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THERMOELECTRIC FIGURE-OF-MERIT EFFECTS ON FLUID FLOW

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Abstract. This work is related to flow of an electro conducting fluid presented thermoelectric figure-of-merit effect, in the presence of magnetic field. The electro conducting thermofluid equation heat transfer with one relaxation time is derived. The flow of electro conducting fluid over a suddenly moved plate is considered. The governing coupled equations in the frame of the boundary layer model are applied to Stokes' first problem with heat sources. Laplace transforms and Fourier transforms techniques are used to get the solution. The inverses of Fourier transforms are obtained analytically. Laplace transforms are obtained using the complex inversion formula of the transform together with Fourier expansion techniques. Numerical results for the temperature distribution and the velocity component are represented graphically. Thermoelectric figure-of-merit effect on the fluid flow is studied.

Nomenclature

(x,y,z)	space coordinates
u	velocity of the fluid along the x-direction
U	velocity of the plate
T	temperature
ρ	density
t	time
p	pressure velocity vector
f	body forces per unit mass
T	temperature
Q	intensity of heat source
$\boldsymbol{H}$	magnetic field intensity vector
$\boldsymbol{B}$	magnetic induction vector
$\boldsymbol{E}$	electric field vector
$\boldsymbol{J}$	conduction electric density vector
S	Seebeck coefficient
Π	Peltier coefficient
H_o	constant component of magnetic field
σ_0	electrical conductivity
μ_0	magnetic permeability
λ	thermal conductivity

1. Introduction

Thermoelectric currents in the presence of magnetic fields can cause pumping or stirring of liquid-metal coolants in nuclear reactors or stirring of molten metal in industrial metallurgy. The interaction between the thermal and magnetohydrodynamic fields is a mutual one owing to alterations in the thermal convection and to the Peltier and Thomson effects (although these are usually small).

referred to as Stokes' first problem.

The study of boundary layer flow of an electrically conducting micropolar fluid past an infinite surface plane, under the influence of a magnetic field, has attracted the interest of many authors [15], and an increasing attention is being devoted to the interaction between magnetic field and fluid flow field in a thermofluid owing to its many applications in science. In all papers quoted above it was assumed that the interactions between the two fields take place by means of the Lorentz force appearing in the equations of motion and by means of a term entering classical Ohm's law and describing the electric field produced by the velocity of a fluid particle, moving in a magnetic field. Usually, in these investigations the heat equation under consideration is taken as the uncoupled not the generalized one. This attitude is justified in many situations since the solutions obtained using any of these equations differ little quantitatively. However, when short time effects are considered, the full-generalized system of equations has to be used a great deal of accuracy is lost.

In the present work, we consider an infinitely long flat plate above which an electro conducting Newtonian fluid exists. Initially both the plate and fluid are assumed to be at rest. Let us suddenly impart a constant velocity to the plate in its own plane in the presence of magnetic field and is applied on the plane surface. A one dimensional problem with a distribution of heat sources is considered. Laplace and Fourier integral transforms are used. The Fourier transforms are inverted analytically. A numerical method is employed for the inversion of the Laplace transforms [16]. Numerical results are given and illustrated graphically for the problem considered.

2. Derivation of general electro conducting fluid equation heat transfer

The phenomenal growth of energy requirements in recent years has been attracting considerable attention all over the world. This has resulted in a continuous exploration of new ideas and avenues in harnessing various conventional energy sources. Such as tidal waves, wind power, geo-thermal energy, etc. It is obvious that in order to utilize geo-thermal energy to a maximum, one should have a complete and precise knowledge of the amount of perturbations needed to generate convection currents in geo-thermal fluid. Also, knowledge of the quantity of perturbations that are essential to initiate convection currents in mineral fluids found in the earth's crust helps one to utilize the minimal energy to extract the minerals. For example, in the recovery of hydro-carbons from underground petroleum are deposits. The use of thermal processes is increasingly gaining importance as it enhances recovery. Heat is being injected into the reservoir in the form of hot water or steam or burning part of the crude in the reservoir can generate heat. In all such thermal recovery processes, fluid flow takes place through a conducting medium and convection currents are detrimental.

The classical heat conduction equation has the property that the heat pulses propagate at infinite speed. Much attention was recently paid to the modification of the classical heat conduction equation, so that the pulses propagate at finite speed. Mathematically speaking, this modification changes the governing partial differential equation from parabolic to hyperbolic type. Cattaneo [17] was the first to offer an explicit mathematical correction of the propagation speed defect inherent in Fourier's heat conduction law. Cattaneo's theory allows for the existence of thermal waves, which propagate at finite speeds. Starting from Maxwell's idea [18] and from paper [17] an extensive amount [19-21] has contributed to elimination of the paradox of instantaneous propagation of thermal disturbances. The approach is known as extend irreversible thermodynamics, which introduces time derivative of the heat flux vector, Cauchy stress tensor and its trace into the classical Fourier law by preserving the entropy principle. The effects of using the Maxwell-Cattaneo model in Stoke's second problem for a viscous fluid were investigated in [20]. They also studied the effects of discontinuous boundary data on the velocity gradients temperature fields occurring in Stoke's first problem for a viscous fluid [21].

$$\rho C_p \frac{D}{Dt}\frac{\partial T}{\partial t} = -\nabla \cdot \frac{\partial \boldsymbol{q}}{\partial t} + \frac{\partial \Phi}{\partial t}. \tag{10}$$

Multiplying (10) by τ_0 and adding to (3) we obtain

$$\rho C_p \frac{D}{Dt}\left(T + \tau_0 \frac{\partial T}{\partial t}\right) = -\nabla \cdot \left(\boldsymbol{q} + \tau_0 \frac{\partial \boldsymbol{q}}{\partial t}\right) + \left(\Phi + \tau_0 \frac{\partial \Phi}{\partial t}\right).$$

Substituting from (9), we get

$$\rho C_p \frac{D}{Dt}\left(T + \tau_0 \frac{\partial T}{\partial t}\right) = \lambda \nabla^2 T - \nabla \cdot \Pi \boldsymbol{J} + \left(\Phi + \tau_0 \frac{\partial \Phi}{\partial t}\right). \tag{11}$$

Taking into account the definition of $\frac{D}{Dt}$ from (5), we arrive at

$$\left[\frac{\partial}{\partial t} + (\boldsymbol{V} \cdot \nabla)\right]\left(T + \tau_0 \frac{\partial T}{\partial t}\right) = \frac{\lambda}{\rho C_p}\nabla^2 T - \frac{1}{\rho C_p}\nabla \cdot \Pi \boldsymbol{J} + \frac{1}{\rho C_p}\left(\Phi + \tau_0 \frac{\partial \Phi}{\partial t}\right). \tag{12}$$

Equation (12) is the generalized energy equation taking into account the relaxation time τ_0. This generalization eliminates the paradox of infinite speed of propagation of heat in thermoelectric conducting fluid.

3. Formulation of the problem

The basic equations in vector form for an electroconducting Newtonian fluid with thermal relaxation in the presence of both magnetic field and heat source are

--- Continuity equation

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \boldsymbol{V}) = 0; \tag{13}$$

--- Momentum equation

$$\rho \frac{\partial \boldsymbol{V}}{\partial t} = \rho \boldsymbol{f} - \operatorname{grad}(p) - \mu \operatorname{curl}\operatorname{curl}(\boldsymbol{V}) + \boldsymbol{F}, \tag{14}$$

where $\boldsymbol{F}$ is the Lorentz force given by

$$\boldsymbol{F} = \boldsymbol{J} \wedge \boldsymbol{B}. \tag{15}$$

--- Generalized equation of heat conduction in the presence of heat source:

$$\rho C_p \frac{D}{Dt}\left(T + \tau_0 \frac{\partial T}{\partial t}\right) = \lambda \nabla^2 T - \nabla \cdot \Pi \boldsymbol{J} + \left(Q + \tau_0 \frac{\partial Q}{\partial t}\right); \tag{16}$$

--- Generalized Ohm's law Eq. (7).

Consider the laminar flow of an incompressible electro conducting fluid above the non-conducting half-space $y > 0$. Taking the positive y-axis of a Cartesian coordinate system in the upward direction, the fluid flow through the half-space $y > 0$ above and in contact with a flat plate occupying xz-plane. A constant magnetic field of strength H_o acts in the z direction. The induced electric current due to the motion of the fluid that is caused by the buoyancy forces dose not distort the applied magnetic field. The previous assumption is reasonably true

of the temperature field we get, the initial and boundary conditions are

$$\left.\begin{array}{llll} t \le 0, & u = 0, \quad T = T_\infty, & \text{everywhere} \\ u = U, & q(0,\mathrm{t}) = \dfrac{1}{2} Q_0 H(t), & \text{at} \quad y = 0 \\ t > 0, & u \to 0, \quad T \to T_\infty, & \text{as} \quad y \to \infty \end{array}\right\}. \tag{24}$$

Using the non-dimensional scheme

$$y^* = \frac{U}{\upsilon} y, \; t^* = \frac{U^2}{\upsilon} t, \; u^* = \frac{u}{U}, \; \Theta = \frac{T - T_\infty}{T_0}, \; Q^* = \frac{\upsilon^2 Q}{\lambda U^2 T_0}, \; \tau_0^* = \frac{U^2}{\upsilon} \tau_0,$$

$$p_r = \frac{C_p \mu}{\lambda}, \; K_0 = \frac{k_0 \sigma_0 B_0 T_0}{\rho U^2}, \; \Pi_0 = \frac{\pi_0 \upsilon \sigma_0 B_0}{\lambda T_0}, \; M = \frac{\upsilon \sigma_0 B_0^2}{\rho U^2}, \tag{25}$$

where p_r is the Prandtl number, M is the magnetic field parameter, and $K_0 \Pi_0 = MZT$.

By introducing the non-dimensional quantities mentioned above, Eqs. (21) and (22) are reduce to the non-dimensional equations, dropping the asterisks for convenience,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - Mu - K_0 \frac{\partial \Theta}{\partial y}, \tag{26}$$

$$p_r \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) \Theta = (1 + ZT_0) \frac{\partial^2 \Theta}{\partial y^2} + \Pi_0 \frac{\partial u}{\partial y} + Q + \tau_0 \frac{\partial Q}{\partial t}, \tag{27}$$

We will also assume that the initial state of the medium is quiescent. Taking Laplace transform, defined by the relation

$$\bar{g}(s) = \int_0^\infty \mathrm{e}^{-st} \, g(t) dt,$$

of both sides Eqs. (26) and (27), we obtain

$$\frac{d^2 \bar{u}}{dy^2} = (s + M) \bar{u} + K_0 \frac{d\bar{\Theta}}{dy}, \tag{28}$$

$$(1 + ZT_0) \frac{d^2 \bar{\Theta}}{dy^2} = s p_r (1 + \tau_0 s) \bar{\Theta} - \Pi_0 \frac{d\bar{u}}{dy} - Q_0 \delta(y) \left(\frac{1 + \tau_0 s}{s} \right). \tag{29}$$

The relevant initial and boundary conditions are

$$\left.\begin{array}{ll} \bar{u}(0,s) = \dfrac{1}{s}, \quad \bar{\Theta}'(0,s) = \dfrac{(1+\tau_0 \mathrm{s})}{2s} Q_0, & \text{at} \quad y = 0 \\ \bar{u} \to 0 \qquad \bar{\Theta} \to 0, & \text{as} \quad y \to \infty \end{array}\right\} \tag{30}$$

The above equation could be written in the following form

$$\frac{d^2 \bar{u}}{dy^2} = a\bar{u} + c \frac{d\bar{\Theta}}{dy}, \tag{31}$$

Then, we have

$$\bar{\Theta}_c = \frac{\Theta_1}{p^2 + p_1^2} + \frac{\Theta_2}{p^2 + p_2^2}, \tag{40}$$

where

$$\Theta_1 = \frac{A_1 a - (A_1 - A_2\,\mathrm{r})\,p_1^2}{p_1^2 - p_2^2}, \quad \Theta_2 = \frac{A_1 a - (A_1 - A_2\,\mathrm{r})\,p_2^2}{p_2^2 - p_1^2}$$

and

$$\bar{u}_s = \frac{p u_1}{p^2 + p_1^2} + \frac{p u_2}{p^2 + p_2^2}, \tag{41}$$

where

$$u_1 = \frac{A_1 c + A_2 (p_1^2 - \Omega)}{p_1^2 - p_2^2}, \quad u_2 = \frac{A_1 c + A_2 (p_2^2 - \Omega)}{p_2^2 - p_1^2}, \tag{42}$$

and p_1^2, p_2^2 satisfy the relations

$$p_1^2 + p_2^2 = a - cr + \Omega, \quad p_1^2 p_2^2 = a\Omega.$$

Applying the boundary conditions in (30), hence, we obtain

$$A_{10} = r\sqrt{\frac{2}{\pi}} \cdot \frac{1}{s} + \sqrt{\frac{2}{\pi}} \cdot \frac{(1+\tau_0 s)}{2s} - \beta\sqrt{2\pi} \quad \text{and} \quad A_{20} = \sqrt{\frac{2}{\pi}} \cdot \frac{1}{s}.$$

Then, we get

$$\bar{\Theta}_c = \frac{\Theta_{10}}{p^2 + p_1^2} + \frac{\Theta_{20}}{p^2 + p_2^2}, \tag{43}$$

where

$$\Theta_{10} = \frac{A_{10} a - (A_{10} - A_{20} r)\,p_1^2}{p_1^2 - p_2^2}, \quad \Theta_{20} = \frac{A_{10} a - (A_{10} - A_{20} r)\,p_2^2}{p_2^2 - p_1^2},$$

and

$$\bar{u}_s = \frac{p u_{10}}{p^2 + p_1^2} + \frac{p u_{20}}{p^2 + p_2^2}, \tag{44}$$

where

$$u_{10} = \frac{A_{10} c + A_{20} (p_1^2 - \Omega)}{p_1^2 - p_2^2} \quad \text{and} \quad u_{20} = \frac{A_{10} c + A_{20} (p_2^2 - \Omega)}{p_2^2 - p_1^2}.$$

5. The analytic inversion of Fourier Sine and Cosine transforms

By using the inverse transforms defined in (33), (35) and (37) we get

flow. The thermoelectric figure-of–merit effect on the flow of electro conducting fluids over boundaries in the presence of magnetic field is presented.

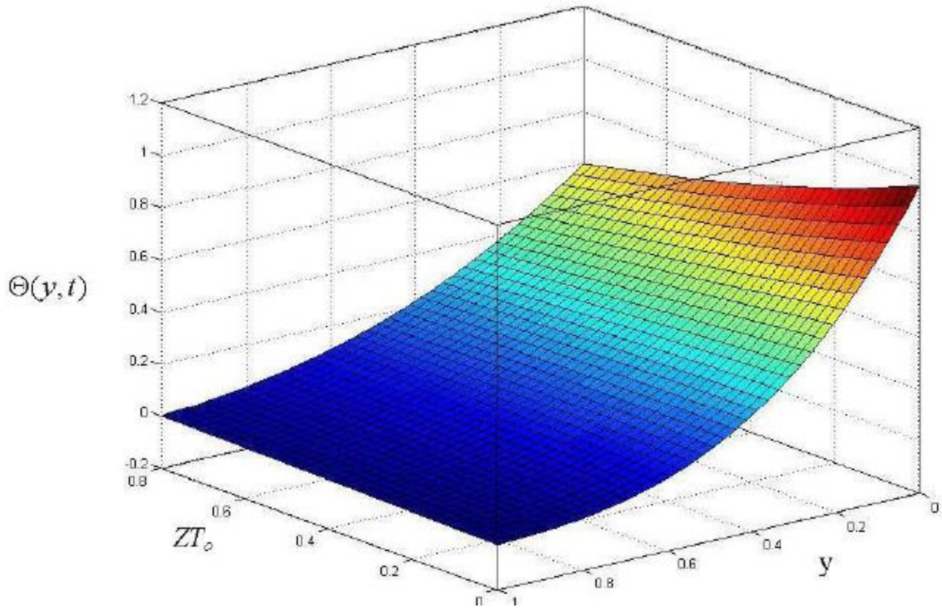

Fig.1. Temperature profile Θ(*y*, *t*) for various values of figure-of-merit ZT_0.

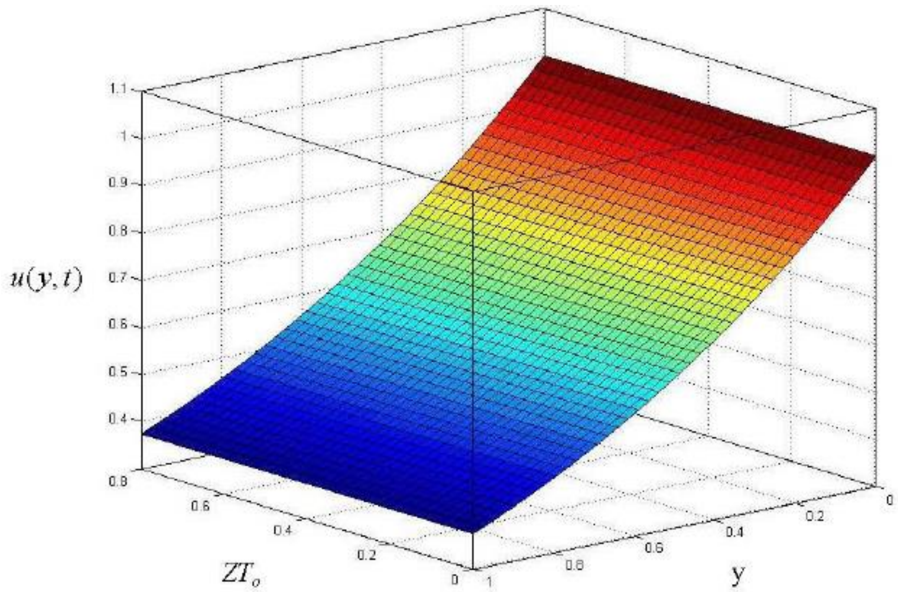

Fig. 2. Velocity profile *u*(*y*, *t*) for various values of figure-of-merit ZT_0.

The result provides a motivation to investigate conducting thermo Newtonian fluids as a new class of applicable thermoelectric materials. Experimental studies confirmed that the one-dimensional model could be used for heat calculation through the fluids over the boundaries**.**

The modification of the heat conduction equation from diffusive to a wave type may be affected either by a microscopic consideration of the phenomenon of heat transport or in a phenomenological way by modifying the classical Fourier's law of heat conduction. The inclusion of the relaxation time and conduction current density modifies the thermal equation, changing it from the parabolic to a hyperbolic type, and thereby eliminating the unrealistic results, that thermal disturbances are realized instantaneously everywhere within the fluid [26].

In this work, the method of direct integration by means of the matrix exponential, which is a standard approach in modern control theory and developed in detail in many texts such as Ogata [27] and Ezzat [28-30], is introduced in the field of generalized thermoelasticity and thermofluid and applied to specific problem for coupled system.

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