, it is possible to design the structure of these nuclei and explain also their stability (Fig. 3).

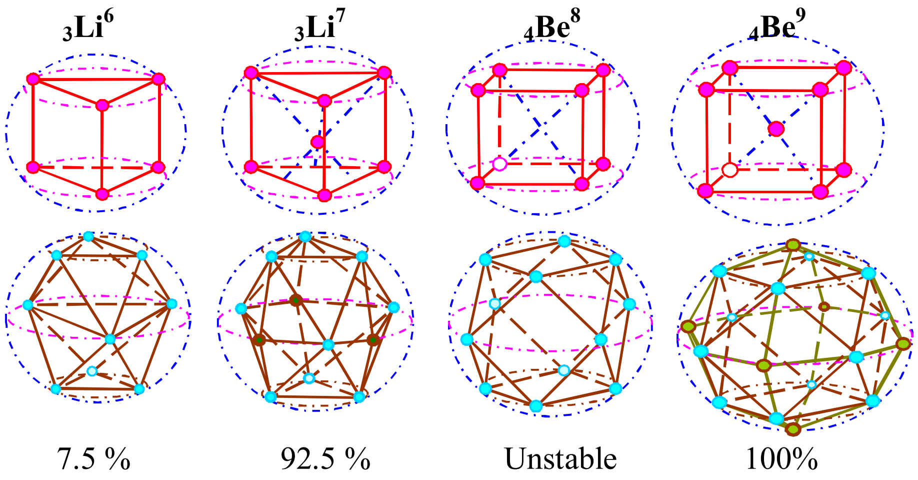


Fig. 3. Proton cells (red) and tertion nets (brown) of lithium, beryllium, and their isotopes

These results deserve further comment. The radioactive isotope $_1H^3$ is also known as tritium with the symbol T; its nucleus is named triton. Combining triton with helium 3 through the use of reaction $_1H^3 + _2He^3 \rightarrow\ _3Li^6$, one can obtain lithium $_3Li^6$. The further reaction $n + _3Li^6 \rightarrow\ _3Li^7$ gives lithium 7. Here a neutron penetrates into lithium $_3Li^6$, where it decays producing a proton and an electron. It is reasonable to accept that a neutron inside lithium 6 is similar to a hydrogen atom. It is known that spectral-line splitting in an electric field (Stark effect) depends on the principal quantum number n. For hydrogen, if n=1, there is no splitting at all; if n=2, there appear three states of equal energy [15]. By analogy with the hydrogen atom, it is valid to say that formally the core-neutron electron gives rise to 2s, $2p_x$, $2p_y$ orbitals, producing a 'valent state' of the core neutron. This state corresponds to the excited sp^2 state, where each of three valent tertions is not in s- or p-state, but in a hybridized state, which can be obtained by mixing a single 2s-state with two 2p-states. The latter is described by a wave function being a linear combination of s- and p-functions. At that, three sp^2 orbitals are located on a plane normal to the three-fold axis of symmetry of lithium. The negatively charged particles (tertions) of the core neutron correspond to three sp^2 orbitals; they are incorporated in the existing tertion net lithium 6 and are painted brown-green in Fig. 3. As a result, there forms a denser tertion net. The proton cell, having a shape of a regular triangle prism, becomes a body-centered one. Such structure is more abundant.

Similar to the algorithm developed for lithium, consider reaction $n + _4Be^8 \rightarrow\ _4Be^9$. Here a neutron penetrates into beryllium $_4Be^8$, where it decays. As a result, there forms a denser tertion net, and the proton cell, having a shape of a cube, becomes a body-centered cube (Fig. 3). While beryllium 8 is unstable, the abundance of $_4Be^9$ is 100%. Therefore its structure has super stability. To my mind, stability is ensured by two factors. The first is the packing density of the proton cell; the second is the density of the coat of mail (tertion net). Both factors take place in the case of beryllium 9, giving its structure super stability and abundance of 100%.

One further comment should be made. Hitherto it was assumed that the negative particle charge is ⅓. From the results obtained for beryllium 9, it follows that the core neutron decays into a proton and six negatively charged particles, having a charge $^1/_6$. The difference may be attributed to the Stark effect, where spectral-line splitting depends on the principal quantum number n. If n=2, there are three states of equal energy, for n=3, the number of states becomes six [15]. Formally the neutronic electron gives rise to $2p_x$, $2p_y$, and $2p_z$ orbitals, producing a 'valent state' of the core neutron.

Now consider potassium. I hope that after the rather detailed introduction a reader can easily catch the author's meaning. Designing the structures, it is necessary to bear in mind that the structures obtained must satisfy "the principle of least complexity", i.e. they are the simplest among all possible.

4. Isomers of potassium and their isotopes

The name derives from the English "potash" or "pot ashes" because it is found in caustic potash (KOH). The chemical symbol K derives from the Latin kalium via the Arabic qali for alkali. It was first isolated by the British chemist Humphry Davy in 1807 from the electrolysis of potash [11]. What do we know about potassium? There are two stable isotopes of potassium: $_{19}K^{39}$ (93.2581%) and $_{19}K^{41}$ (6.7302%), and one isotope $_{19}K^{40}$, has a very large half-decay period being equal to $1.277\cdot10^6$ [14].

The problem is how to explain these data in the framework of a unified model. Previously [3-7] the nuclei were separated into two main types: basic nuclei having equal numbers of protons and neutrons, and isotopes having one or more additional core neutrons. A better understanding can be gained if we begin with basic nuclei having equal numbers of protons and neutrons. Consider again the parallels between fullerenes and nuclei.

Isomers of fullerene C_{38}. From the periodic table of fullerenes [16,17], it follows that there is only one perfect fullerene of the mass 38, having 3-fold S-symmetry. To it, one needs to add the semi-perfect fullerene having rotation-reflection 4-fold symmetry. The reason is connected with the fact that this semi-perfect fullerene has almost the same energy as the perfect one. The fullerenes, which were considered, were designed previously [17,18]. With knowledge of the fullerene structure, it is possible to design nuclear structures. They can be obtained in various ways through the use of the most probable geometrically compatible reactions. Consider some possible reactions.

Joining deuteron to argon. One of the possible reactions is illustrated in Fig. 4. It can be written as $d + {}_{18}Ar^{36}(1) \rightarrow {}_{19}K^{38}(1)$. The structure of argon was obtained in [7]. Here a deuteron is incorporated into the surface of the basic nucleus of argon. From Figure 4 it follows that for argon only four protons from thirty-six take part in the reaction. They are pink. The nucleus obtained contains one square, ten pentagons, and ten hexagons. The graph representation of this reaction is shown in Fig. 5.

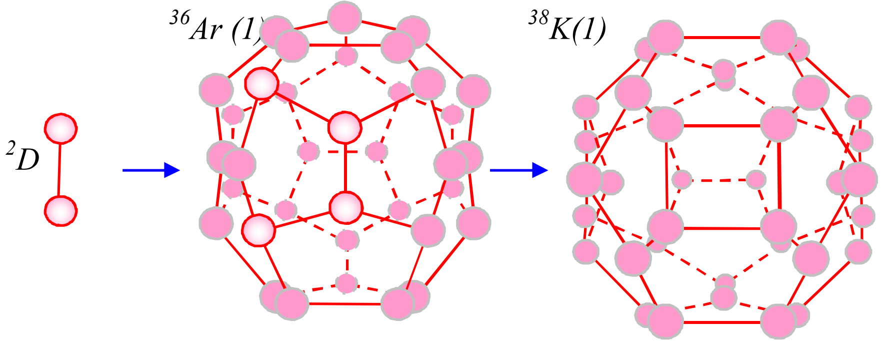


Fig. 4. Attachment of deuteron to argon (1) and formation of potassium (1)

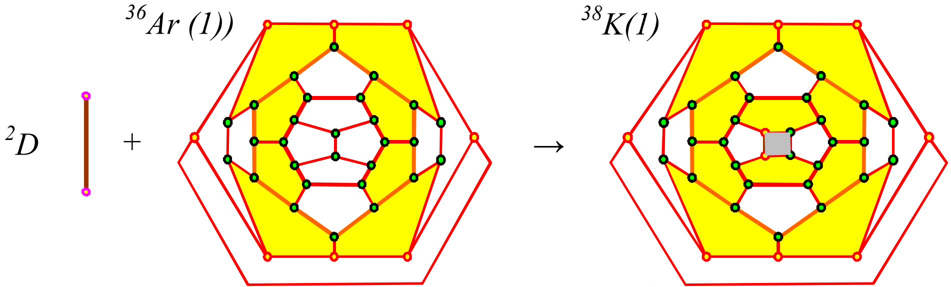


Fig. 5. Graph of nuclear reaction $d + {}_{18}Ar^{36}(1) \rightarrow {}_{19}K^{38}(1)$

Joining three-fold symmetry-cupola oxygen to two-fold-symmetry sodium. Consider the reaction ^{16}O + ^{22}Na → ^{38}K (2). It consists in joining a cupola of three-fold symmetry to a spheroid having two-fold symmetry (Fig. 6). Their structures were obtained previously [5,6,19].

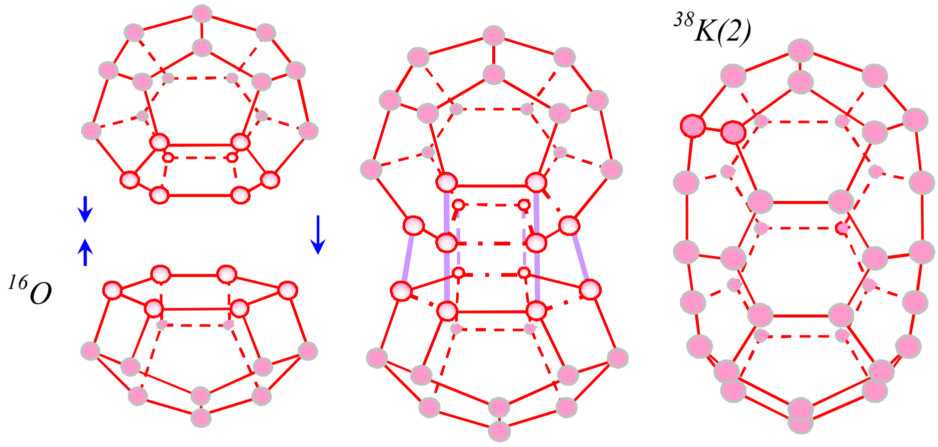


Fig. 6. Fusion of oxygen with sodium and formation of potassium (2): neutral protons (dark pink spheres), reacting protons (light pink spheres), proton bonds (red lines), intermediate-compound bonds to be broken (red dot lines), new bonds to be formed (lilac firm lines)

The graph representation of this reaction is shown in Fig. 7. To fully appreciate the nuclear reactions discussed above, the electronic structure (tertion net) of the potassium isomers is presented in Fig. 8. Similar to beryllium 8, these basic nuclei are unstable.

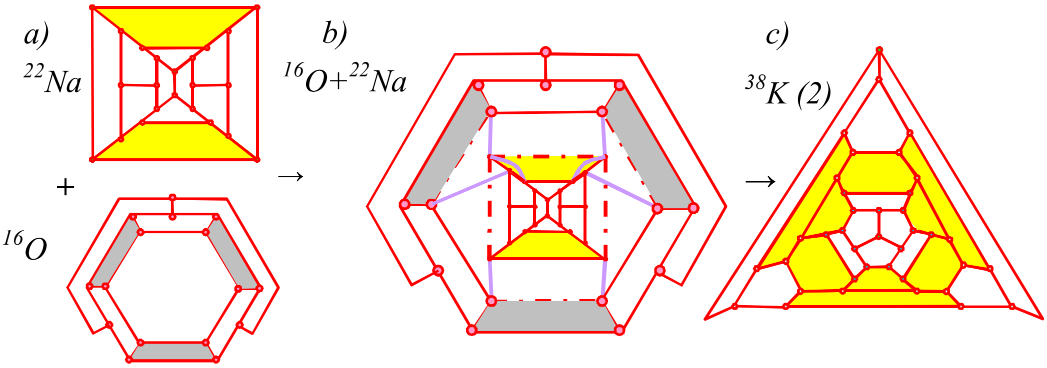


Fig. 7. Graph of nuclear reaction $_{8}O^{16} + {}_{11}Na^{22} \rightarrow {}_{19}K^{38}$(2). Embedding the sodium graph into the oxygen graph: a) separate graphs, b) embedding, c) graph of potassium (2)

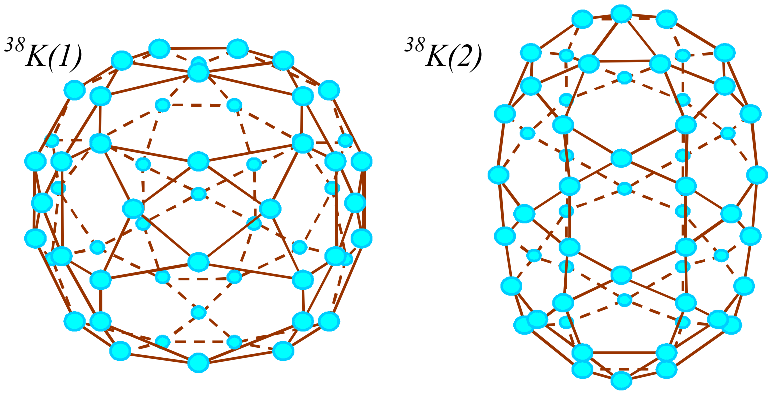


Fig. 8. Electronic structure of potassium isomers

One-neutron-core isotopes. From the results obtained for the basic nuclei, it follows that the isotope ${}_{19}K^{39}$ having different symmetry is able to form two isomers. They can be got by incorporating a neutron, n + ${}_{19}K^{38}$ → ${}_{19}K^{39}$. The protons cells and tertion nets of the isomers are shown in Fig. 9.

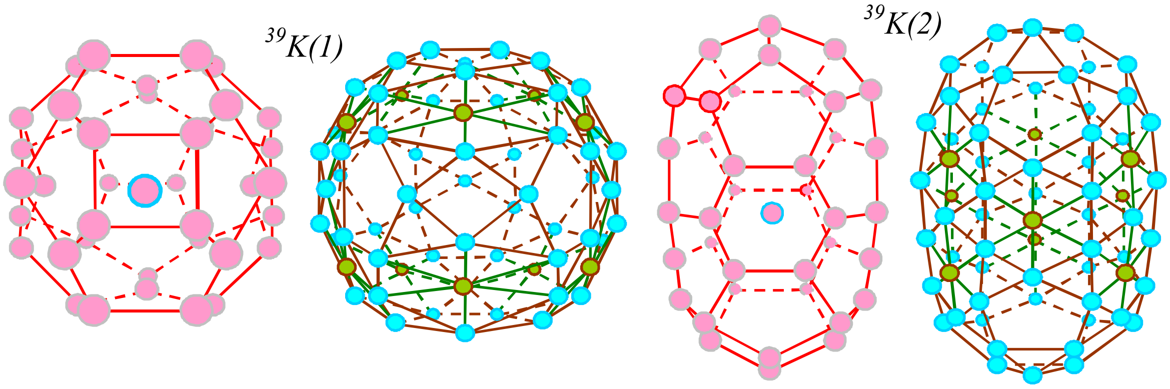


Fig. 9. Protonic cells and tertion nets of the isomers of potassium isotope ${}_{19}K^{39}$

It should be emphasized the following. Extra core neutrons of isotope isomers are inside the corresponding nuclei. Here they are subjected to the electric field of their surroundings. It turned out that, due to the Stark effect [15], the decomposition of an internal core neutron into a proton and tertions differs from that of an external neutron; namely, the number of components of electron splitting depends on the symmetry of the surroundings. It is reasonable to leave the name "tertian" for an electron component, but change its charge. In our case, the internal core neutron's tertions have the charge $^{1}/_{10}$ *e* for ^{39}K (1) and $^{1}/_{9}$ *e* for ^{39}K (2). The regular thing is that the number of such tertions is equal to the number of hexagons in the tertion net of a basic nucleus.

Two-neutron-core isotopes. It is possible to obtain two isomers of the isotope ${}_{19}K^{40}$ through the use of the reaction n + ${}_{19}K^{39}$ → ${}_{19}K^{40}$ (Fig. 10).

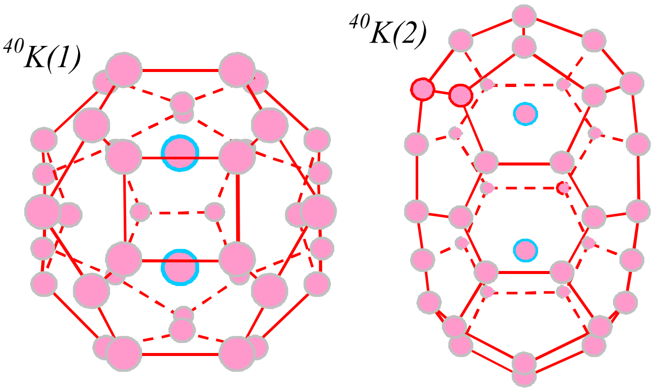


Fig. 10. Protonic cells of the isomers of potassium isotope ${}_{19}K^{40}$

Consider isotopes ^{40}K. From Figure 10 it follows that two core neutrons are removed from one another at the distance larger than the parameter of the proton cell. When the neutrons decompose into protons and tertions under the action of the external surroundings, the protons fell in the minima of the double-well potential directed along the main symmetry axis. However, now the protons are only slightly connected with one another. It means that they behave themselves almost independently sending their tertions into the existing coat of mail. As a result, the shape of tertion net does not change. It looks like in Fig. 9, but the tertions produced by the core neutrons now have double charge, $^{1}/_{5}$ *e* for ^{40}K (1) and $^{2}/_{9}$ *e* for ^{40}K (2), respectively.

Three-neutron-core isotope. From the results obtained for silicon, sulfur, and argon [7], it follows that, if the isotopes contain more than two internal neutrons, they can form clusters inside the nucleus. However, the cluster symmetry must be compatible with that of the external surroundings. In our case, only the isotope ^{40}K(2) of three-fold S-symmetry is able to incorporate one more neutron and generate a core of compatible symmetry. There are two possible isomers of the core: a linear chain and an equilateral triangle (Fig. 11). The approximate position of the "neutron" triangle can be found in a purely geometric way as it is shown in Fig. 11, at the right. In the first case, the tertions of the third neutron have the charge ½ *e*, and in the second one ⅓ *e*. The structure of the coat of mail (the external tertion net) remains unchanged.

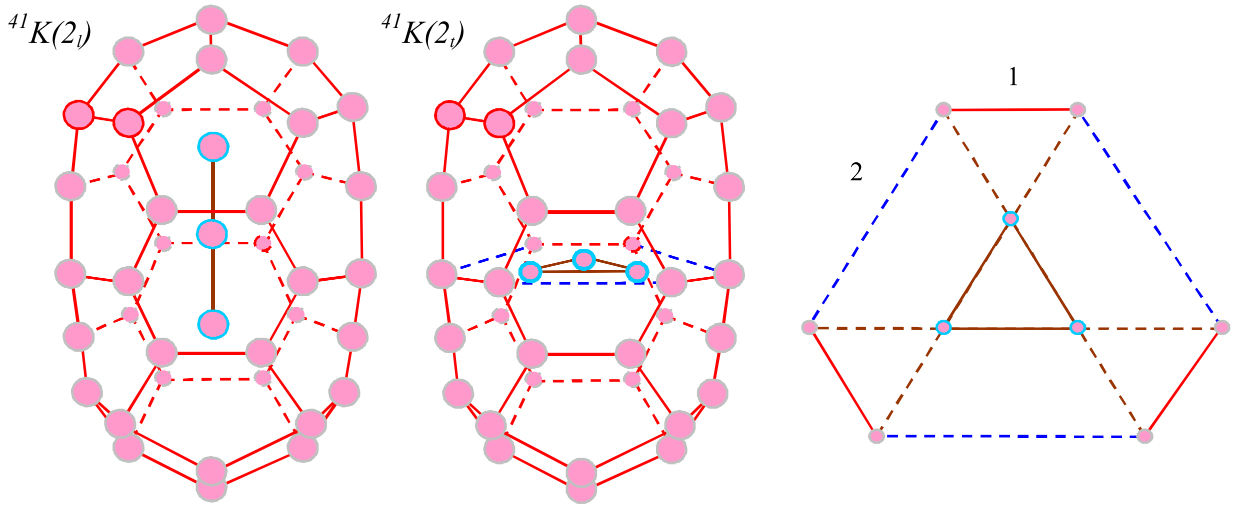


Fig. 11. Protonic cell of isotope ^{41}K(2) and the equatorial section (at the right)

These results deserve more consideration. Up to now, when there were two internal core neutrons, they were decomposed into protons and tertions by the external surroundings, the protons being tightly connected with the electronic coat of mail and maybe only slightly connected to one another. Now it is seen that the internal protons form the linear or triangle core, they being tightly connected to one another. In the second case, the triangle-core structure resembles that of helium-3 proposed previously [3]. The fact that there are two stable core structures sets one thinking that helium-3 may also be realized as an isomer having the liner shape. Moreover, one is inclined to think that the existence of the two isomers provokes the appearance of two superfluid phases of helium-3.

5. Isomers of calcium and their isotopes

The name derives from the Latin *calx* for "lime (CaO)" or "limestone ($CaCO_3$)" in which it was found. It was first isolated by the British chemist Humphry Davy in 1808 with help from the Swedish chemist Jöns Jacob Berzelius and the Swedish court physician M.M. af Pontin [11].

There are five stable isotopes of calcium: $_{20}Ca^{40}$ (96.947%), $_{20}Ca^{42}$ (0.647%), $_{20}Ca^{43}$ (0.135%), $_{20}Ca^{44}$ (2.086%), $_{20}Ca^{46}$ (0.004%) and $_{20}Ca^{48}$ (>2·10^{18} y, 0.187%). Besides, there is isotope $_{20}Ca^{41}$, having a very large half-decay period equal to 1.4·10^{5} y, and isotope $_{20}Ca^{45}$ with an average half-decay period of 164 diurnals [14]. The problem is the same as before, how to explain these data in the framework of a unified model. Consider again the parallels between fullerenes and nuclei. A better understanding can be gained if we begin with basic nuclei having equal numbers of protons and neutrons.

Isomers of fullerene C_{40}. From the periodic table of fullerenes [16,17], it follows that there are three perfect fullerenes of the mass 40. They belong to 4-fold-symmetry, 5-fold, and tetrahedral. To them, one needs to add the two perfect fullerenes having rotation-reflection 4-fold symmetry and tetrahedral one, and moreover, the imperfect fullerene having 3-fold S-symmetry (simultaneously the perfect fullerene of 2-fold symmetry). The reason is connected with the fact that the latter fullerene has an energy which is close to that of the perfect ones and even less. The fullerenes, which are considered, have been designed previously [20,21]. They can be obtained through the use of the most probable geometrically compatible reactions in different ways.

With knowledge of the fullerene structure, it is possible to obtain nuclear structures. They can be designed in various ways through the use of the most probable geometrically and physically compatible reactions. By analogy with the fullerenes, consider some possible nuclear reactions.

Joining deuteron to potassium of three-fold symmetry. The possible reaction is presented in Fig. 12. It can be written as $d + {}_{19}K^{38}(2) \rightarrow {}_{20}Ca^{40}(1)$, the structure of potassium being obtained above. Here a deuteron is incorporated into a basic nucleus of potassium. From Figure 12 it follows that for potassium only four protons from thirty-eight take part in the reaction. They are pink. The nucleus obtained contains six pentagons and ten hexagons. The graph representation of this reaction is shown in Fig. 13.

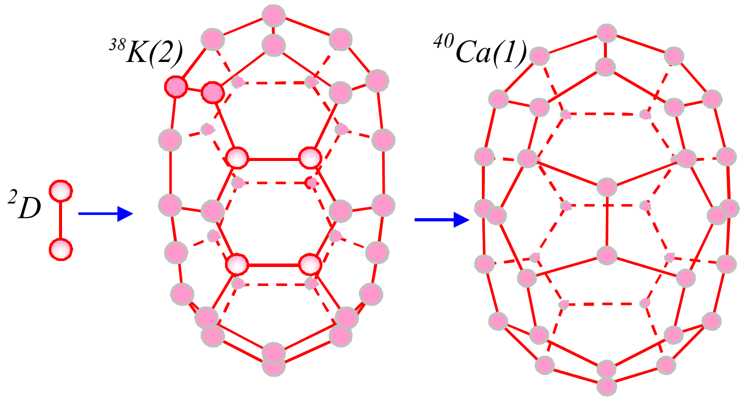


Fig. 12. Attachment of deuteron to potassium (2) and formation of calcium (1)

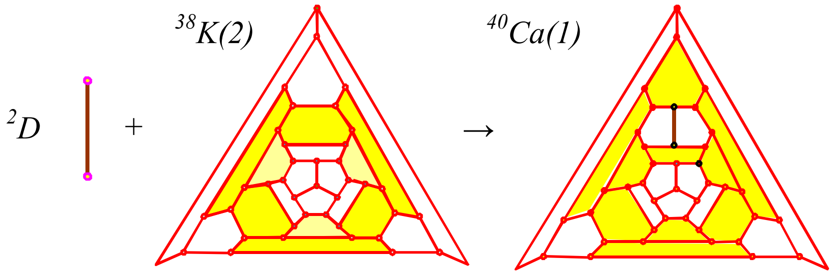


Fig. 13. Graph representation of the nuclear reaction $d + {}_{19}K^{38}(2) \rightarrow {}_{20}Ca^{40}(1)$

Joining two deuterons to argon of four-fold symmetry. The reaction is illustrated in Fig. 14 and can be written as $d + \rightarrow {}_{18}Ar^{36}(4\text{-fold}) + d \rightarrow {}_{20}Ca^{40}(2)$. Here two deuterons are incorporated in series into one of the basic nucleus of sulfur having four-fold symmetry. From Figure 14 it follows that for argon eight protons from thirty-six take part in the reaction. They are pink. The nucleus obtained contains twelve pentagons and five hexagons. The graph representation of the two-stage reaction is shown in Fig. 15.

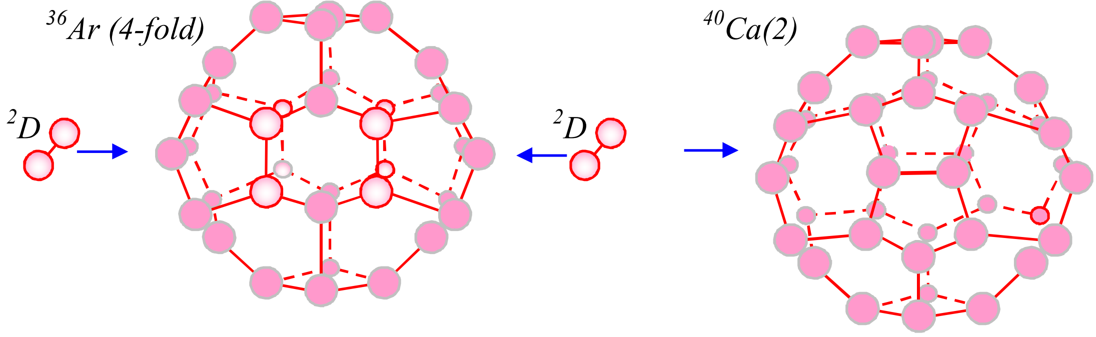


Fig. 14. Attachment of two deuterons to argon (4-fold) and formation of calcium (2)

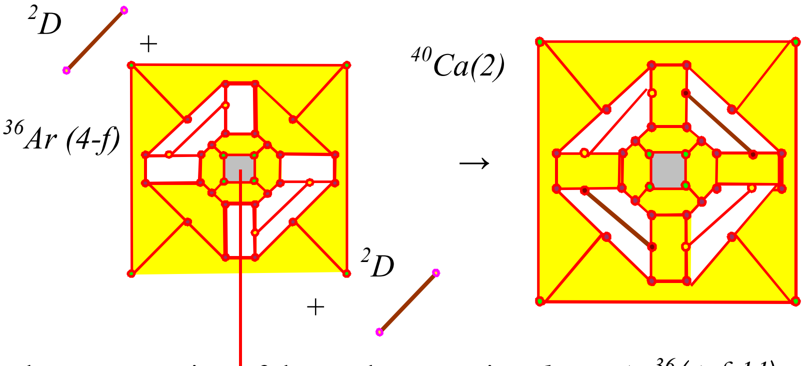


Fig. 15. Graph representation of the nuclear reaction $d + {}_{18}Ar^{36}(4\text{-}fold) \rightarrow {}_{20}Ca^{40}(2)$

Joining deuteron to potassium having rotation-reflection 4-fold symmetry. One of the possible reactions is illustrated in Fig. 16. It can be written as $d + {}_{19}K^{38}(1) \rightarrow {}_{20}Ca^{40}(3)$, the structure of potassium 38 (1) being obtained above. Here a deuteron is incorporated into a basic nucleus of potassium. From the figure, it follows that for potassium only four protons from thirty-eight take part in the reaction. They are pink. The nucleus obtained contains two squares, eight pentagons, and twelve hexagons. The graph of this reaction is shown in Fig. 17.

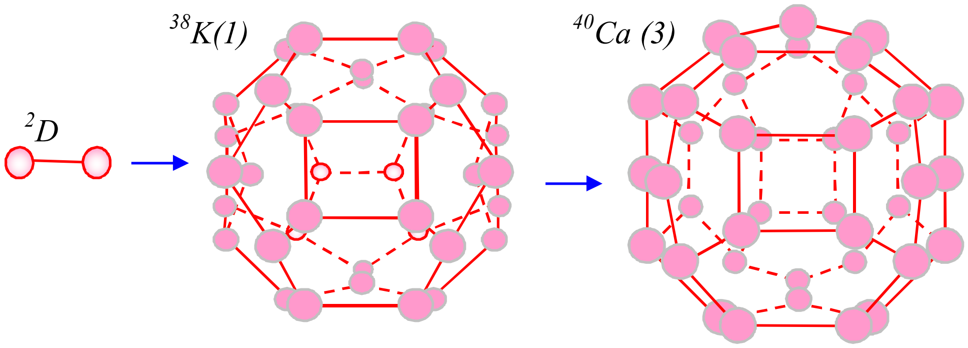


Fig. 16. Attachment of deuteron to potassium (1) and formation of calcium (3)

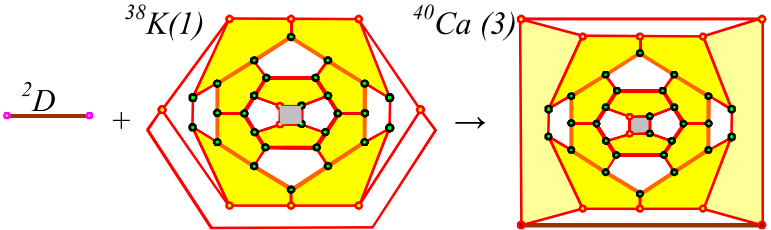


Fig. 17. Graph representation of the nuclear reaction $d + {}_{18}K^{38}(1) \rightarrow {}_{20}Ca^{40}(3)$

To fully appreciate the nuclear reactions discussed above, the electronic structure (tertion net) of the calcium isomers is presented in Fig. 18.

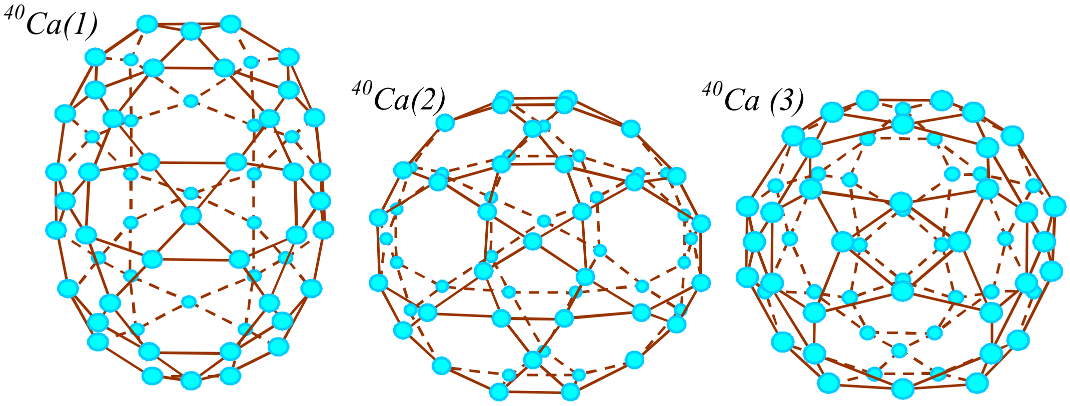


Fig. 18. Electronic structure of calcium isomers

Nuclear polymerization, folding, and fusion. Consider the parallels between polymers and nuclei. By analogy with biosynthesis [22], one inclines to think that there is the following chain of nuclear reactions creating α-process (Fig. 19). At first, two single alpha-particles (a) combine forming a dimer (b). Then the dimer adds another alpha-particle producing a linear trimer. The trimer has one proton, which is slightly connected with the trimer through the use of only one bond (c). Then the trimer attaches one more alpha-particle creating a four-link chain with two slightly connected protons (d). The process is continuing producing a five-link chain (e). Similar to the interactions of electronic and atomic degrees of freedom in molecules [23], the interaction of tertions (they are not shown in the figure) and protons leads to internal rotation [24] of the slightly connected protons (f). This structure is able to fold up creating the neon nucleus of five-fold symmetry (g) in the shape of a cupola Therefore calcium isomers of five-fold symmetry can be also obtained by means of the following reactions

$$^{4}\mathrm{He}+{}^{4}\mathrm{He}\to({}^{4}\mathrm{He}{}^{4}\mathrm{He})+{}^{4}\mathrm{He}\to({}^{4}\mathrm{He}{}^{4}\mathrm{He}{}^{4}\mathrm{He})+{}^{4}\mathrm{He}\to({}^{4}\mathrm{He}{}^{4}\mathrm{He}{}^{4}\mathrm{He}{}^{4}\mathrm{He})+{}^{4}\mathrm{He}\to{}^{20}\mathrm{Ne}$$
$$\to{}^{20}\mathrm{Ne}+{}^{20}\mathrm{Ne}\to{}^{40}\mathrm{Ca}.$$

It should be emphasized that there are two ways of neon cupola fusion: mirror symmetry and rotation-reflection. As a result, there appear two isomers of calcium. The protons cells and tertion nets of the isomers are shown in Fig. 20. The graphs of the nuclear cells are presented in Fig. 21.

Fusion of two neon-dodecahedron nuclei. There are three isomers of neon 20: classical dodecahedron of pentagons, (tetra-hexa)$_3$-penta$_6$-dodecahedron, and (tetra-hexa)$_6$ dodecahedron [5]. Among the fullerenes having similar shapes, the classical dodecahedron has the highest probability of formation [19]. For this reason, it is worthwhile to investigate the reaction ${}_{10}\mathrm{Ne}^{20}+{}_{10}\mathrm{Ne}^{20}\to{}_{20}\mathrm{Ca}^{40}$, where both constituents are classical dodecahedra. The reaction is illustrated in Fig. 22.

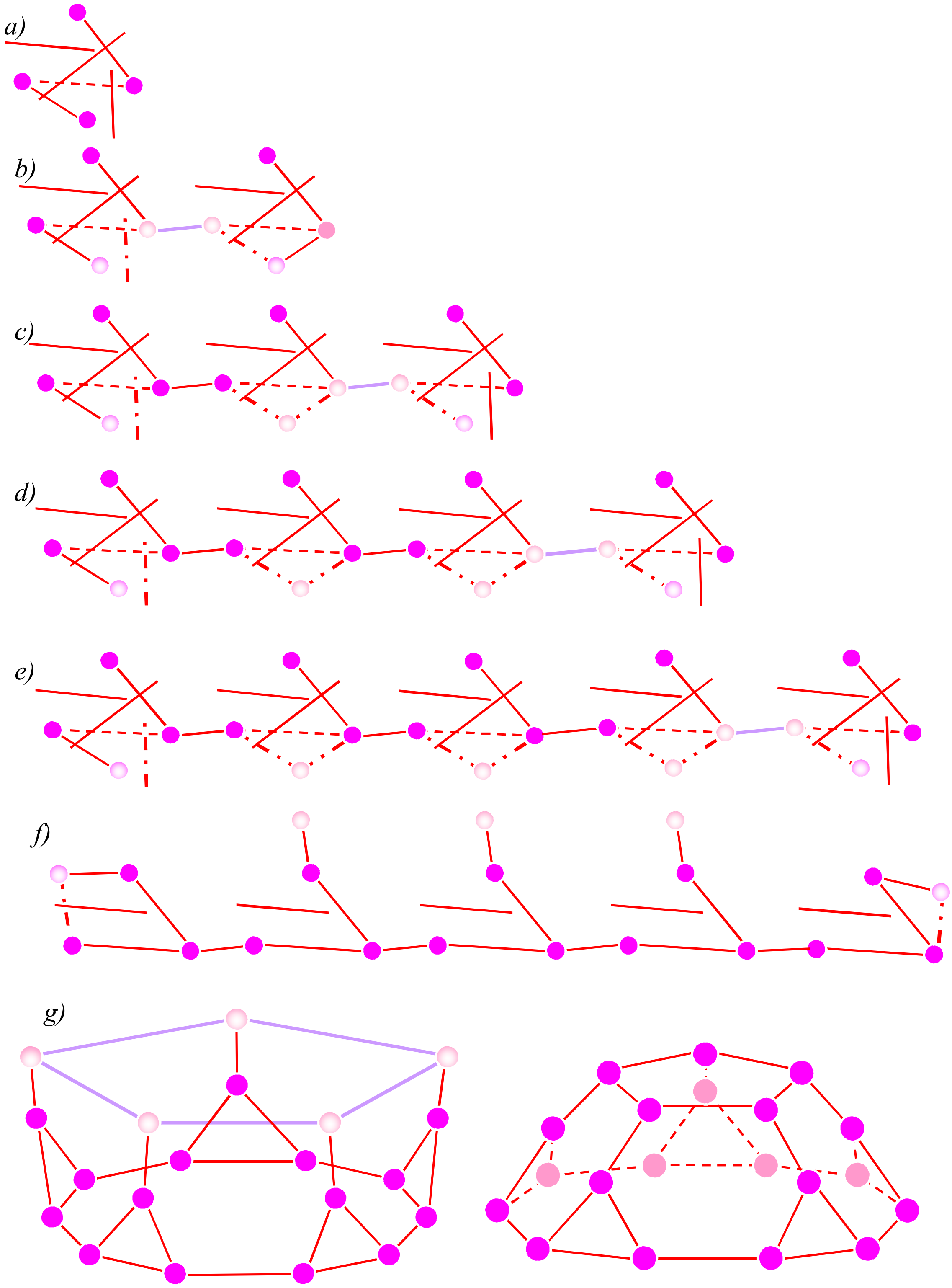


Fig. 19. Generation of cupola-neon-20 isomer: a) single α-particle; b) dimer formation; c) trimer formation with one slightly connected proton; d) four-link chain with two slightly connected protons; e) five-link chain with three slightly connected protons; f) internal rotation of five protons, g) folding of the five-link chain and formation of a five-fold symmetry cupola

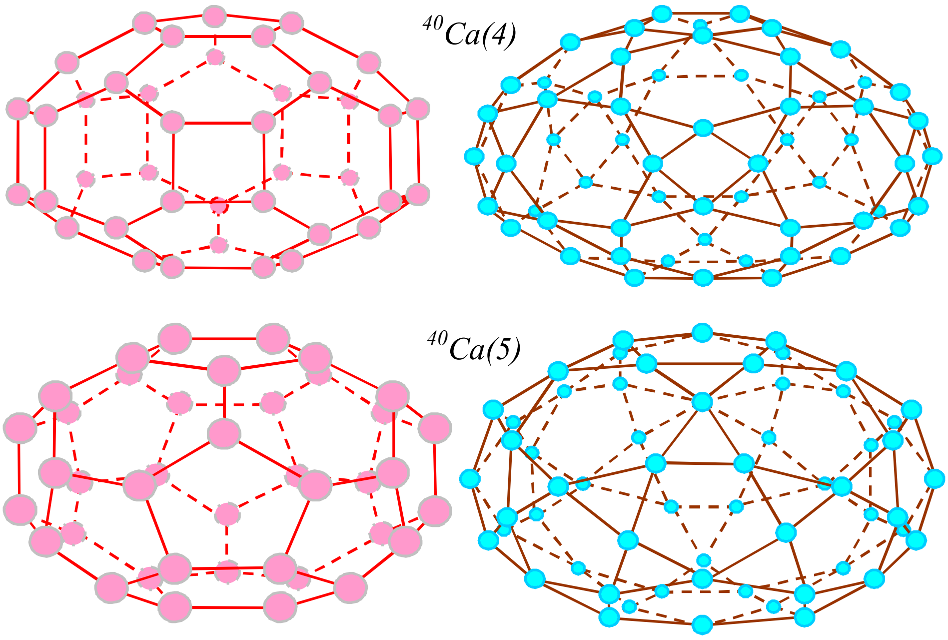


Fig. 20. Protonic and electronic structure of the isomers of calcium $_{20}Ca^{40}$ (4 & 5)

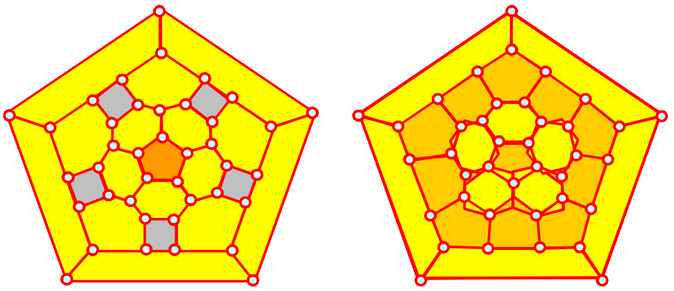


Fig. 21. Graph of the tetra$_5$–penta$_2$–hexa$_{15}$ polyhedron at the left and that of the penta$_{12}$–hexa$_{10}$ polyhedron at the right

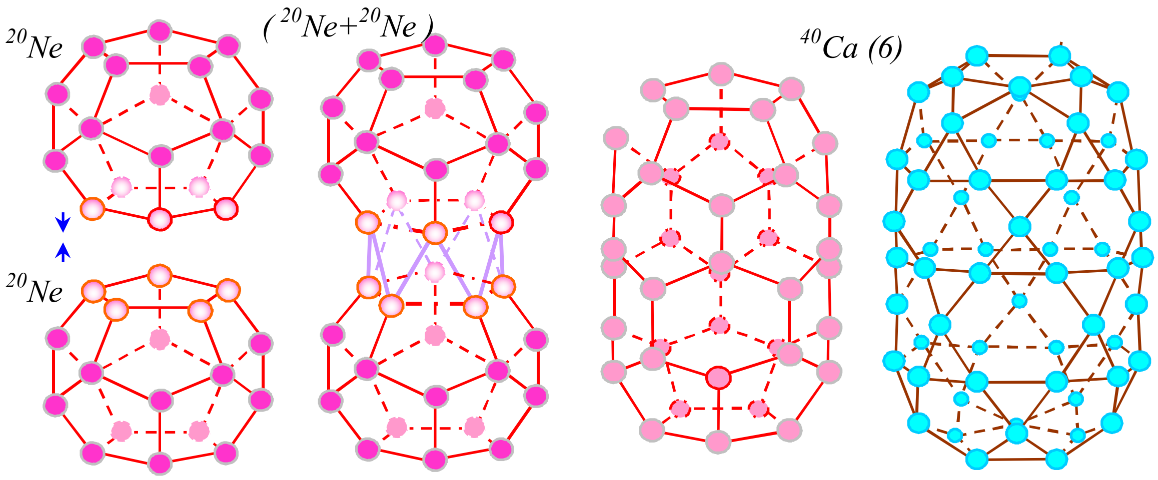


Fig. 22. Fusion of two neon nuclei and formation of calcium (5)

From the figure, it follows that for each neon nucleus only five protons from twenty take part in the reaction. They are pink. The nucleus obtained contains twelve pentagons and ten hexagons; it has five-fold symmetry. The graph of the reaction is shown in Fig. 23.

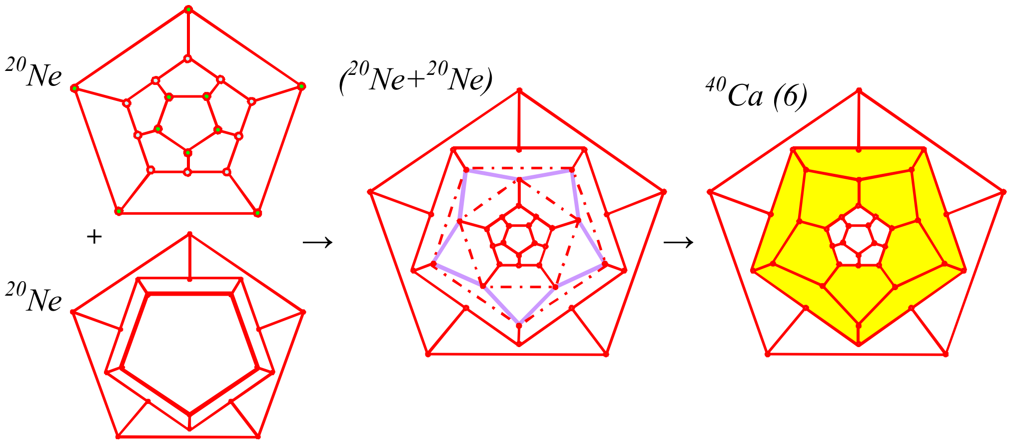


Fig. 23. Graph representation of the nuclear reaction ${}_{10}\mathrm{Ne}^{20} + {}_{10}\mathrm{Ne}^{20} \rightarrow {}_{20}\mathrm{Ca}^{40}$(5)

Joining carbon of six-fold symmetry with tetrahedral silicon. The reaction is illustrated in Fig. 24 and can be written as ${}_{6}\mathrm{C}^{12} + {}_{12}\mathrm{Si}^{28} \rightarrow {}_{20}\mathrm{Ca}^{40}$(7). The nucleus obtained consists of pentagons and hexagons only. The graph representation of this reaction is shown in Fig. 25. It should be emphasized that the first reacting component has six-fold symmetry whereas the second one refers to tetrahedral symmetry. By analogy with designing the fullerenes of tetrahedral symmetry [25], it was assumed that this reaction would give a nucleus of tetrahedral symmetry. However, contrary to the expectation, the nucleus obtained had no tetrahedral symmetry. Under such circumstances, I should look again at the fullerene science.

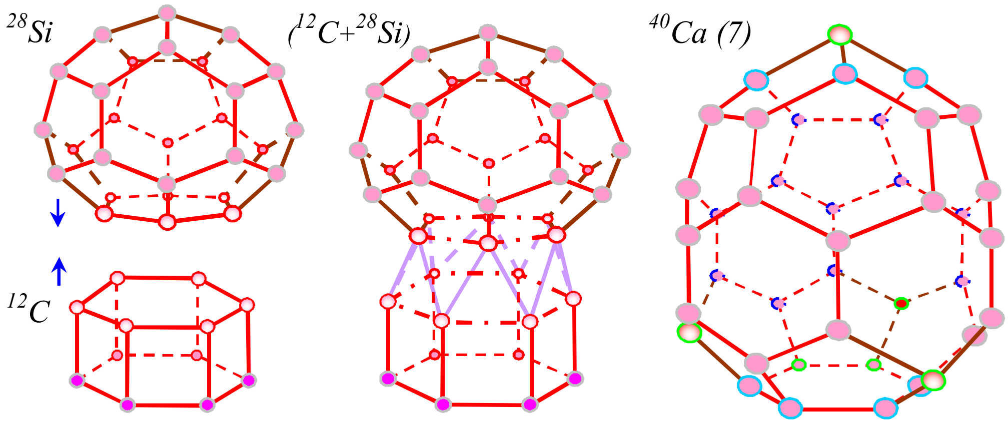


Fig. 24. Fusion of carbon with silicon and formation of calcium (7)

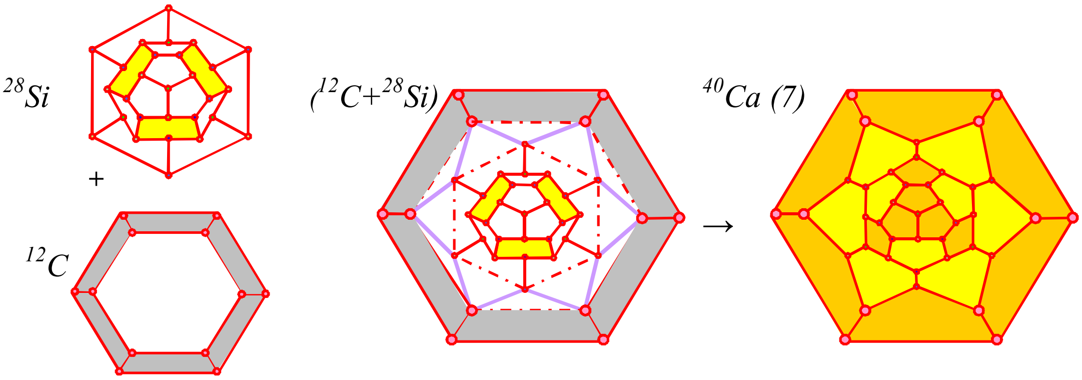


Fig. 25. Graph representation of the nuclear reaction ${}_{6}\mathrm{C}^{12} + {}_{12}\mathrm{Si}^{28} \rightarrow {}_{20}\mathrm{Ca}^{40}$(7)

Stone-Wales transformation of fullerenes. Sometimes an asymmetric fullerene is undergone to Stone-Wales transformation [26] for obtaining a more stable and symmetric isomer [27-30]. The transformation [26] was designed for "spheroidal molecules of sp^2 hybridized carbon atoms with different arrangements of the hexagonal and pentagonal rings. The rearrangement formally requires two sigma bonds to be broken and new bonds to be formed."

Stone-Wales transformation of nuclei. However, the scheme [26] shows only an isolated event. When one has to deal with a whole molecule or with a whole nucleus, where many such events take place, for clarity's sake, it will be more illustrative and informative to use graph language. The Stone-Wales' graph transformation for calcium (4) is shown in Fig. 26. Here the broken edges (bonds) are given using red dot lines; new edges are specially marked, they being painted lilac; the other color designations are the same as before. Really, using three Stone-Wales' transformations we obtained the tetrahedral nucleus of calcium (5). The space shape of protonic cells and tertion nets is shown in Fig. 27.

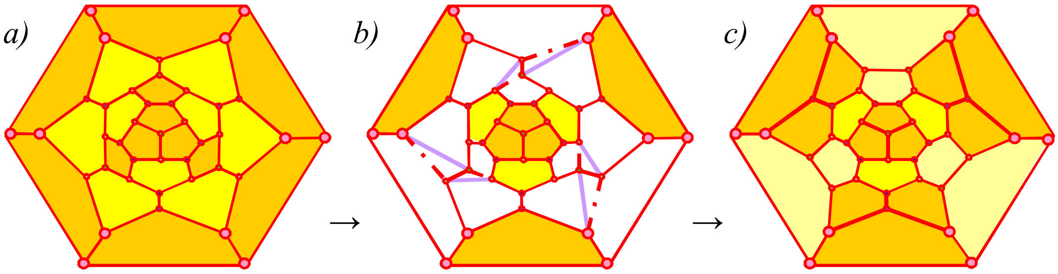


Fig. 26. Graph representation of three Stone-Wales' transformations

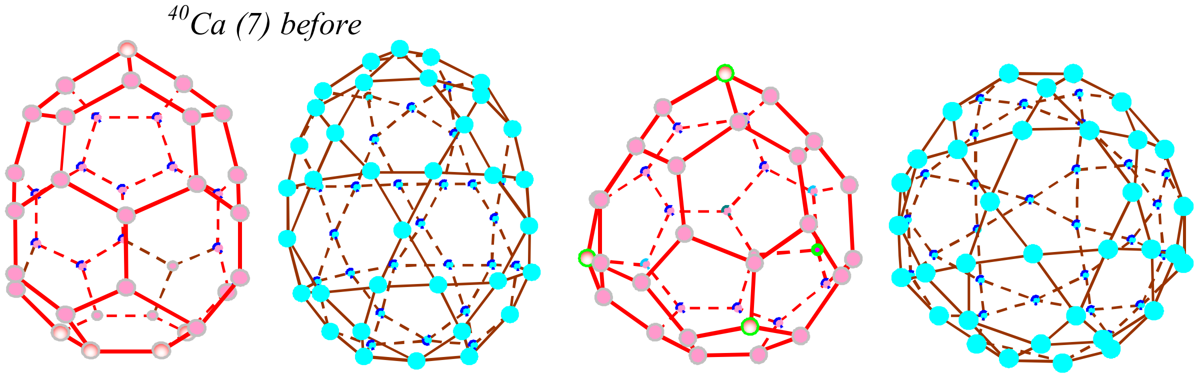
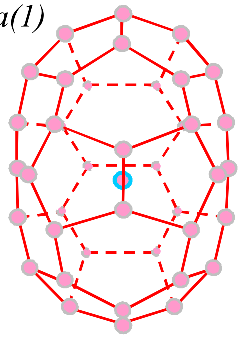


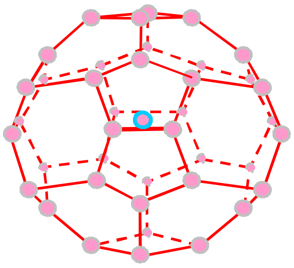
Fig. 27. Protonic cell and tertion net of the calcium isomers 7 and 8

One-neutron-core isotopes. The results obtained for the basic nuclei $_{20}Ca^{40}$ show that it is able to form seven isomers. Therefore for the isotope $_{20}Ca^{41}$ there is also such possibility. The corresponding isomers can be got by incorporating a neutron into the basic nucleus according to reaction $n + {}_{20}Ca^{40} \rightarrow {}_{20}Ca^{41}$. The protons cells and tertion nets of the isomers obtained are shown in Fig. 28.

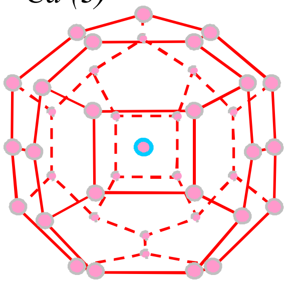
$^{41}Ca(1)$



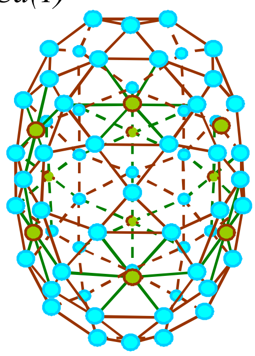
$^{41}Ca(2)$



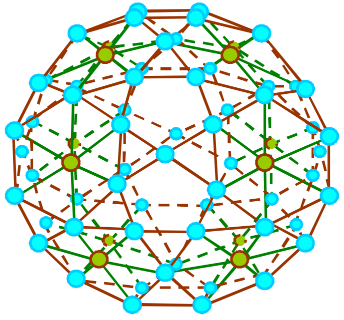
$^{41}Ca\ (3)$



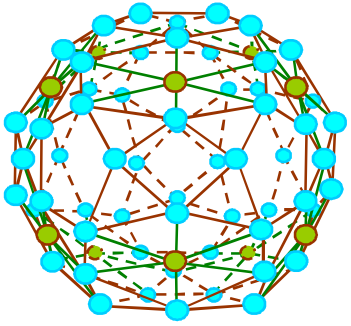
$^{41}Ca(1)$



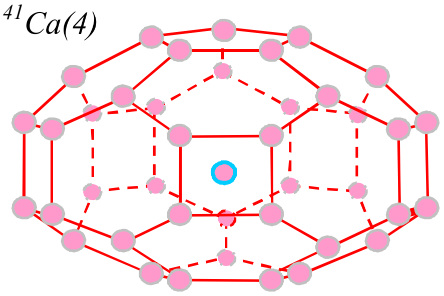
$^{41}Ca(2)$



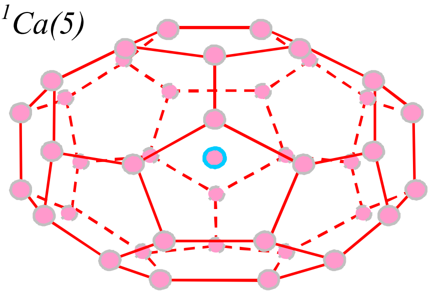
$^{41}Ca(3)$



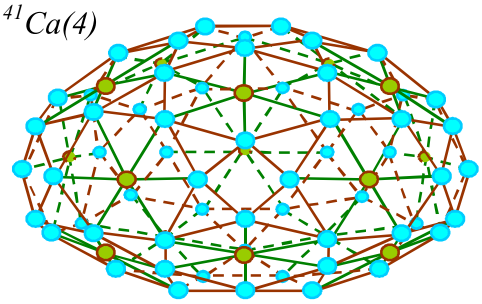
$^{41}Ca(4)$



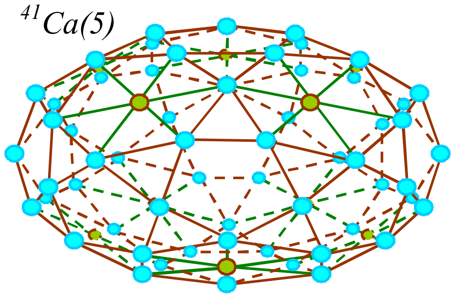
$^{41}Ca(5)$



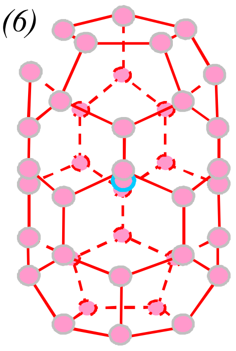
$^{41}Ca(4)$



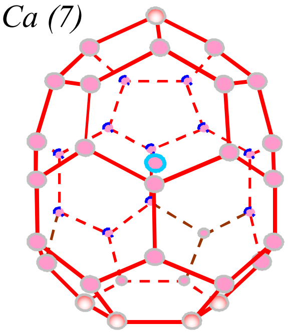
$^{41}Ca(5)$



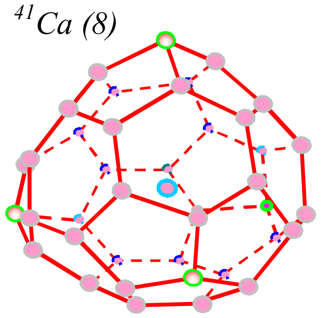
$^{41}Ca\ (6)$



$^{41}Ca\ (7)$



$^{41}Ca\ (8)$



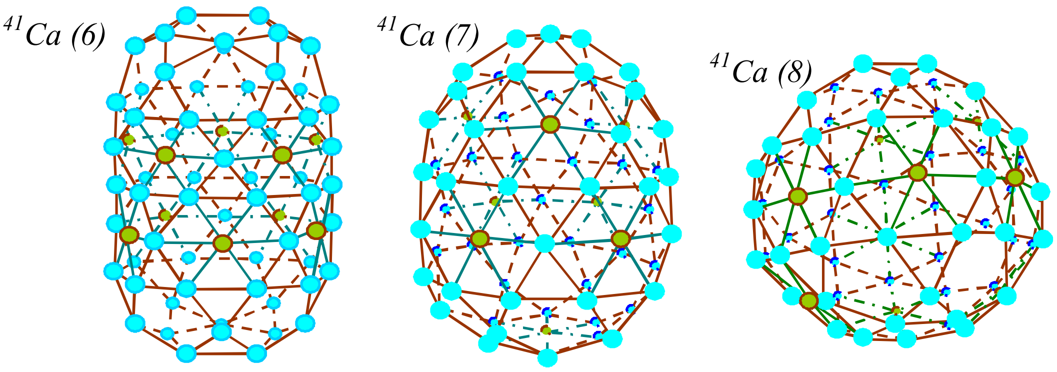
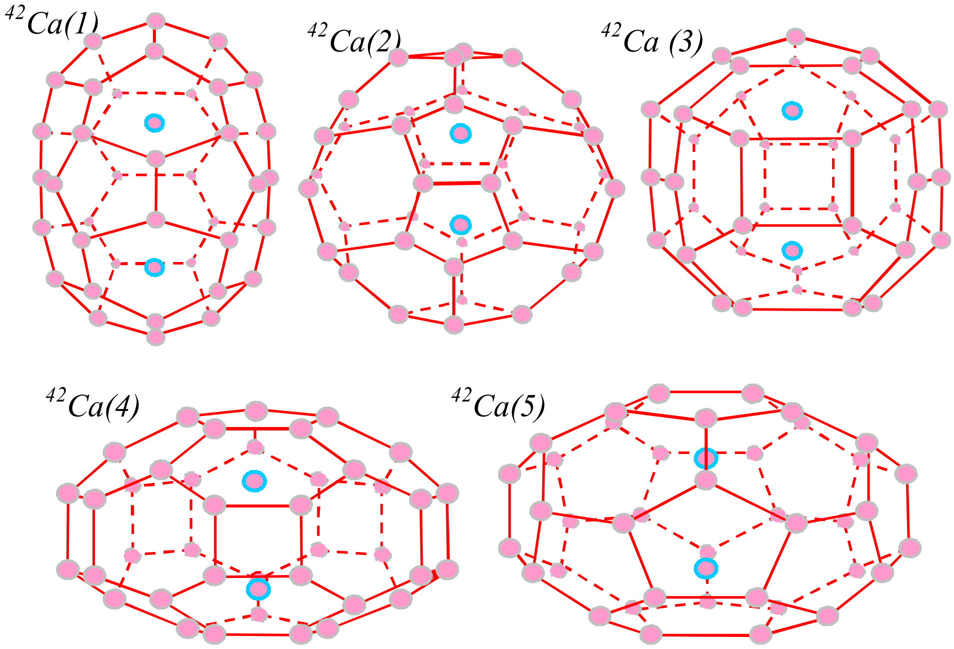


Fig. 28. Protonic cells and tertion nets of the isomers of calcium isotope $_{20}$Ca^{41}.

It should be emphasized the following. Core extra neutrons of isotope isomers are inside the corresponding nuclei. Here they are subjected to the electric field of their surroundings. It turned out that, due to the Stark effect [15], the decomposition of an internal core neutron into a proton and tertions differs from that of an external neutron; namely, the number of electronic-splitting components depends on the symmetry of the surroundings. It is reasonable to leave the name "tertian" for an electron component, but change its charge. In our case the tertions of the core neutrons have the charge $^{1}/_{10}$ *e* for ^{41}Ca (1), $^{1}/_{12}$ *e* for ^{41}Ca (2) and ^{41}Ca (3), $^{1}/_{15}$ *e* for ^{41}Ca (4), $^{1}/_{10}$ *e* for ^{41}Ca (5), ^{41}Ca (6), ^{41}Ca (7) and ^{41}Ca (8). The regular thing is that the number of such tertions is equal to the number of hexagons in the tertion net of a basic nucleus.

Two-neutron-core isotopes. It is possible to obtain seven isomers of the isotope $_{20}$Ca^{42} through the use of the reaction n + $_{20}$Ca^{41} → $_{20}$Ca^{42} (Fig. 29).



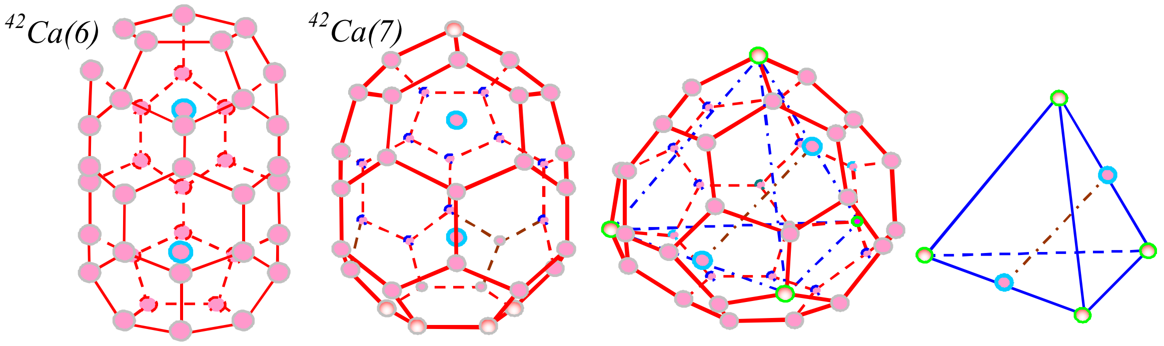
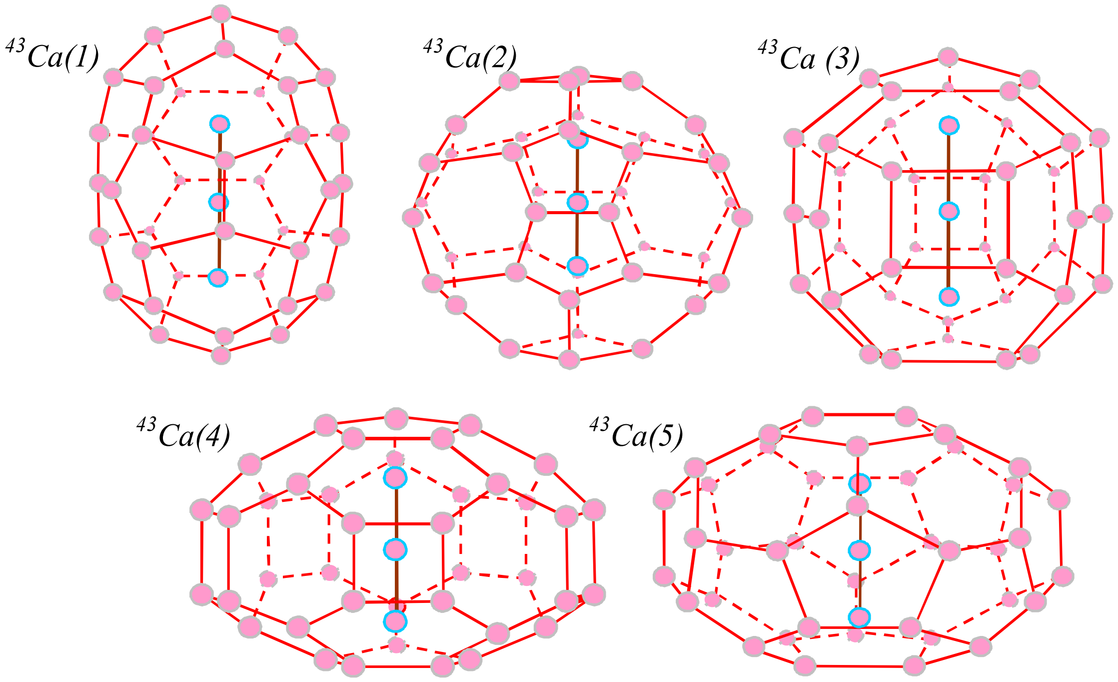


Fig. 29. Protonic cells of the isomers of calcium isotope $_{20}\text{Ca}^{4}$

Consider isotopes ^{42}Ca. From Figure 29 it follows that two internal neutrons are removed from one another at the distance larger than the parameter of the proton cell. When there are two internal neutrons, the latter decomposes into protons and tertions by the external surroundings. The protons are localized in the minima of the double-well potential directed along the main symmetry axis. However, now the protons are only slightly connected with one another. It means that they behave themselves almost independently sending their tertions into the coat of mail. As a result, the shape of tertion net does not change. It looks like in Fig. 28, but the neutron's tertions now have double charge, $^1/_5$ e for ^{42}Ca (1), $^1/_6$ e for ^{42}Ca (2) and ^{42}Ca (3), $^2/_{15}$ e for ^{42}Ca (4), $^1/_5$ e for ^{42}Ca (5), ^{42}Ca (6), ^{42}Ca (7) and ^{42}Ca (8), respectively.

Three-neutron-core isotopes. From the results obtained for silicon, sulfur, argon [7], and potassium, it follows that, if the isotopes contain more than two internal neutrons, they can form clusters inside the nucleus. However, the cluster symmetry must be compatible with that of the external surroundings. The simplest structure of the core is a linear chain of three protons lying along the main axis of symmetry. Therefore the seven isotopes having such a core can be generated. Besides the isotope ^{42}Ca(7$_t$) with a triangle core can be created (Fig. 30).



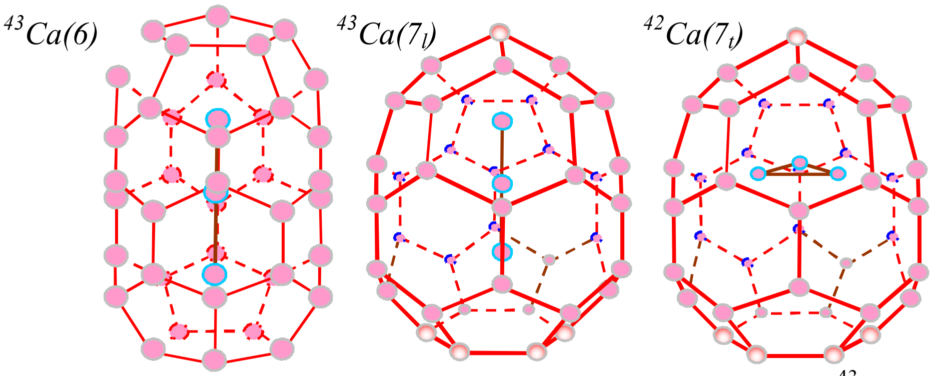


Fig. 30. Protonic cells of the isomers of calcium isotope $_{20}Ca^{43}$

Four-neutron-core isotopes. In view of the symmetry restrictions considered above, only two isomers from eight, ^{44}Ca(2) of four-fold symmetry and ^{44}Ca(8) of tetrahedral one can be formed (Fig. 31). The cores of these isotopes have symmetry compatible with the surroundings.

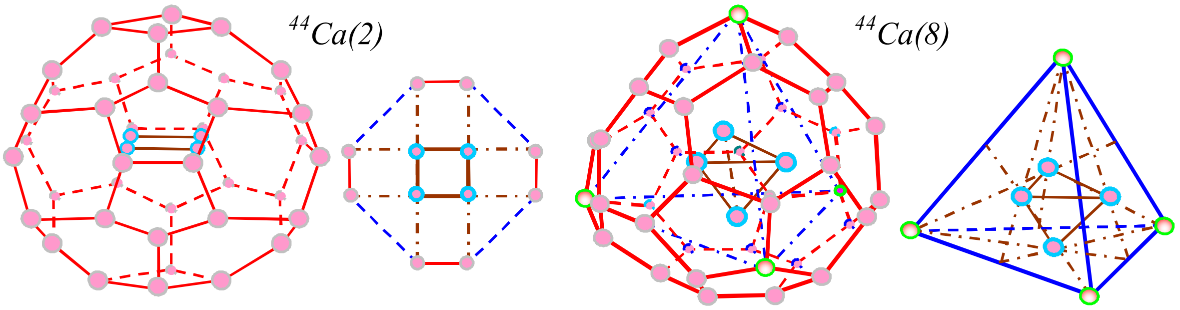


Fig. 31. Protonic cells of the isomers of calcium isotope $_{20}Ca^{44}$

Five-neutron-core isotopes. Similar to the four-neutron-core isotopes, in view of the symmetry restrictions, only two isomers from eight, ^{45}Ca(4) and ^{45}Ca(5) of five-fold symmetry can be formed (Fig. 32). The cores of these isotopes have symmetry compatible with the surroundings.

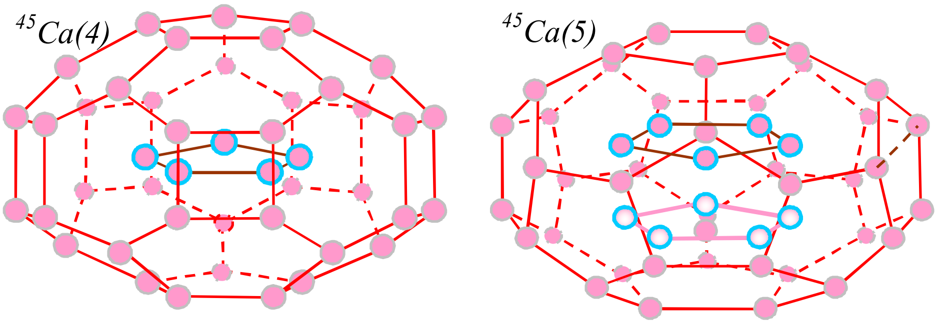


Fig. 32. Protonic cells of the isomers of calcium isotope $_{20}Ca^{45}$

Internal rotation. The isotope ^{45}Ca(5) has a rotation-reflection axis of five-fold symmetry. The core of isotope ^{45}Ca(5) can occupy two different positions inside the nucleus; one in the upper part of the nucleus, and another identical position in the lower part can be achieved through a spiral motion of the core. The core's protons in the lower part of nucleus are specially marked in the figure; they and their bonds are pale pink. Similar to the molecules [1], one can consider such different space core forms of one and the same nucleus as *nuclear* *conformers*. In our case, there are two conformers. One can continue the analogy and come to the conclusion that there exists internal rotation of some cores inside some nuclei.

Six-neutron-core isotope. Only one isomer from eight, ^{46}Ca(7), can be formed. However, it has two different space cores which have symmetry compatible with the surroundings (Fig. 33).

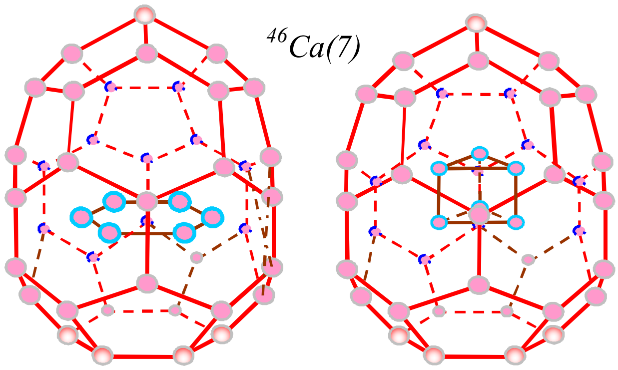


Fig. 33. Protonic cells of the calcium isotopes $_{20}Ca^{46}$

Eight-neutron-core isotope. As before, only one isomer from eight, ^{48}Ca(2), can be formed. However, it has two different space cores which have symmetry compatible with the surroundings (Fig. 34).

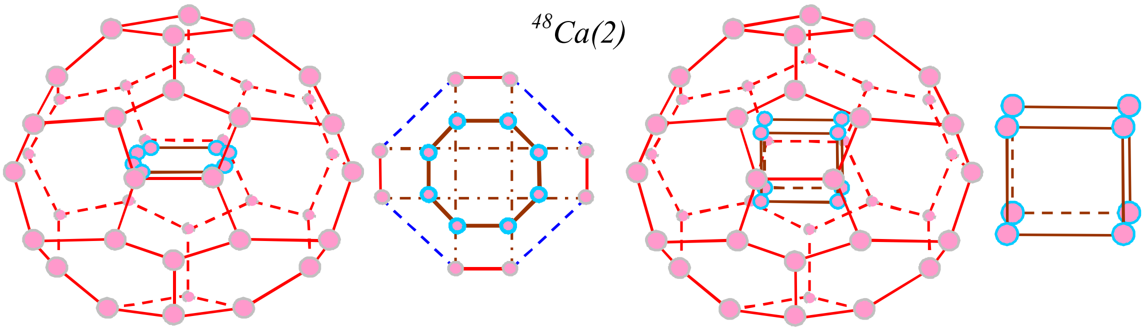


Fig. 34. Protonic cells of the calcium isotopes $_{20}Ca^{48}$

Summary. The results obtained deserve further comment. From the figures above it follows that the nucleus cores are compact. What is more interesting, is the fact that in some cases the cores have the shape of the first elements of the periodic system; namely, helium-3, helium-4, lithium-6, and beryllium-8. As for the electronic structure, it is the same one as for the isotopes with a lesser number of internal core neutrons. Really, although each next neutron above two ones gives an additional electron or some tertions, they are going for constructing the core; the external coat of mail remains without changing.

6. Isomers of scandium

The name derives from the Latin *scandia* for Scandinavia, where the mineral was found. It was first isolated by the Swedish chemist Lars-Fredrik Nilson in 1879 in an ytterbium sample. In the same year, the Swedish chemist Per Theodore Cleve proved that scandium was Mendeleev's hypothetical element "eka-boron" [11].

What do we know about scandium? There are 21 elements that are considered to be monoisotopic. Among them: Be, F, Na, Al, P, and Sc; the first five ones were investigated before now. With the exception of beryllium, all monoisotopic elements have odd numbers of protons, even numbers of neutrons, and, therefore, odd atomic numbers. Scandium has one stable isotope $_{21}Sc^{45}$ (100%) [14].

The problem is how to explain these data in the framework of a unified model. Previously [3-7] the nuclei were separated into two main types: basic nuclei having equal numbers of protons and neutrons, and isotopes having one or more additional neutrons. A better understanding can be gained if we begin with basic nuclei having equal numbers of protons and neutrons. Consider again the parallels between fullerenes and nuclei.

Isomer of fullerene C₄₂. From the periodic table of fullerenes [16,17], it follows that there is only one perfect fullerene of the mass 42, having 3-fold T-symmetry. The fullerene was designed previously [17,18]. With knowledge of the fullerene structure, it is possible to obtain a nuclear structure. It can be designed in various ways through the use of the most probable geometrically compatible reactions. Consider one of the possible reactions.

Joining two deuterons to potassium of three-fold S-symmetry. The reaction is presented in Fig. 35. It can be written as $_{19}K^{38}(2)+d+d \rightarrow {}_{20}Ca^{40}(1)$, the structure of potassium being obtained above. Here two deuterons are incorporated into a basic nucleus of potassium. From the figure, it follows that for potassium only six protons from thirty-eight take part in the reaction. They are pink. The nucleus obtained contains two trigons, six pentagons, and ten hexagons. The graph representation of this reaction is shown in Fig. 36.

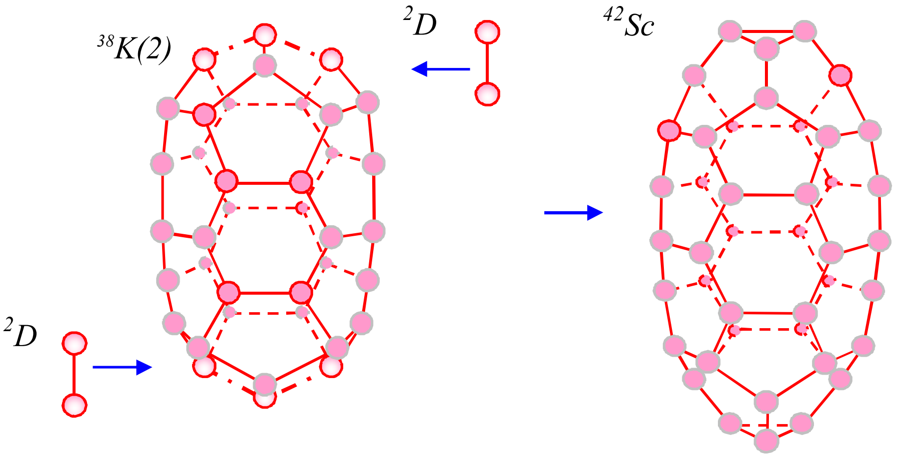


Fig. 35. Attachment of two deuterons to potassium (2) and formation of scandium

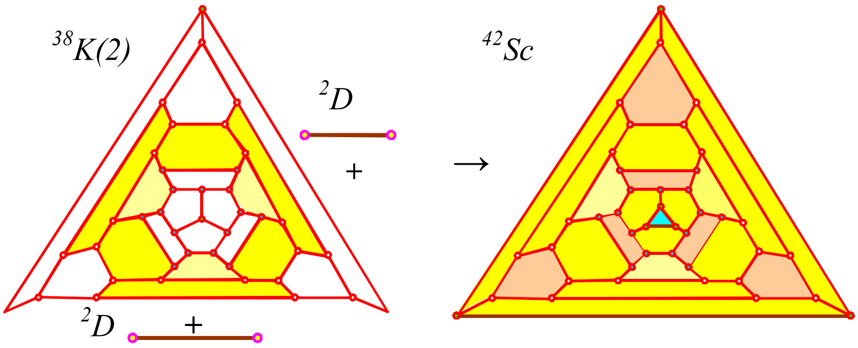


Fig. 36. Graph representation of the nuclear reaction $_{19}K^{38}(2)+d+d \rightarrow {}_{21}Sc^{42}(1)$

Three-neutron-core isotope. The only stable isotope ^{45}Sc can be designed from the basic nucleus ^{42}Sc of three-fold T-symmetry by incorporating three neutrons into the center of the basic nucleus. From the results obtained for silicon, sulfur, and argon [7], it follows that the isotopes, containing more than two internal neutrons, form clusters inside the nucleus. With all this going on, the cluster symmetry must be compatible with that of the external surroundings. In our case, it means that the core must have the shape either of a linear chain of three protons, lying along the main axis of symmetry or of an equilateral triangle (Fig. 37). The approximate position of the neutron triangle can be found in a purely geometric way as it is shown in Fig. 37, at the right. The tertion nets of the isotopes ^{42}Sc and ^{45}Sc are shown in Fig. 38.

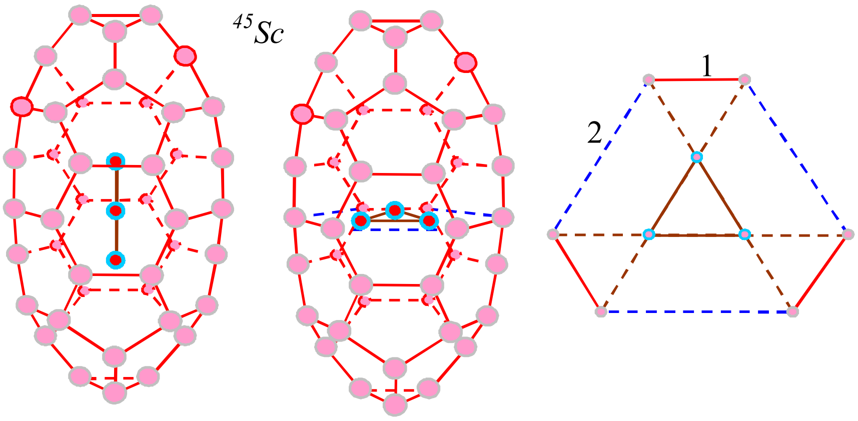


Fig. 37. Protonic cells of isotope ^{45}Sc and the equatorial section at the right (100%)

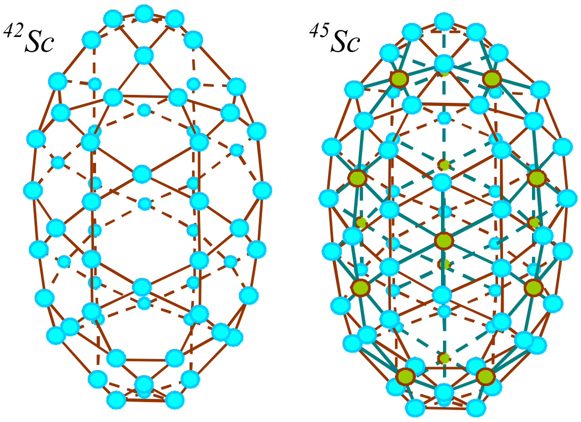


Fig. 38. Electronic structure of scandium isotopes

These results deserve more consideration. One can see that the internal protons of ^{45}Sc form the linear or triangle core, they being tightly connected to one another. Three internal neutrons give three electrons. It is reasonable to assume that one of them remains in the internal core forming two tertions of the charge ½ *e* or three tertions of the charge ⅓ *e*. The other two electrons can be sent to the external electronic coat of mail (tertion net) where they decompose into tertions. The number of tertions must be equal to that of the hexagons of the tertion net of the basic nucleus. In our case, there are fifteen hexagonal faces of the coat of mail. Therefore the tertions sent have the charge $^2/_{15}$ *e*. At that, the tertion-net hexagons became face-centered.

7. Isomers of titanium and their isotopes

The name derives from the Latin *titans*, who were the mythological "first sons of the earth". It was originally discovered by the English clergyman William Gregor in the mineral ilmenite ($FeTiO_3$) in 1791. He called this iron titanite menachanite and the element menachin, for the Menachan parish where it was found. It was rediscovered in 1795 by the German chemist Martin Heinrich Klaproth, who called it titanium because it had no characteristic properties to use as a name. Titanium metal was first isolated by the Swedish chemists Sven Otto Pettersson and Lars Fredrik Nilson [11].

There are five stable isotopes of titanium: $_{22}\mathrm{Ti}^{46}$ (8.0%), $_{22}\mathrm{Ti}^{47}$ (7.30%), $_{22}\mathrm{Ti}^{48}$ (73.8%), $_{22}\mathrm{Ti}^{49}$ (5.5%), $_{22}\mathrm{Ti}^{50}$ (5.4%). Besides, there is isotope $_{22}\mathrm{Ti}^{44}$, having the rather large half-decay period being equal to 47.3 y [14].

From the periodic table of fullerenes [16,17], it follows that there is only one perfect fullerene of mass 44, having 3-fold S-symmetry. The fullerene was designed previously [17,18]. With knowledge of the fullerene structure, one is able to design a nuclear structure. The problem is the same as before, how to design the nuclear structure of titanium's isotopes and isomers. A better understanding can be gained again if we begin with the basic nucleus having equal numbers of protons and neutrons. Consider some possible reactions.

Joining three deuterons to potassium of three-fold S-symmetry. The possible reaction is presented in Fig. 39. It can be written as $_{19}K^{38}(2)+d+d+d\rightarrow{}_{22}Ti^{44}(2)$. Here three deuterons are incorporated into a basic nucleus of potassium having three-fold S-symmetry. From Figure 39 it follows that for potassium only four protons from thirty-eight take part in the reaction. They are pink. The structure of the titanium design is shown in Fig. 39 on the right. The nucleus obtained contains twelve pentagons and twelve hexagons and has three-fold S-symmetry. The reaction graph is presented in Fig. 40.

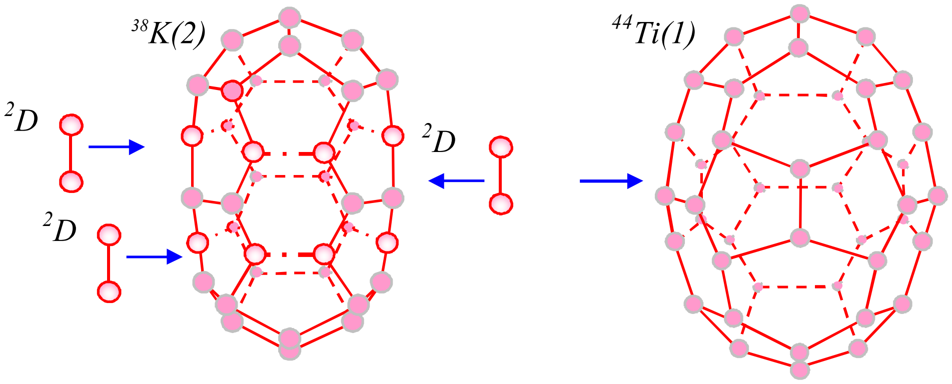


Fig. 39. Attachment of three deuterons to potassium (2) and formation of titanium (1)

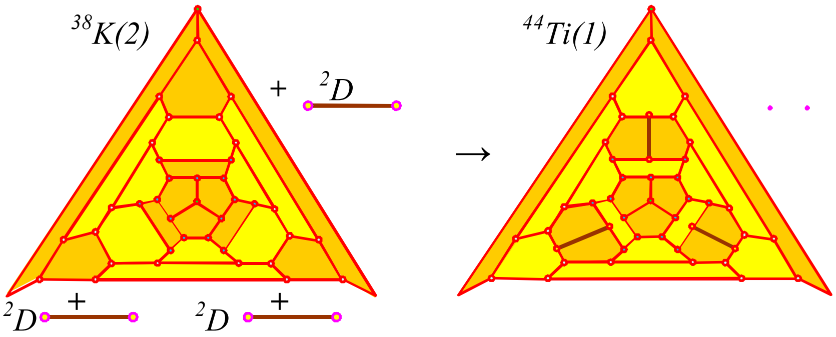


Fig. 40. Graph representation of the nuclear reaction $_{19}K^{38}(2)+d+d+d\rightarrow{}_{22}Ti^{44}(1)$

Joining two α-particles to argon of six-fold symmetry. The reaction is presented in Fig. 41. It can be written as $_{18}Ar^{36}(2)+\alpha+\alpha\rightarrow{}_{22}Ti^{44}(1)$. Here two alpha-particles are incorporated into a basic nucleus of argon having six-fold symmetry. From Figure 39 it follows that for argon twelve protons from thirty-six take part in the reaction. They are pink. The structure of the titanium design is shown in Fig. 41 on the right. The nucleus obtained contains twelve pentagons and twelve hexagons and has also three-fold S-symmetry. The graph of this reaction is presented in Fig. 40.

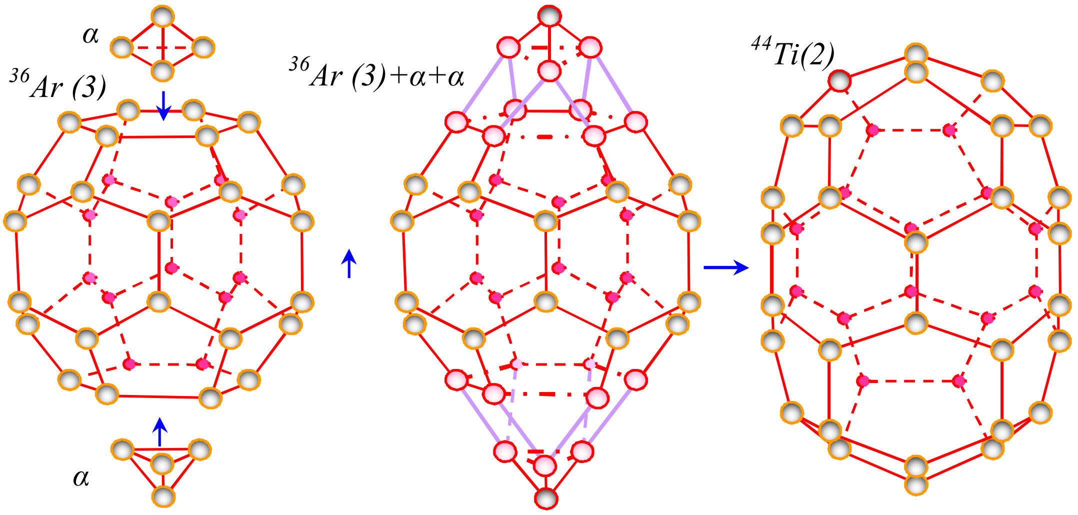


Fig. 41. Attachment of two alpha-particles to argon (3) and formation of titanium (2)

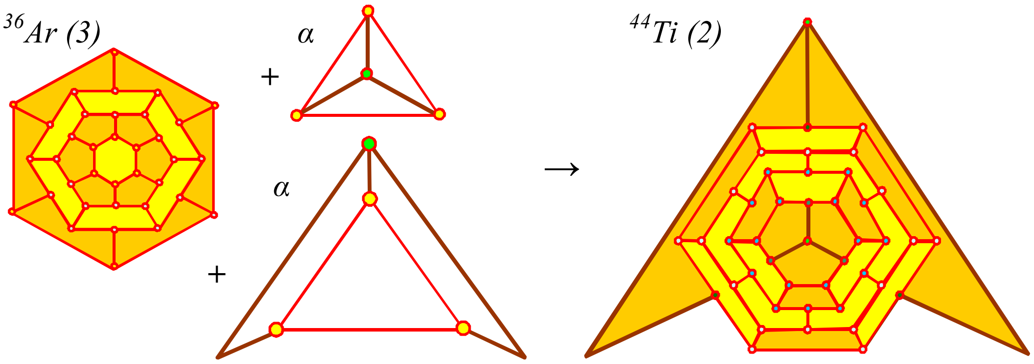


Fig. 42. Graph representation of the nuclear reaction $_{18}Ar^{36}(3)$+α+α→ $_{22}Ti^{44}(2)$

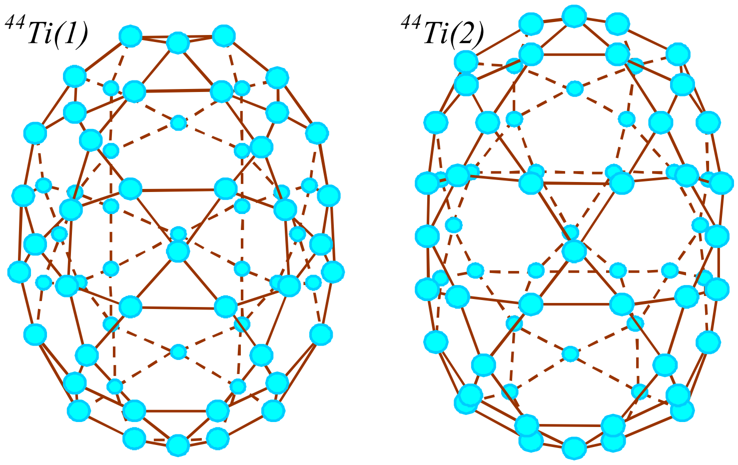


Fig. 43. Electronic structure of two basic titanium isomers

Neutron-core isotopes of titanium isomers. As pointed out above, there are five stable isotopes of titanium: $_{22}\mathrm{Ti}^{46}$, $_{22}\mathrm{Ti}^{47}$, $_{22}\mathrm{Ti}^{48}$, $_{22}\mathrm{Ti}^{49}$, and $_{22}\mathrm{Ti}^{50}$. They can be got by successive incorporating neutrons: $_{22}Ti^{44}+n\rightarrow{}_{22}Ti^{45}+n\rightarrow{}_{22}Ti^{46}+n\rightarrow{}_{22}Ti^{47}+n\rightarrow{}_{22}Ti^{48}+n\rightarrow{}_{22}Ti^{49}+n\rightarrow{}_{22}Ti^{50}$. The protons cells of the isotopes are shown in Figs. 44 and 45.

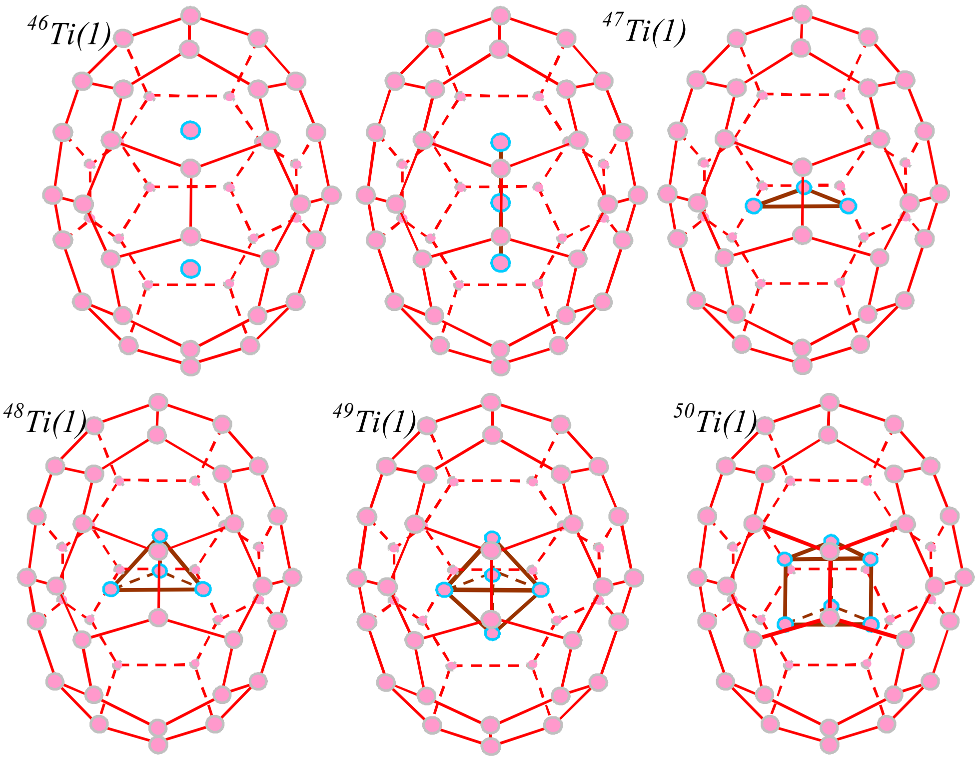


Fig. 44. Isotopes of titanium-1 isomer

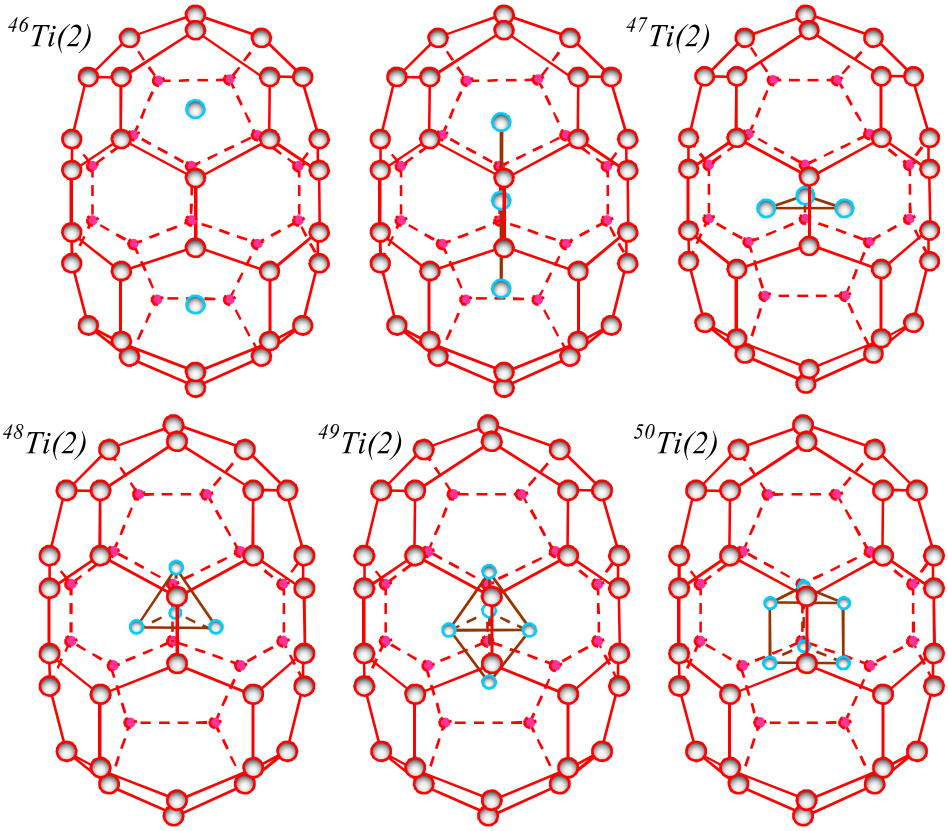


Fig. 45. Isotopes of titanium-2 isomer

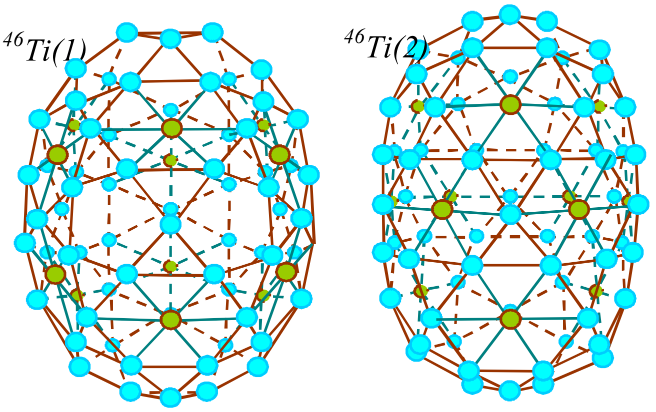


Fig. 46. Tertion nets of the isomers of titanium isotope $_{22}Ti^{46}$

From the results obtained previously, it follows that the isotopes, containing two internal core neutrons, decompose into two protons and two electrons. The latter decomposes into the tertion, the number of which and hence the charge being depended on the structure of the external coat of arms (tertion net). Being removed from one another at the distance larger than the parameter of the proton cell, the core protons do not interact with each other and send their tertions to the external coat of mail. Since there are twelve hexagons, each face-centered hexagon has a tertion of charge $^1/_6$ *e*.

If the isotopes contain more than two internal neutrons, the latter form clusters (core) inside the nucleus. However, the core symmetry must be compatible with that of the external surroundings. From the figures above it follows that the nucleus cores are compact. What is more interesting, is the fact that the cores have the shape of the first elements of the periodic system; namely, helium-3, helium-4, and lithium-6. At the same time, the electronic structure is unchangeable; it is the same one as for the isotopes with two internal core neutrons. This is no surprise. Although each next neutron above two ones gives an additional electron (or three tertions), its tertions are going for fastening the formed proton to the already existing core, so the external coat of mail remains without changing.

8. Summary

By analogy with fullerenes, nuclear geometry has been designed. The nuclei can be classed into two groups: basic nuclei having equal numbers of protons and neutrons and isotopes having one, two, or more additional neutrons. The latter ensures the mechanical stability of the structure with respect to thermal vibrations and shear stresses. If the number of additional neutrons exceeds two, the latter can form a core whose symmetry is compatible with that of the basic nuclei.

On the basis of the geometric approach, it is possible to design the structure of potassium, calcium, scandium, and titanium isomers together with their isotopes, within the framework of one and the same assumptions.

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