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A methodology for estimating the damage growth rates in layered composites using special fatigue accumulation rules

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Abstract. The main provisions of existing methods for estimating the delamination growth rates in layered composites under cyclic tension are presented. It is noted that the considered methods do not allow one to estimate the growth rates of various damage modes during the accumulation of fatigue. As a definite alternative to the presented methods, a methodology for such estimates is proposed using special rules of separate fatigue accumulation. Verification of the ratios of the proposed methodology is carried out on the example of calculated estimates of the growth rates of three damage modes (delamination, matrix cracking, and splitting and ply cracking around open hole edges) of three types of specimens made of laminates of various carbon plastics. It is noted that the accuracy of the obtained estimates depends on the accuracy of the approximation of the experimental data, on the basis of which the search for relations for the special rules of separate fatigue accumulation is conducted.
Keywords: layered composites, delamination, matrix cracking, splitting and ply cracking, damage growth rates, fatigue accumulation rules

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Introduction

Estimating the evolution of different types (modes) of fatigue damage in layered polymer composite materials (PCM) is a rather complex and time-consuming problem in mechanics of fatigue failure of PCM. Numerous studies consider this problem, primarily for *delamination* in PCM.

Studies on determining the predominant failure mode throughout the developing delamination are focused on the *mechanics of interlayer fracture*, primarily aimed at finding the deformation energy per unit area of the delamination increment [1]. This parameter is called the *elastic energy release rate into the tip of the crack G.* Three failure modes are generally considered: mode I (due to breaking stresses); mode II (due to transverse shear stresses) and mode III (due to longitudinal shear stresses). Modes I and II are viewed as the most critical, so they are given the closest attention in constructing test methods and in computational studies [2–3].

A number of equations are given in [2–7] to estimate the delamination rate under cyclic loading. For example, the following equation is presented in [3] for estimating the delamination rate for a mixed fracture mode (I+II):

$$\frac{dL}{dn} = m_1\left(\frac{G_I}{G_{Ic}}\right)^{n_1} + m_2\left(\frac{G_{II}}{G_{IIc}}\right)^{n_2}, \tag{1}$$

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where $\frac{dL}{dn}$ is the delamination per one cycle of fatigue loading; G_I is the effective release rate of elastic energy in mode I for the given loading cycle; G_{II} is the effective release rate of elastic energy in mode II for the given loading cycle; G_{Ic} is the critical release rate of elastic energy in mode I, which is a characteristic of crack resistance (determined experimentally, as a rule, less than the value of G_{Ic} for static fracture); G_{IIc} is the critical release rate of elastic energy for mode II, which is a characteristic of crack resistance (determined experimentally, as a rule, less than G_{IIc} for static destruction); m_1, m_2, n_1, n_2 are the material constants.

Analyzing Eq. (1), we can conclude that it is rather complex, requiring a significant amount of preliminary experimental data; for this reason, it remains a challenge to adapt this equation for engineering estimates of delamination initiation/growth in specific layered materials.

The fundamentals of the physical model describing the damage evolving under cyclic loading in layered PCM have major differences with the methods of linear mechanics of interlayer fracture, as discussed in [8–10].

Fatigue damage is determined in this model by variable values of the parameter D. At the start of life, $D_i = 0$, unless the damage D_i was introduced during manufacturing or at an earlier point in the loading cycle. Cyclic loading produces an increase in damage from D_i to D_f, accompanied by catastrophic failure of the composite laminate.

It is assumed in [8–10] that the accumulation rate of fatigue damage depends on the amplitude of cyclic stresses, the load asymmetry coefficient and the instantaneous value of the parameter D. Then, $\frac{dD}{dn} = f(\Delta\sigma, R, D)$.

The main problem with this approach is that the function f is unknown. The above studies primarily propose to determine this function in terms of the relationship between the variation in Young's modulus during tensile testing of the composite laminate and the accumulated damage D.

It is assumed that the relationship between the parameter D and Young's modulus for the given layered composite can be written as follows: $E = E_0 \cdot g(D)$, where E is the instantaneous value of the elastic modulus; E_0 is the initial value of the elastic modulus for undamaged material; $g(D)$ is a function.

The *delamination* rates in layered PCM *under cyclic tensile loads* were estimated in [11] using the expression:

$$f(\Delta\sigma, R, D) = \frac{dD}{dn} = -2.857\left(\frac{1}{E_0}\frac{dE}{dn}\right), \tag{2}$$

whose right-hand side is estimated from the experimental data.

Similar data are presented, for example, in [12] for quasi-isotropic laminate made of XAS/914 $[45/90/\text{-}45/0]_s$ carbon fiber under cyclic tension with $R = 0.1$. The study also gives an example for approximating the experimental data considered using an equation similar to the well-known Paris equation:

$$\frac{dD}{dn} = 9.2 \times 10^{-5} \times \left(\frac{\Delta\sigma}{\sigma_{UTS}}\right)^{6.4}, \tag{3}$$

where σ_{UTS} = 550 MPa is the ultimate tensile strength of the given laminate (data from [12]).

An equation similar in form to the well-known Collipriest equation was proposed in [13] for the same laminate:

$$\frac{dD}{dn}=C\left[\frac{\lg\frac{\Delta\bar{\sigma}}{\bar{\sigma}_{th}}}{\lg\frac{\bar{\sigma}_{c}}{\Delta\bar{\sigma}}}\right]^{m}, \tag{4}$$

where $C, m, \bar{\sigma}_c, \bar{\sigma}_{th}$ *are* empirical constants depending on the type and properties of the layered PCM.

Thus, if the empirical constants $C, m, \bar{\sigma}_c, \bar{\sigma}_{th}$, are known, each value of $\Delta\bar{\sigma}$ corresponds to a specific delamination rate dD/dn.

The experience accumulated suggests that Eqs. (2)–(4) can be successfully applied to engineering estimates of *delamination* rates in quasi-isotropic laminates *under cyclic tension.*

However, we should also consider the following:

1. Eq. (2) contains a rather important parameter, dE/dn, which is absent from Eqs. (3)–(4). Thus, Eqs. (3)–(4) virtually do not account for the effect of the variation in the stiffness of the layered composite on the delamination rate *throughout fatigue accumulation*. Furthermore, Eqs. (3)–(4) do not allow to estimate the growth rates for *different damage modes* (*other than delamination*), which is also naturally of considerable interest for research.

2. Similar conclusions can be drawn about the methods of linear mechanics of interlayer fracture outlined in [2–7].

3. The methods outlined in [2–13] are also largely inapplicable to solving another crucial problem in damage assessment, that is, the *progression* of different damage modes in one PCM.

An alternative to these methods, making it possible to estimate the growth rates of various fatigue damage modes throughout fatigue accumulation and the sequential progression of different damage modes in one PCM, is the procedure we constructed for such estimates using *special rules for linear summation of fatigue damage*.

Main hypotheses for fatigue damage accumulation in layered composites

A range of hypotheses are proposed in foreign studies to describe the accumulation of fatigue damage in layered PCM (for example, 15 such hypotheses are discussed in [14]). Unfortunately, no data are available in Russian and foreign sources regarding the procedures for applying these hypotheses in engineering calculations for fatigue analysis; there are also insufficient data to confirm the hypotheses experimentally.

Below is a brief overview of several hypotheses most commonly used at present to assess the accumulation of fatigue damage during cyclic loading of layered PCM.

Palmgren–Miner rule. The simplest hypothesis of fatigue damage accumulation used in fatigue analysis of metal samples and structural elements is the linear summation rule (Palmgren–Miner rule):

$$D=\sum\frac{n_i}{N_i}=1, \tag{5}$$

where D is the accumulated fatigue damage, $D = 1$ upon fracture of the given element; n_i is the number of loading cycles for the stress level σ_i; N_i is the number of loading cycles before up to fracture of the given element for the stress level σ_i.

Notably, relation (5) is often used for calculating the fatigue life of elements made of layered PCM. On the other hand, numerous studies (for example, [15–16]) suggest that applying this rule in fatigue analysis of samples and elements made of layered PCM can produce substantial errors in the computational results.

For this reason, many researchers are inclined to conclude that the rules that can satisfactorily describe fatigue damage accumulation during cyclic loading of PCM should be sought within the framework of the Marco–Starkey theory on *nonlinear* accumulation. This hypothesis can account for the effects from sequentially applying loads of different magnitude, so it holds much promise for fatigue analysis of elements made of layered PCM.

Marco–Starkey model. The Marco-Starkey theory is based on the following assumptions:

1. Damage curves for the amplitude of symmetric stresses with any magnitude can be described by the relation:

$$D_i = \left(\frac{n_i}{N_i}\right)^{q_i},$$

where D_i is the accumulated fatigue damage; n_i is the number of loading cycles with the amplitude of symmetric stresses σ_{ai}; N_i is the number of loading cycles before fracture of the given element with the same amplitude of symmetrical stresses σ_{ai}; q_i is an exponent *depending on the stress level*.

2. A specimen loaded by symmetrical stresses in any sequence is fractured when the total value of D reaches unity. A specialized procedure has been developed for accumulation of fatigue damage from one level to another.

Hwang–Han model. The *Hwang–Han* model [17] can predict *linear* growth of accumulated damage in layered PCM *depending on the stress level and regardless of the loading history.* While this model is in many respects similar to the Marco–Starkey model, it cannot account for the effects from sequentially applying loads with different magnitudes.

The main relations of the Hwang–Han model are written as follows:

$$D = \sum_{i=1}^{k} D_i = \sum_{i=1}^{k}\left[\left(\frac{n_i}{N_i}\right)^{c_i}\right] = 1, \; D_i = \left(\frac{n_i}{N_i}\right)^{c_i} = \frac{E_0 - E(n_i)}{E_0 - E_f},$$

where n_i is the number of loading cycles of the layered PCM for the stress level σ_i; N_i is the number of loading cycles up to fracture of the layered PCM for the stress level σ_i; E_0 is the elastic modulus of the undamaged material; $E(n_i)$ is the elastic modulus of the material during fatigue accumulation; E_f is the elastic modulus of the material at the time of fatigue fracture; c_i is the parameter of the relationship *depending on the stress level*.

Howe–Owen model. A model of nonlinear damage accumulation was proposed in [18] by *Howe and Owen* to formulate a law for accumulation of fatigue damage that can be used in practical calculations for fatigue in layered PCM:

$$D = \sum_{i=1}^{k}\left[A\left(\frac{n_i}{N_i}\right) + B\left(\frac{n_i}{N_i}\right)^2\right], \tag{6}$$

where D is the accumulated fatigue damage of the given element, $D = 1$ upon fracture; n_i is the number of loading cycles for the stress level σ_i; N_i is the number of loading cycles up to fracture of the given element for the stress level σ_i. A and B are the parameters of the relation *independent of the stress level*.

The values of parameters A and B can be determined by linear regression analysis of known experimental data on the fatigue life of layered PCM specimens (elements) under complex loading. The values of these parameters are determined from the results of at least 2 series of tests with different parameters of complex loading.

A modification of the relation (6) is proposed in [9]:

$$D = \sum_{i=1}^{k}\left[A\left(\frac{n_i}{N_i}\right) + B\left(\frac{n_i}{N_i}\right)^c\right], \tag{7}$$

containing three parameters A, B and c, determined using an iterative procedure aimed at fitting the experimental data to the relation (7).

We can observe the following from the results of the analysis.

1. All the models considered propose an aggregate or cumulative approach to deal with accumulation of generalized or finite damage and do not describe *linear summation for different damage modes*.
2. We should note that linear summation for different modes of fatigue damage deserves separate consideration, as it is a matter of considerable interest for studies into sequential fatigue failure and analysis of the growth rates of different damages in layered PCM.
3. The Howe–Owen model (6) is the *basis* for searching for the hypotheses accounting for summation of fatigue damage for different damage modes. According to [18–21], this hypothesis also proved to be versatile, as it is applicable for various elements made of layered PCM, and for various types of cyclic loading. Unfortunately, no data are openly available on the *practical* applications of the Marco–Starkey and Hwang–Han models to describe the summation rules for fatigue damage in PCM.

Procedure for estimating the growth rates of various damage modes using special rules for linear fatigue accumulation

The procedure proposed for estimating the growth rates of various damage modes during cyclic loading of layered PCM comprises the following steps.

The computational case of regular uniaxial cyclic loading is loading with a constant amplitude. The procedure is formulated as follows for the given computational case.

1. The damage mode is specific to the given specimen or element made of layered PCM under the cyclic loading conditions considered.
2. The experimental data $D = f(n/N)$ obtained using non-destructive testing methods are assumed to be known (for measuring one of the material properties, for example, Young's modulus, electrical conductivity, light scattering, X-ray absorption, ultrasonic attenuation, acoustic emission detection).
3. The following function is sought for by fitting the experimental data:

$$D = f_1(\sigma, R, n/N)\,, \tag{8}$$

this function is in fact the special rule for summation of fatigue damage for the given damage mode and the cyclic loading conditions in layered PCM.

4. The growth rate of the given damage mode is defined as the derivative of the function (8):

$$D' = \frac{dD}{dn} = f_2(\Delta\sigma, R, n)\,. \tag{9}$$

Graphically, function (9) can be constructed as $D' = f(n)$ or as $D' = f(n/N)$.

The computational case of irregular uniaxial cyclic loading is loading with variable amplitudes. The procedure is formulated as follows for the given computational case.

1. The damage mode is specific to the given specimen or element made of layered PCM under the cyclic loading conditions considered.
2. The given cyclic loading can be represented as a certain loading block repeated during fatigue tests of the specimen or element.
3. The experimental data obtained by non-destructive testing methods are assumed to be known; fitting these data can yield the function:

$$D = f_3(n_{bl.} / N_{bl.})\,, \tag{10}$$

which is in fact the special rule for summation of fatigue damage for the given damage mode and the cyclic loading conditions of the specimen or element.

4. The growth rate of the given damage mode is defined as the derivative of the function (10):

$$D' = \frac{dD}{dn} = f_4(n_{bl.} / N_{bl.}) . \quad (11)$$

Verification of the proposed relations

The amount of the experimental data serving as a basis for constructing functions (8) and (10) is fairly limited. Nevertheless, such data are found in some publications.

The relations (8)–(9) were verified for an example where experimental data were processed for two types of specimens:

1. Specimens of T800/5245 $[(\pm 45.0_2)_2]_s$ carbon fiber laminate [22–23].
2. Specimens of laminate based on chopped carbon fiber and polyester [18].

Specimens made of T800/5245 $[(\pm 45.0_2)_2]_s$ carbon fiber were tested by cyclic tension with a maximum stress of 1000 MPa and R = 0.1 [22]. The damage mode considered is cracking of the matrix in the inner 45° layers. According to [23], the fatigue life of the specimens was N_f= 1,500,000 cycles.

Experimental data for CD_i-n/N are given in [22] (CD_i *is* the crack density in the laminate matrix after n loading cycles. These data can be rewritten in the D-n/N coordinates, where $D=CD_i/CD_{fr}$, CD_{fr} is the crack density in the laminate matrix during fracture.

Similar data are given in Fig. 1, also showing a trend line for the approximation of the experimental data.

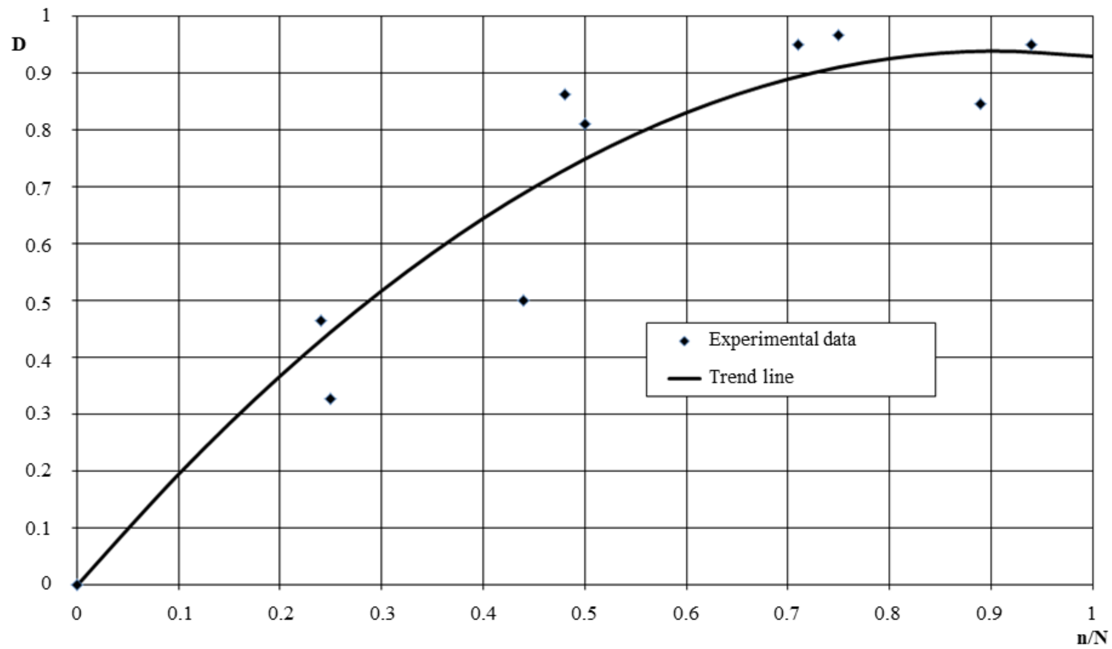

Fig. 1. Accumulated damage in T800/5245 $[(\pm 45.0_2)_2]_s$ carbon fiber laminate specimens during cyclic tensile tests with a maximum stress of 1000 MPa and R= 0.1

The equation of the trend line is a second-degree polynomial (the confidence value of the approximation is R^2 = 0.9099):

$$D = -1.136 \cdot \left(\frac{n}{N_f} \right)^2 + 2.065 \cdot \left(\frac{n}{N_f} \right). \quad (12)$$

Then the cracking rate of the matrix:

$$D' = \frac{dD}{dn} = -2.272 \cdot \left(\frac{n}{N_f^{\,2}} \right) + 2.065 \cdot \left(\frac{1}{N_f} \right) = -1.010 \cdot 10^{-12} \cdot n + 1.376 \cdot 10^{-6}. \tag{13}$$

Function (13) is plotted as a dependence $D' = f(n)$ in Fig. 2, and as a dependence $D' = f(n/N)$ in Fig. 3.

Specimens made of laminate incorporating chopped carbon fiber and polyester based on the data from [18] were subjected to cyclic tensile tests. The damage modes considered are matrix cracking and delamination. The fatigue life of the specimens is not known and assumed to reach a conditional level N_f = 1,000,000 cycles for studying the damage growth rates.

The *D-n/N* dependences are plotted in Fig. 4 for the given damage modes based on the data from [18].

The figure also shows the trend lines for the approximation of the experimental data.

The equation of the trend line is a second-degree polynomial (the confidence value of the approximation is R^2=0.9926):

$$D = -0.795 \cdot \left(\frac{n}{N_f} \right)^2 + 1.521 \cdot \left(\frac{n}{N_f} \right) + 0.253\,. \tag{14}$$

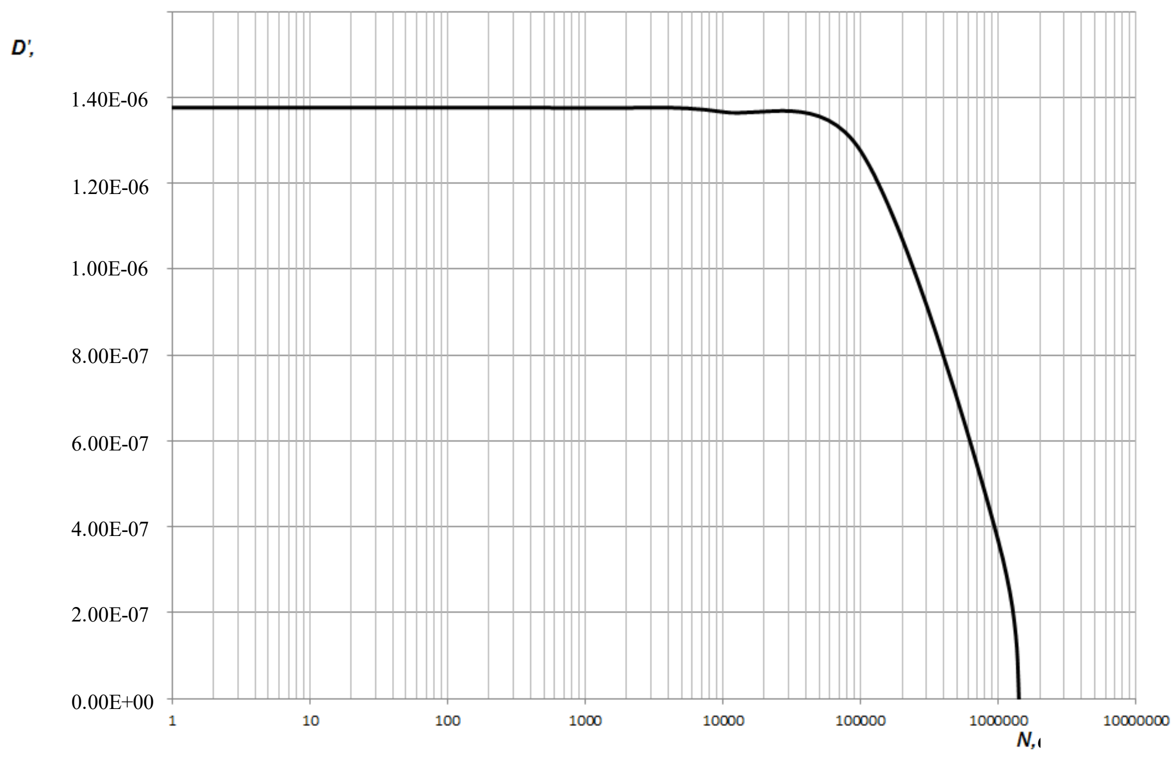

Fig. 2. Matrix cracking rate $D' = f(n)$ in specimens made of T800/5245 [(±45.0$_2$)$_2$]$_s$ carbon fiber laminate

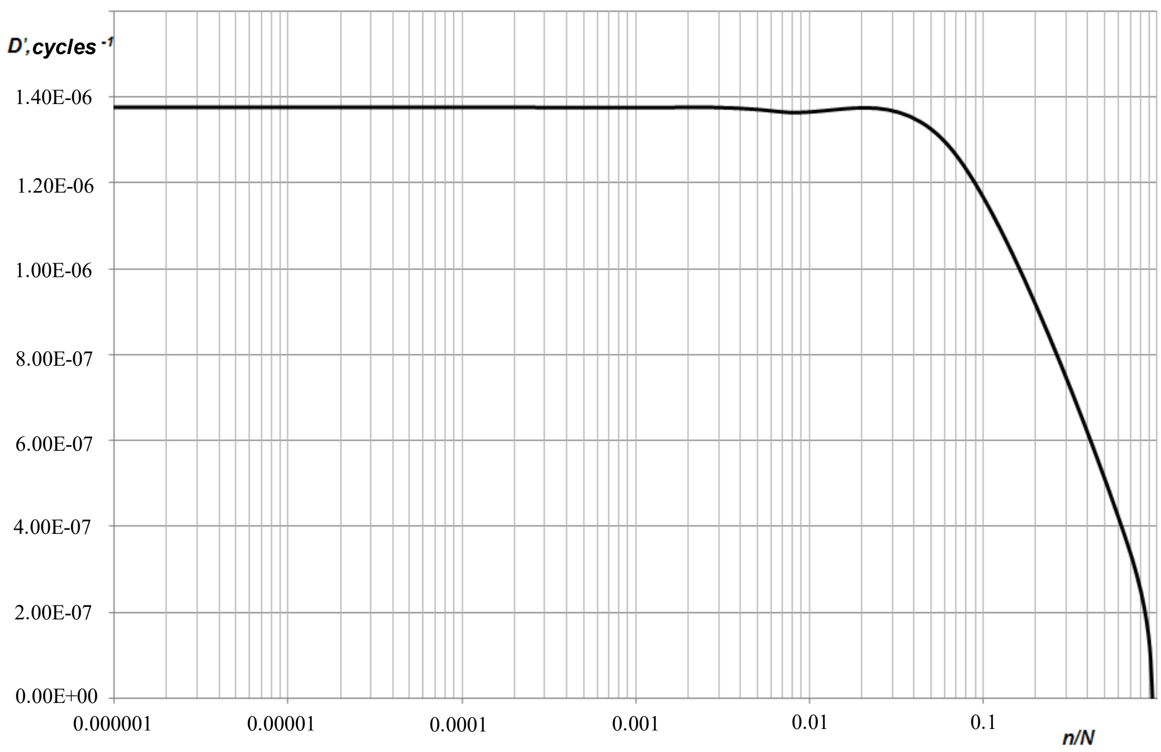

Fig. 3. Matrix cracking rate $D' = f(n/N)$ in specimens made of T800/5245 [(±45.0$_2$)$_2$]$_s$ carbon fiber laminate

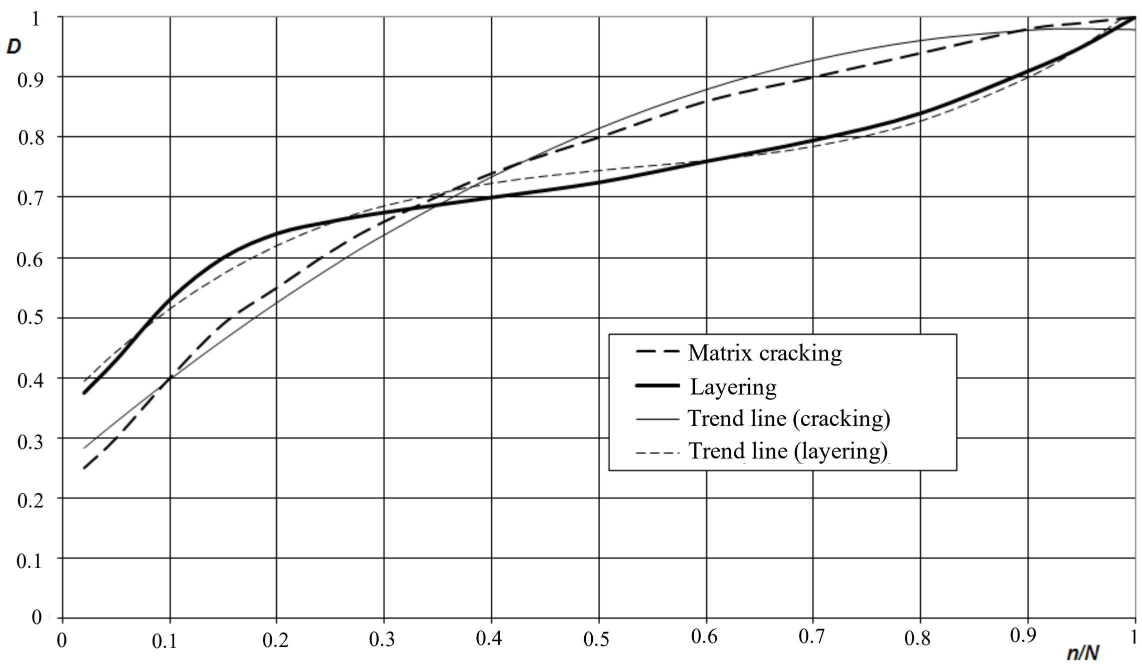

Fig. 4. Accumulated damage in specimens made of laminate based on chopped carbon fiber and polyester

Then the cracking rate of the matrix is:

$$D' = \frac{dD}{dn} = -1.590 \cdot \left(\frac{n}{N_f^{\,2}} \right) + 1.521 \cdot \left(\frac{1}{N_f} \right) = -1.590 \cdot 10^{-12} \cdot n + 1.521 \cdot 10^{-6} . \qquad (15)$$

The equation of the trend line for delamination is a third-degree polynomial (the confidence value of the approximation is R^2=0.9922):

$$D = 1.945 \cdot \left(\frac{n}{N_f} \right)^3 - 3.153 \cdot \left(\frac{n}{N_f} \right)^2 + 1.864 \cdot \left(\frac{n}{N_f} \right) + 0.358 . \qquad (16)$$

Then the delamination rate is:

$$D' = \frac{dD}{dn} = 5.835 \cdot \left(\frac{n}{N_f^{\,3}} \right)^2 - 6.306 \cdot \left(\frac{n}{N_f^{\,2}} \right) + 1.864 \cdot \left(\frac{1}{N_f} \right) = \qquad (17)$$
$$= 5.835 \cdot 10^{-18} \cdot n^2 - 6.306 \cdot 10^{-12} \cdot n + 1.864 \cdot 10^{-6}$$

Functions (14) and (16) are plotted in Fig. 4, functions (15) and (17) are plotted in Fig. 5.

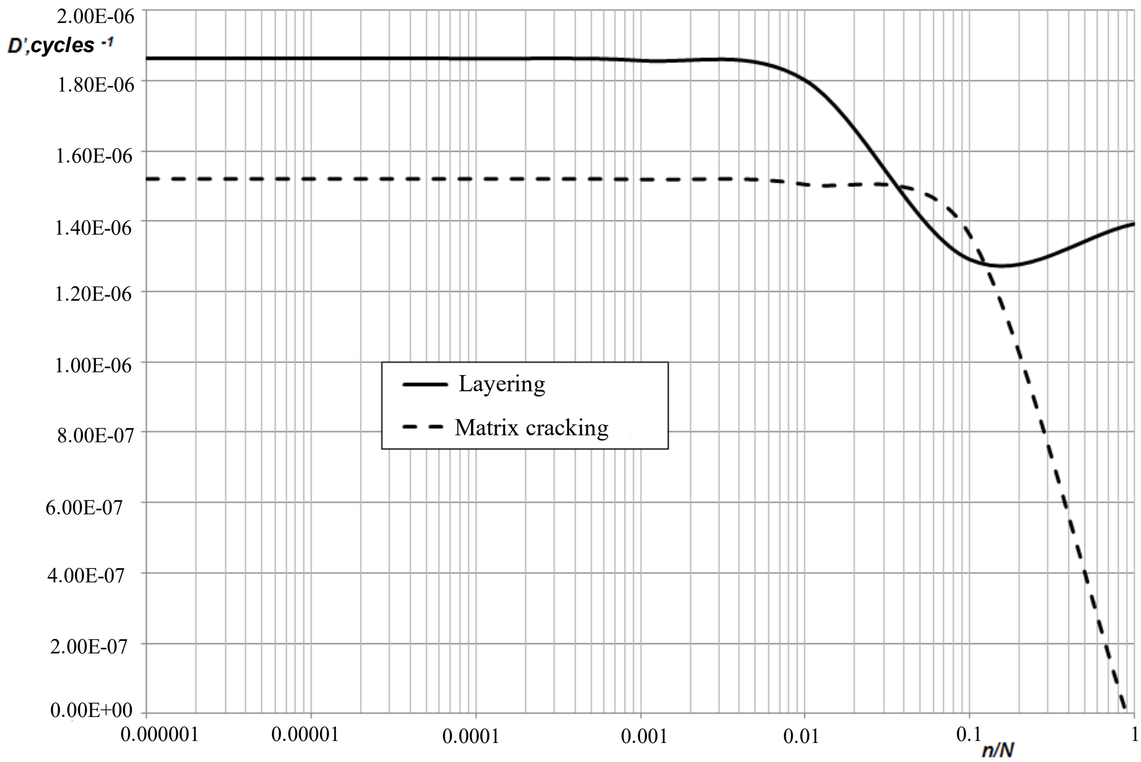

Fig. 5. Delamination and cracking rates of the matrix $D' = f(n/N)$ in laminate specimens based on chopped carbon fiber and polyester

Proposed approach applied to the computational case of irregular uniaxial cyclic loading.

Relations (10)–(11) were verified for processing experimental data and computational estimates of the rates and durations of damage growth for free-hole specimens made of AS4/3501-6 $[0/\pm45/90]_{s4}$ carbon fiber under predominantly tensile loads using block loading for the air stage from the TWIST standardized quasi-random software [24]. The loading blocks for the the air stage from the TWIST program are given in Table 1.

Table 1. Loading block for the air stage from the TWIST program

Level	Amplitude	Number of cycles

of loading	of loading	of loading
1	$1.6\times\sigma_m$	1
2	$1.5\times\sigma_m$	2
3	$1.3\times\sigma_m$	5
4	$1.15\times\sigma_m$	18
5	$0.99\times\sigma_m$	52
6	$0.84\times\sigma_m$	152
7	$0.68\times\sigma_m$	800
8	$0.53\times\sigma_m$	4170
9	$0.37\times\sigma_m$	34800
10	$0.22\times\sigma_m$	358665

(σ_m is the mean stress of the air stage)

The tests were carried out with the mean stress σ_m = 134.4 MPa. According to [25], the fatigue life of the specimens was N_{bl_f} = 10 blocks.

The damage mode considered is splitting of carbon fibers in the free hole zone.

The split length *SL* of the laminate fibers was measured in the free hole zone during the tests. The dependence $SL = f(n_{bl})$ was constructed based on the test results in [25]; here n_{bl} is the number of loading blocks before failure.

The dependence presented in [25] can be reconstructed with a small error in the coordinates "$D - n_{bl} / N_{bl}$", where $D = SL / SL_f$; SL_f is the critical split length of composite fibers at the time of fatigue failure in the specimen; N_{bl} is the number of loading blocks before failure. In this case, the parameter *D* can be regarded as the accumulated fatigue damage of the specimen.

A similar relationship is shown in Fig. 6. also showing a trend line for the approximation of the experimental data.

The equation of the trend line is a second-degree polynomial (the confidence value of the approximation is R^2=0.9694):

$$D = -1.063 \cdot \left(\frac{n_{bl}}{N_{bl\_f}} \right)^2 + 1.968 \cdot \left(\frac{n_{bl}}{N_{bl\_f}} \right) + 0.081 . \tag{18}$$

The fiber splitting rate is then

$$D' = \frac{dD}{dn} = -2.126 \cdot \left(\frac{n_{bl}}{{N_{bl\_f}}^2} \right) + 1.968 \cdot \left(\frac{1}{N_{bl\_f}} \right) = -2.126 \cdot 10^{-2} \cdot n_{bl} + 1.968 \cdot 10^{-1} . \tag{19}$$

Function (18) is plotted in Fig. 6, function (19) is plotted in Fig. 7.

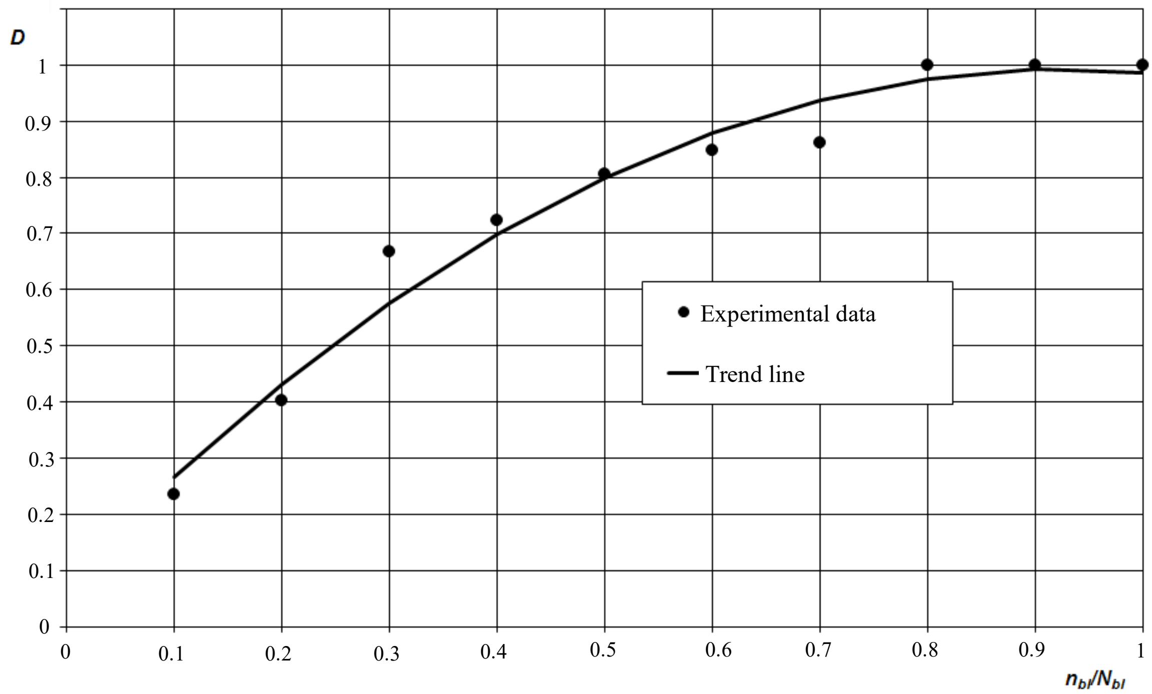

Fig. 6. Accumulated damage in free-hole specimens made of AS4/3501-6 $[0/\pm45/90]_{s4}$ carbon fiber subjected to block loading for the air stage from the TWIST program

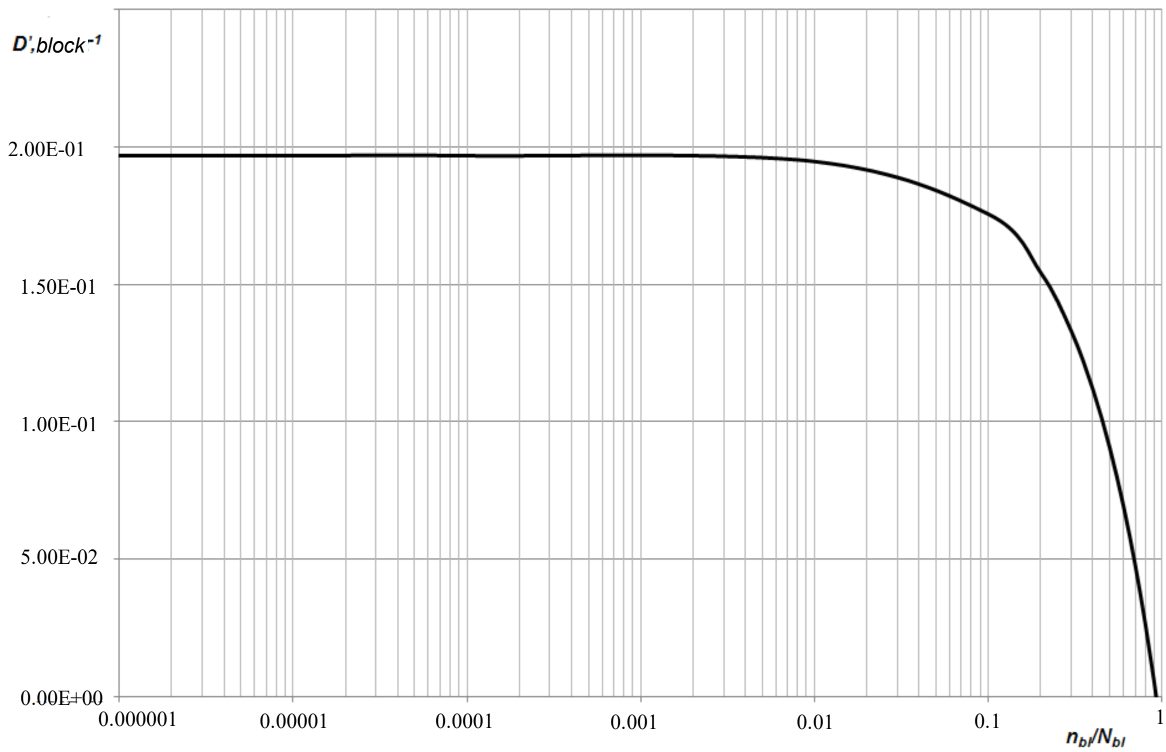

Fig. 7. Fiber splitting rate in the free-hole zone of AS4/3501-6 $[0/\pm45/90]_{s4}$ carbon fiber specimens

Discussion

Analyzing the dependences shown in Figs. 1–7, we can draw the following conclusions.

1. The calculated matrix cracking rate in internal 45° layers in T800/5245 $[(\pm45.0_2)_2]_s$ carbon fiber laminate specimens under cyclic tension with a maximum stress of 1000 MPa and R = 0.1 is the highest at the initial site of fatigue accumulation, up to the value $n/N \approx 0.03$. The cracking rate then drops sharply, nearly amounting to zero at $n/N = 1.0$ (see Fig. 3). Fracture criterion of laminate: $n/N = 1.0$. Taking the equality for the matrix cracking rate $D' = 0$ with $n/N = 1.0$ at the time of laminate failure and solving Eq. (13) in the form $D' = -1.010 \cdot 10^{-12} \cdot n + 1.376 \cdot 10^{-6} = 0$, we can find the cracking duration $n = N_{gr}$ = 1,366,000 cycles, which is slightly less than the fatigue life of the specimens N_f = 1,500,000 cycles.

2. The calculated matrix cracking rate in laminate specimens based on chopped carbon fiber and polyester is the highest at the initial site of fatigue accumulation, up to $n/N \approx 0.03$. The cracking rate then drops sharply, nearly amounting to zero at $n/N = 1.0$ (see. Fig. 5). Fracture criterion of laminate: $n/N = 1.0$. Taking the equality for the cracking growth rate of the matrix at the time of laminate failure $D' = 0$ with $n/N = 1.0$ and solving Eq. (15) in the form $D' = -1.590 \cdot 10^{-12} \cdot n + 1.521 \cdot 10^{-6} = 0$, we can find the cracking duration $n = N_{gr}$ = 956,600 cycles, which is slightly less than the fatigue life of the specimens N_f=1,000,000 cycles.

3. The calculated delamination rate in laminate specimens based on chopped carbon fiber and polyester is the highest at the initial site of fatigue accumulation (approximately up to the value $n/N \approx 0.005$), subsequently dropping sharply but increasing slightly at the final site of fatigue accumulation, see Fig. 5. Fracture criterion of laminate: $n/N = 1.0$. Taking the equality for the delamination rate $D' = 1.39 \cdot 10^{-6}$ with $n/N = 1.0$ at the time of laminate failure (see Fig. 5) and solving Eq. (17) in the form $D' = 5.835 \cdot 10^{-18} \cdot n^2 - 6.306 \cdot 10^{-12} \cdot n + 1.864 \cdot 10^{-6} = 1.39 \cdot 10^{-6}$, we can find the delamination duration $n = N_{gr}$ = 999,400 cycles, which is slightly less than the fatigue life of the specimens N_f = 1,000,000 cycles.

4. An important conclusion about the progression of the damage considered can be drawn for laminate specimens based on chopped carbon fiber and polyester. Analyzing the data in Figs. 4–5, we can assume that delamination triggered matrix cracking at the initial stage of fatigue accumulation. The delamination rate exceeds the matrix cracking rate over almost the entire site of fatigue accumulation, see Fig. 5. We can also conclude that delamination rather than matrix cracking is the main reason behind the failure in the laminate.

5. The calculated fiber splitting rate in the free-hole zone of AS4/3501-6 $[0/\pm45/90]_{s4}$ carbon fiber specimens subjected to block loading from the TWIST software is maximum at the initial site of fatigue accumulation site, up to $n_{bl}/N_{bl} \approx 0.01$. The growth rate of the damage considered then drops sharply and is almost zero at $n_{bl}/N_{bl} = 1.0$, see Fig. 7. Fracture criterion of laminate: $n_{bl}/N_{bl} = 1.0$. Taking the equality for the fiber splitting rate $D' = 0$ with $n_{bl}/N_{bl} = 1.0$ at the time of specimen failure and solving Eq. (19) in the form $D' = -2.126 \cdot 10^{-2} \cdot n_{bl} + 1.968 \cdot 10^{-1} = 0$, we can find the fiber splitting duration $n_{bl} = N_{bl\_gr}$ = 9.3 blocks, which is slightly less than the fatigue life of the specimens N_{bl_f} = 10 blocks.

6. Evidently, the accuracy of the estimates obtained largely depends on the approximation accuracy of experimental data used to formulate the rules for linear summation of fatigue damage. In view of the reliability values obtained for the approximations, we can argue that

satisfactory accuracy can be achieved for the computational estimates of the growth rates of the given damage modes.

7. The computational estimates of damage durations in the given specimens turn out to be somewhat lower than their fatigue durations, so we can conclude that the proposed approach produces somewhat conservative estimates.

Conclusion

This paper outlines the key principles for estimating the delamination rates in layered PCM subjected to cyclic tensile tests, described in [1–13]. Notably, the methods considered do not allow estimating the growth rates of *various damage modes throughout fatigue accumulation*, which is of undoubted interest for research.

An alternative to these methods, making it possible to estimate the growth rates of various fatigue damage modes throughout fatigue accumulation and the sequential progression of different damage modes in one PCM, is the procedure we constructed for such estimates using *special rules of linear summation of fatigue damage*.

The Howe–Owen model was chosen as a *basis* for searching for such hypotheses [18]. According to [18–21], this model also proved to be versatile, as it is applicable for various elements made of layered PCM, and for various types of cyclic loading.

We verified the relations constructed for the proposed methodology for the case of computational estimates of three damage modes (delamination, matrix cracking and fiber splitting in the free-hole zone) for three types of specimens from laminates of various carbon fiber plastics.

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