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ELASTODYNAMIC RESPONSE DUE TO MECHANICAL FORCES IN A MICROSTRETCH THERMOELASTIC MEDIUM WITH MASS DIFFUSION

Rajneesh Kumar[1], Arvind Kumar[2*]

[1]Department of Mathematics, Kurukshetra University, Kurukshetra, Haryana, India

[2]Department of Mathematics, Punjab Technical University, Jalandhar, Punjab, India

*e-mail: math_arvi@yahoo.in

Abstract. The present investigation deals with the 2-dimensional deformation in a homogeneous microstretch thermoelastic medium with mass diffusion due to mechanical forces. The normal mode analysis technique is used to obtain the components of displacement, microrotation, microstress, mass concentration and temperature change due to normal force and tangential force. The numerical computation has been performed (using matlab software) on the resulting quantities. The computed numerical results are shown graphically to depict the effect of microstretch, diffusion and relaxation times. Some particular cases of interest are also deduced from the present investigation.

1. Introduction

The linear theory of elasticity has a great importance in the study of elastic properties and stress analysis of most common structural material, e.g. steel. To a lesser extent, linear theory of elasticity also describes the behavior of other common materials like coal, concrete and wood. But this theory is not applicable to synthetic materials of the elastomers and polymer type e.g. PVC. The linear theory of micropolar elasticity is sufficient to explain the behavior of such materials.

The micropolar theory of elasticity and theory of micropolar elastic solids with stretch was presented by Eringen [1, 2]. He derived the equations of motion, constitutive equations and boundary conditions for a class of micropolar solids which can stretch and contract. This model introduced and explained the motion of certain class of rigid chopped fibers, liquid crystals, granular and composite materials. This theory considered the intrinsic rotations of the microstructures and supported body and surface couples.

Eringen [3] developed a theory of thermo-microstretch elastic solids in which he included microstructural expansions and contractions. The material points of microstretch solids can stretch and contract independently of their translation and rotations. For example, composite materials reinforce with chopped elastic fibers, porous medium with pores filled with gases, asphalt or other inclusions are characterized as microstretch solids. A comprehensive review on this subject is given by Eringen [4]. Microstretch theory is generalization of the theory of micropolar elasticity and a special case of micromorphic theory. Thus a microstretch elastic solid possess seven degrees of freedom, three for translation, three for rotation (as in micropolar elasticity) and one for stretch, required by substructures. Such a model can catch more information about the micro deformation inside a material point, which is more suitable for modeling the overall properties of the foam matrix in the case of foam composites.

Modern structural elements are often subjected to temperature change of such magnitude that their material properties may no longer be regarded as having constant values i.e. the thermal and mechanical properties of the materials vary with temperature, so the temperature-dependence of the material properties must be taken into consideration in the thermal stress analysis of these elements.

The physical applications are encountered in the context of problems such as ground explosions and oil industries. This problem is also useful in field of geo-mechanics, mechanical engineering and civil engineering where interest is in various phenomenon's' occurring in earthquakes and measurement of stress and temperature distribution due to presence of mechanical sources.

Diffusion is defined as the spontaneous movement of the particles from a high concentration region to the low-concentration region, and it occurs in response to a concentration gradient expressed as the change in the concentration due to change in position. Thermal diffusion utilizes the transfer of heat across a thin liquid or gas to accomplish isotope separation. Today, thermal diffusion remains a practical process to separate isotopes of noble gases e.g., xenon and other light isotopes e.g., carbon for research purposes. In most of the applications, the concentration is calculated using what is known as Fick's law. This is a simple law which does not take into consideration the mutual interaction between the introduced substance and the medium into which it is introduced or the effect of temperature on this interaction. However, there is a certain degree of coupling with temperature and temperature gradients as temperature speeds up the diffusion process. The thermodiffusion in elastic solids is due to coupling of fields of temperature, mass diffusion and that of strain in addition to heat and mass exchange with the environment. Nowacki [5-8] developed the theory of thermoelastic diffusion by using coupled thermoelastic model. Dudziak and Kowalski [9] and Olesiak and Pyryev [10], respectively, discussed the theory of thermodiffusion and coupled quasi stationary problems of thermal diffusion for an elastic layer. They studied the influence of cross-effects arising from the coupling of the fields of temperature, mass diffusion, and strain due to which the thermal excitation results in additional mass concentration and that generates additional fields of temperature. Cicco [11] discussed the stress concentration effects in microstretch elastic bodies. Ezzat and Awad [12] adopted the normal mode analysis technique to obtain the temperature gradient, displacement, stresses and micro-rotation. Kumar et al. [13] investigated the disturbance due to force in normal and tangential direction and porosity effect by using normal mode analysis in fluid saturated porous medium. Fundamental solution in the theory of thermomicrostretch elastic diffusive solids was developed by Kumar and Kansal [14].

This present problem deals with the 2-dimensional deformation in a homogeneous microstretch thermoelastic medium with mass diffusion due to mechanical forces. The normal mode analysis technique is used to obtain the expressions for the displacement components, couple stress, temperature, mass concentration and microstress distribution due to various sources. Microstretch, mass diffusion and relaxation effects are shown on the considered domain graphically. Some special cases have been deduced from the present investigation.

Basic equations. Following Eringen [3], Sherief et al. [15] and Kumar & Kansal [14], the basic equations for homogeneous, isotropic microstretch generalized thermoelastic diffusive solids in the absence of body force, body couple, stretch force and heat source are given by:

$$(\lambda+\mu)\nabla(\nabla.\boldsymbol{u})+(\mu+K)\nabla^2\boldsymbol{u}+K\nabla\times\boldsymbol{\phi}+\lambda_0\nabla\phi^*-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T-\beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla C=\rho\ddot{\boldsymbol{u}}, \qquad (1.1)$$

$$(\gamma\nabla^2-2K)\boldsymbol{\phi}+(\alpha+\beta)\nabla(\nabla.\boldsymbol{\phi})+K\nabla\times\boldsymbol{u}=\rho j\ddot{\boldsymbol{\phi}}, \qquad (1.2)$$

$$(\alpha_0\nabla^2-\lambda_1)\phi^*-\lambda_0\nabla.\boldsymbol{u}+\nu_1\left(1+\tau_1\frac{\partial}{\partial t}\right)T+\nu_2\left(1+\tau^1\frac{\partial}{\partial t}\right)C=\frac{\rho j_0}{2}\ddot{\phi}^*, \tag{1.3}$$

$$K^*\nabla^2T=\rho c^*\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T+\beta_1T_0\left(\frac{\partial}{\partial t}+\varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)\nabla.\boldsymbol{u}+\nu_1T_0\left(\frac{\partial}{\partial t}+\varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)\phi^*+aT_0\left(\frac{\partial}{\partial t}+\gamma_1\frac{\partial^2}{\partial t^2}\right)C, \tag{1.4}$$

$$D\beta_2\nabla^2(\nabla.\boldsymbol{u})+Da\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla^2T+\left(\frac{\partial}{\partial t}+\varepsilon\tau^0\frac{\partial^2}{\partial t^2}\right)C-Db\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla^2C=0, \tag{1.5}$$

$$t_{ij}=(\lambda_0\phi^*+\lambda u_{r,r})\delta_{ij}+\mu(u_{i,j}+u_{j,i})+K(u_{j,i}-\epsilon_{ijk}\phi_k)-\beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\delta_{ij}T-\beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)\delta_{ij}, \tag{1.6}$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}+b_0\epsilon_{mji}\phi^*_{,m}. \tag{1.7}$$

Here λ, μ, α, β, γ, K, λ_0, λ_1, α_0, b_0 are material constants; ρ is mass density; $\boldsymbol{u}=(u_1,u_2,u_3)$ is the displacement vector, $\boldsymbol{\phi}=(\phi_1,\phi_2,\phi_3)$ is the microrotation vector; ϕ^* is the scalar microstretch function; T is temperature; T_0 is the reference temperature of the body chosen; C is the concentration of the diffusion material in the elastic body; K^* is the coefficient of the thermal conductivity; c^* is the specific heat at constant strain; D is the thermoelastic diffusion constant; a is the coefficient describing the measure of thermo diffusion; b is the coefficient describing the measure of mass diffusion effects; j is the microinertia; $\beta_1=(3\lambda+2\mu+K)\alpha_{t1}$; $\beta_2=(3\lambda+2\mu+K)\alpha_{c1}$; $\nu_1=(3\lambda+2\mu+K)\alpha_{t2}$; $\nu_2=(3\lambda+2\mu+K)\alpha_{c2}$; α_{t1}, α_{t2} are coefficients of linear thermal expansion; α_{c1}, α_{c2} are coefficients of linear diffusion expansion; j_0 is the microinertia for the microelements; t_{ij} are components of stress; m_{ij} are components of couple stress; λ_i^* is the microstress tensor; e_{ij} are components of strain; e_{kk} is the dilatation; δ_{ij} is Kroneker delta function; τ^0, τ^1 are the diffusion relaxation times, and τ_0, τ_1 are thermal relaxation times with $\tau_0\geq\tau_1\geq0$. Here $\tau^0=\tau^1=\tau_0=\tau_1=\gamma_1=0$ for Coupled Thermoelastic theory (CT) model; $\tau_1=\tau^1=0$, $\epsilon=1$, $\gamma_1=\tau_0$ for Lord-Shulman (LS) model; and $\varepsilon=0$, $\gamma_1=\tau^0$, where $\tau^0>0$ for Green-Lindsay (GL) model.

In the above equations symbol (",") followed by a suffix denotes differentiation with respect to spatial coordinates and a superposed dot (" ˙ ") denotes the derivative with respect to time respectively.

2. Formulation of the problem

We consider a micropolar generalized thermoelastic with mass diffusion medium with rectangular Cartesian coordinate system $OX_1X_2X_3$ with x_3-axis pointing vertically downward the medium.

For two dimensional problems the displacement vector and microrotation vector have been considered of the form:

$$\boldsymbol{u}=(u_1,0,u_3),\ \boldsymbol{\phi}=(0,\phi_2,0), \tag{2.1}$$

For further consideration it is convenient to introduce in equations (1.1)-(1.5) the dimensionless quantities are defined as:

$$u_r'=\frac{\rho\omega^*c_1}{\beta_1T_0}u_r,\ x_r'=\frac{\omega^*}{c_1}x_r,\ t'=\omega^*t,\ \phi^{*\prime}=\frac{\rho c_1^2}{\beta_1T_0}\phi^*,\ T'=\frac{T}{T_0},\ \tau_1'=\omega^*\tau_1,\ \tau_0'=\omega^*\tau_0,\ \gamma_1'=\omega^*\gamma_1,\ t_{ij}'=\frac{1}{\beta_1T_0}t_{ij},\ \omega^*=\frac{\rho c^*c_1^2}{K^*},\ \phi_i'=\frac{\rho c_1^2}{\beta_1T_0}\phi_i,\ \tau^{1\prime}=\omega^*\tau^1,\ c_1^2=\frac{\lambda+2\mu+k}{\rho},\ c_2^2=\frac{\mu+k}{\rho},\ c_3^2=\frac{\gamma}{\rho j},\ c_4^2=\frac{2\alpha_0}{\rho j_0},\ \varepsilon=\frac{\gamma^2T_0}{\rho^2c^*c_1},\ m_{ij}^*=\frac{\omega^*}{c\beta_1T_0}m_{ij},\quad C'=\frac{\beta_2}{\rho c_1^2}C. \tag{2.2}$$

With the aid of equation (2.2) and (2.3), the equations (1.1)-(1.5) reduce to:

$$a_1\frac{\partial e}{\partial x_1}+a_2\nabla^2u_1-a_3\frac{\partial\phi_2}{\partial x_3}+a_4\frac{\partial\phi^*}{\partial x_1}-\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_1}-a_5\left(1+\tau^1\frac{\partial}{\partial t}\right)\frac{\partial C}{\partial x_1}=\ddot{u}_1, \tag{2.3}$$

$$a_1\frac{\partial e}{\partial x_3}+a_2\nabla^2 u_3+a_3\frac{\partial \phi_2}{\partial x_1}+a_4\frac{\partial \phi^*}{\partial x_3}-\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x_3}-a_5\left(1+\tau^1\frac{\partial}{\partial t}\right)\frac{\partial C}{\partial x_3}=\ddot{u}_3, \tag{2.4}$$

$$\nabla^2\phi_2-2a_6\phi_2+a_6\left(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1}\right)=a_7\ddot{\phi}_2, \tag{2.5}$$

$$\nabla^2\phi^*-a_8\phi^*-a_9 e+a_{10}\left(1+\tau_1\frac{\partial}{\partial t}\right)T+a_{11}\left(1+\tau^1\frac{\partial}{\partial t}\right)C=a_{12}\ddot{\phi}^*, \tag{2.6}$$

$$\nabla^2 T=a_{13}\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T+a_{14}\left(\frac{\partial}{\partial t}+\varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)e+a_{15}\left(\frac{\partial}{\partial t}+\varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)\phi^*+a_{16}\left(\frac{\partial}{\partial t}+\gamma_1\frac{\partial^2}{\partial t^2}\right)C, \tag{2.7}$$

$$\nabla^2 e+a_{17}\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla^2 T+a_{18}\left(\frac{\partial}{\partial t}+\varepsilon\tau^0\frac{\partial^2}{\partial t^2}\right)C-a_{19}\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla^2 C=0, \tag{2.8}$$

Here $a_1=\frac{\lambda+\mu}{\beta_1 T_0}$, $a_2=\frac{\mu+K}{\beta_1 T_0}$, $a_3=\frac{K}{\rho c_1^2}$, $a_4=\frac{\lambda_0}{\rho c_1^2}$, $a_5=\frac{\rho c_1^2}{\beta_1 T_0}$, $a_6=\frac{K c_1^2}{\gamma\omega^{*2}}$, $a_7=\frac{\rho j c_1^2}{\gamma}$, $a_8=\frac{\lambda_1 c_1^2}{\alpha_0\omega^{*2}}$, $a_9=\frac{\lambda_0\rho c_1^4}{\beta_1 T_0\alpha_0\omega^{*2}}$, $a_{10}=\frac{\upsilon_1\rho c_1^4}{\beta_1\alpha_0\omega^{*2}}$, $a_{11}=\frac{\nu_2\rho^2 c_1^6}{\beta_1\beta_2 T_0\alpha_0\omega^{*2}}$, $a_{12}=\frac{\rho c_1^2 j_0}{2\alpha_0}$, $a_{13}=\frac{\rho c^* c_1^2}{\omega^* K^*}$, $a_{14}=\frac{\beta_1 c_1^2}{\omega^* K^*}$, $a_{15}=\frac{\upsilon_1\beta_1 T_0}{\omega^*\rho K^*}$, $a_{16}=\frac{a\rho c_1^4}{\omega^*\beta_2 K^*}$, $a_{17}=\frac{aT_0}{\beta_2}$, $a_{18}=\frac{\rho c_1^4}{\omega^* D\beta_2^2}$, $a_{19}=\frac{b\rho c_1^2}{\beta_2^2}$.

The displacement components u_1 and u_3 are related to potential functions ϕ and ψ as:

$$u_1=\frac{\partial\phi}{\partial x_1}-\frac{\partial\psi}{\partial x_3},\ u_3=\frac{\partial\phi}{\partial x_3}+\frac{\partial\psi}{\partial x_1}. \tag{2.9}$$

Using the relation (2.9), in the equations (2.3)-(2.8), we obtain:

$$(a_1+a_2)\nabla^2\phi-\ddot{\phi}+a_4\phi^*-\left(1+\tau_1\frac{\partial}{\partial t}\right)T-a_2\left(1+\tau^1\frac{\partial}{\partial t}\right)C=0, \tag{2.10}$$

$$\left(\nabla^2-a_8-a_{12}\frac{\partial^2}{\partial t^2}\right)\phi^*-a_9\nabla^2\phi+a_{10}\left(1+\tau_1\frac{\partial}{\partial t}\right)T+a_{11}\left(1+\tau^1\frac{\partial}{\partial t}\right)C=0, \tag{2.11}$$

$$a_{13}\left(1+\tau_0\frac{\partial}{\partial t}\right)\dot{T}+a_{14}\left(\frac{\partial}{\partial t}+\varepsilon\tau_0\frac{\partial^2}{\partial t^2}\right)\nabla^2\phi+a_{15}\left(1+\varepsilon\tau_0\frac{\partial}{\partial t}\right)\dot{\phi}^*+a_{16}\left(1+\gamma_1\frac{\partial}{\partial t}\right)\dot{C}-\nabla^2 T=0, \tag{2.12}$$

$$\nabla^4\phi+a_{17}\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla^2 T+a_{18}\left(\frac{\partial}{\partial t}+\varepsilon\tau^0\frac{\partial^2}{\partial t^2}\right)C-a_{19}\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla^2 C=0, \tag{2.13}$$

$$a_2\nabla^2\psi-\ddot{\psi}+a_3\phi_2=0, \tag{2.14}$$

$$\nabla^2\phi_2-2a_6\phi_2-a_6\nabla^2\psi=a_7\ddot{\phi}_2. \tag{2.15}$$

Here $\nabla^2=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2}$ is the Laplacian operator.

3. Solution of the problem

The solution of the considered physical variables can be decomposed in terms of the normal modes as in the following form:

$$\{\phi,\psi,T,\phi_2,\phi^*,C\}(x_1,x_3,t)=\{\bar{\phi},\bar{\psi},\bar{T},\overline{\phi_2},\overline{\phi^*},\bar{C}\}(x_3)e^{i(kx_1-\omega t)}. \tag{3.1}$$

Here ω is the angular velocity and k is a complex constant.
Making use of (3.1), the relations (2.10)-(2.15) yields:

$$\left[b_1\left(\frac{d^2}{dx_3{}^2}-k^2\right)+\omega^2\right]\bar{\phi}+b_2\overline{\phi^*}+b_3\bar{T}+b_4\bar{C}=0, \tag{3.2}$$

$$b_5\left(\frac{d^2}{dx_3{}^2}-k^2\right)\bar{\phi}+\left(\frac{d^2}{dx_3{}^2}+b_6\right)\overline{\phi^*}+b_7\bar{T}+b_8\bar{C}=0, \tag{3.3}$$

$$b_9\left(\frac{d^2}{dx_3{}^2}-k^2\right)\bar{\phi}+b_{10}\overline{\phi^*}+\left(b_{11}-\frac{d^2}{dx_3{}^2}\right)\bar{T}+b_{12}\bar{C}=0, \tag{3.4}$$

$$\left(\frac{d^2}{dz^2}-k^2\right)^2\bar{\phi}+b_{13}\left(\frac{d^2}{d{x_3}^2}-k^2\right)\bar{T}+\left(b_{14}+b_{15}\left(\frac{d^2}{d{x_3}^2}-k^2\right)\right)\bar{C}=0, \quad (3.5)$$

$$\left[b_{16}\left(\frac{d^2}{d{x_3}^2}-k^2\right)+\omega^2\right]\bar{\psi}+b_{17}\overline{\phi_2}=0, \quad (3.6)$$

$$b_{20}\left(\frac{d^2}{d{x_3}^2}-k^2\right)\bar{\psi}+\left[b_{18}\left(\frac{d^2}{d{x_3}^2}-k^2\right)+b_{19}\right]\overline{\phi_2}=0, \quad (3.7)$$

Here,

$b_1=\frac{\rho c_1^2}{\beta_1 T_0}$, $b_2=\frac{\lambda_0}{\rho c_1^2}$, $b_3=-(1-i\omega\tau_1)$, $b_4=-b_1(1-i\omega\tau^1)$, $b_5=-\frac{\lambda_0\rho c_1^4}{\beta_1 T_0\alpha_0\omega^{*2}}$, $b_6=-k^2-\frac{\lambda_1 c_1^2}{\alpha_0\omega^{*2}}+\frac{\omega^2\rho j_0 c_1^2}{2\alpha_0}$, $b_7=\frac{\upsilon_1\rho c_1^4}{\beta_1\alpha_0\omega^{*2}}(1-i\omega\tau_1)$, $b_8=\frac{\nu_2\rho^2 c_1^6}{\beta_1\beta_2 T_0\alpha_0\omega^{*2}}$, $b_9=-\frac{\beta_1 c_1^2}{\omega^* K^*}(i\omega+\tau_0\epsilon\omega^2)$, $b_{10}=-\frac{\upsilon_1\beta_1 T_0}{\omega^*\rho K^*}(i\omega+\tau_0\epsilon\omega^2)$, $b_{11}=-\frac{\rho c^* c_1^2}{\omega^* K^*}(i\omega+\tau_0\omega^2)+k^2$, $b_{12}=-\frac{a\rho c_1^4}{\omega^*\beta_2 K^*}(i\omega+\gamma_1\omega^2)$, $b_{13}=\frac{aT_0}{\beta_2}(1-i\omega\tau_1)$, $b_{14}=-\frac{\rho c_1^4}{\omega^* D\beta_2^2}(i\omega+\tau^0\epsilon\omega^2)$, $b_{15}=-\frac{b\rho c_1^2}{\beta_2^2}(1-i\omega\tau^1)$, $b_{16}=\frac{\mu+K}{\beta_1 T_0}$, $b_{17}=-\frac{K}{\rho c_1^2}$, $b_{18}=\frac{\gamma}{\gamma\rho j c_1^2}$, $b_{19}=-\frac{2K}{\rho j\omega^{*2}}+\omega^2$, $b_{20}=-\frac{Kc_1^2}{j\omega^{*2}\beta_1 T_0}$.\ (3.8)

On solving equations (3.2)-(3.5), we obtain:

$$\left[\mathrm{A}_0\frac{d^8}{d\,{x_3}^8}+\mathrm{A}_1\frac{d^6}{d\,{x_3}^6}+\mathrm{A}_2\frac{d^4}{d\,{x_3}^4}+\mathrm{A}_3\frac{d^2}{d\,{x_3}^2}+\mathrm{A}_5\right](\bar{\phi},\overline{\phi^*},\bar{T},\bar{C})=0, \quad (3.9)$$

And on solving equations (3.6)-(3.7), we obtain:

$$\left[\frac{d^4}{d\,{x_3}^4}+\mathrm{A}_6\frac{d^2}{d\,{x_3}^2}+\mathrm{A}_7\right](\overline{\phi_2},\bar{\psi})=0, \quad (3.10)$$

The solution of the above system satisfying the radiation conditions that $(\bar{\phi},\bar{\psi},\bar{T},\overline{\phi_2},\overline{\phi^*},\bar{C})\to 0$ as $x_3\to\infty$ are given as following:

$$(\bar{\phi},\overline{\phi^*},\bar{T},\bar{C})=\sum_{i=1}^{4}(1,\alpha_{1i},\alpha_{2i},\alpha_{3i})M_i e^{-m_i x_3}, \quad (3.11)$$

$$(\bar{\psi},\overline{\phi_2})=\sum_{i=5}^{6}(1,\beta_{1i})N_i e^{-m_i x_3}. \quad (3.12)$$

Here M_i and N_i are the functions depending on k and ω; m_i^2 $(i=1,2,3,4)$ are the roots of the equation (3.9) and $m_i^2(i=5,6)$ are the roots of equation (3.10); $\alpha_{1i}=\frac{D_{1i}}{D_{0i}}$; $\alpha_{2i}=\frac{D_{2i}}{D_{0i}}$; $\alpha_{3i}=\frac{D_{3i}}{D_{0i}}$, $i=1,2,3,4$; $\beta_{1i}=-\frac{\delta_3(m_i^2-k^2)}{(m_i^2-k^2)+\delta_4}$, $i=5,6$.

4. Boundary Conditions

We consider normal and tangential force acting on the surface $x_3=0$ along with vanishing of couple stress, microstress and temperature gradient with insulated and impermeable boundary at $x_3=0$. Mathematically this can be written as:

$$t_{33}=-F_1 e^{-(kx_1-\omega t)},\ t_{31}=-F_2 e^{-(kx_1-\omega t)},\ m_{32}=0,\ \lambda_3=0,\ \frac{\partial T}{\partial x_3}=0,\ \frac{\partial C}{\partial x_3}=0. \quad (4.1)$$

Here F_1 and F_2 are the magnitude of the applied force.
Using these boundary conditions and solving the linear equations formed, we obtain:

$$t_{33}=\sum_{i=1}^{6}G_{1i}e^{-m_i x_3}\,e^{-(kx_1-\omega t)},\ i=1,2,\ldots,6, \quad (4.2)$$

$$t_{31}=\sum_{i=1}^{6}G_{2i}e^{-m_i x_3}\,e^{-(kx_1-\omega t)},\ i=1,2,\ldots,6, \quad (4.3)$$

$$m_{32}=\sum_{i=1}^{6}G_{3i}e^{-m_i x_3}\,e^{-(kx_1-\omega t)},\ i=1,2,\ldots,6, \quad (4.4)$$

$$\lambda_3 = \sum_{i=1}^{6} G_{4i} e^{-m_i x_3}\, e^{-(kx_1-\omega t)},\ i = 1,2,\dots,6, \tag{4.5}$$

$$u_1 = \sum_{i=1}^{6} G_{5i} e^{-m_i x_3}\, e^{-(kx_1-\omega t)},\ i = 1,2,\dots,6, \tag{4.6}$$

$$u_3 = \sum_{i=1}^{6} G_{6i} e^{-m_i x_3}\, e^{-(kx_1-\omega t)},\ i = 1,2,\dots,6, \tag{4.7}$$

$$T = \sum_{i=1}^{6} G_{7i} e^{-m_i x_3}\, e^{-(kx_1-\omega t)},\ i = 1,2,\dots,6, \tag{4.8}$$

$$C = \sum_{i=1}^{6} G_{8i} e^{-m_i x_3}\, e^{-(kx_1-\omega t)},\ i = 1,2,\dots,6. \tag{4.9}$$

Here G_{ij}, $i = 1,2,\dots,6$, $j = 1,2,\dots,8$ are the constants.

Case 1-Normal Stress. To obtain the expressions due to normal stress we must set $F_2 = 0$ in the boundary conditions (4.1).

Case I1- Tangential Stress. To obtain the expressions due to tangential stress we must set $F_1 = 0$ in the boundary conditions (4.1).

Particular cases

(i) If we take $\tau_1 = \tau^1 = 0$, $\varepsilon = 1$, $\gamma_1 = \tau_0$, in Eqs. (4.2)- (4.9), we obtain the corresponding expressions of stresses, displacements and temperature distribution for L-S theory.

(ii) If we take $\varepsilon = 0$, $\gamma_1 = \tau^0$ in Eqs. (4.2)- (4.9), the corresponding expressions of stresses, displacements and temperature distribution are obtained for G-L theory.

(iii) Taking $\tau^0 = \tau^1 = \tau_0 = \tau_1 = \gamma_1 = 0$ in Eqs. (4.2) - (4.9), yield the corresponding expressions of stresses, displacements and temperature distribution for Coupled theory of thermoelasticity.

Special cases

(a) Microstretch Thermoelastic Solid. If we neglect the diffusion effect in Equations (4.2) - (4.9), we obtain the corresponding expressions of stresses, displacements and temperature for microstretch thermoelastic solid.

(b) Micropolar Thermoelastic Diffusive Solid. If we neglect the microstretch effect in Equations (4.2) - (4.9), we obtain the corresponding expressions of stresses, displacements and temperature for micropolar thermoelastic diffusive solid.

5. Numerical Results and Discussions

The analysis is conducted for a magnesium crystal-like material. The values of physical constants are: $\lambda = 9.4 \times 10^{10}\ Nm^{-2}$, $\mu = 4.0 \times 10^{10}\ Nm^{-2}$, $K = 1.0 \times 10^{16}\ Nm^{-2}$, $\rho = 1.74 \times 10^{3}\ Kgm^{-3}$, $j = 0.2 \times 10^{-19} m^2$, $\gamma = 0.779 \times 10^{-9}\ N$.

Thermal and diffusion parameters are given by

$c^* = 1.04 \times 10^3\ JKg^{-1}K^{-1}$, $K^* = 1.7 \times 10^6\ Jm^{-1}s^{-1}K^{-1}$, $\alpha_{t1} = 2.33 \times 10^{-5} K^{-1}$, $\alpha_{t2} = 2.48 \times 10^{10} K^{-1}$, $T_0 = 0.298 \times 10^3\ K$, $\tau_0 = 0.02$, $\tau_1 = 0.01$, $\alpha_{c1} = 2.65 \times 10^{-4} m^3 Kg^{-1}$, $\alpha_{c2} = 2.83 \times 10^{-4} m^3 Kg^{-1}$, $a = 2.9 \times 10^4 m^2 s^{-2} K^{-1}$, $b = 32 \times 10^5\ Kg^{-1} m^5 s^{-2}$, $\tau^1 = 0.04$, $\tau^0 = 0.03$, $D = 0.85 \times 10^{-8}\ Kgm^{-3}s$.

And, the microstretch parameters are taken as: $j_0 = 0.19 \times 10^{-19} m^2$, $\alpha_0 = 0.779 \times 10^{-9}\ N$, $b_0 = 0.5 \times 10^{-9}\ N$, $\lambda_0 = 0.5 \times 10^{10}\ Nm^{-2}$ $\lambda_1 = 0.5 \times 10^{10}\ Nm^{-2}$.

The computations were carried out for a single value of $\omega = 10$ and on the surface of the plane $x_3 = 1$. The trends for the normal stress t_{33}, tangential couple stress m_{32}, tangential stress t_{31} and microstress λ_3 on the surface of plane $x_3 = 1$ due to applied concentrated and uniformly distributed normal sources are shown in Figs. 1-7. The comparison of three theories of generalized thermoelasticity, namely, Lord-Shulman (LS), Green-Lindsay (GL) and Coupled theory (CT) has been shown in graphs. Also the diffusion effect and microstretch effect has been shown in graphs.

Throughout these graphs LS stands for LS theory for microstretch thermoelastic medium with mass diffusion, GL stands for GL theory for microstretch thermoelastic solid with mass diffusion and CT means CT theory for microstretch thermoelastic solid with mass diffusion. In Figures 1-4 the effect of mass concentration is shown for the three theories and in Figs. 5-7 the effect of microstretch is shown for LS, GL and CT theories of thermoelasticity.

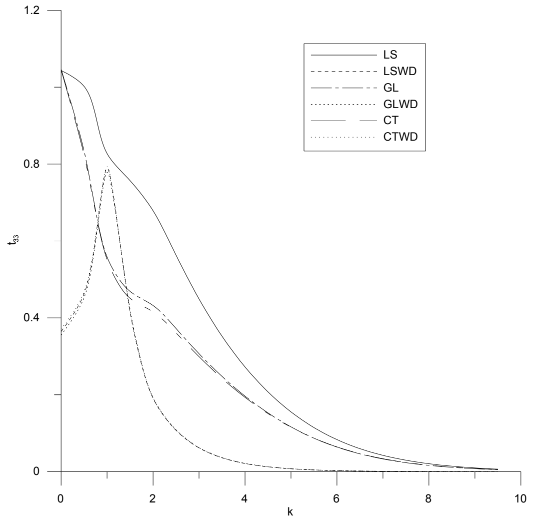

Fig. 1. Normal stress t_{33} as a function of wave number *k*.

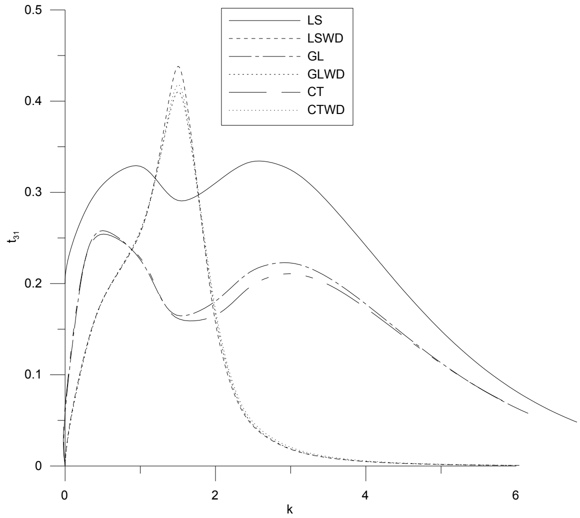

Fig. 2. Tangential stress t_{31} as a function of wave number *k*.

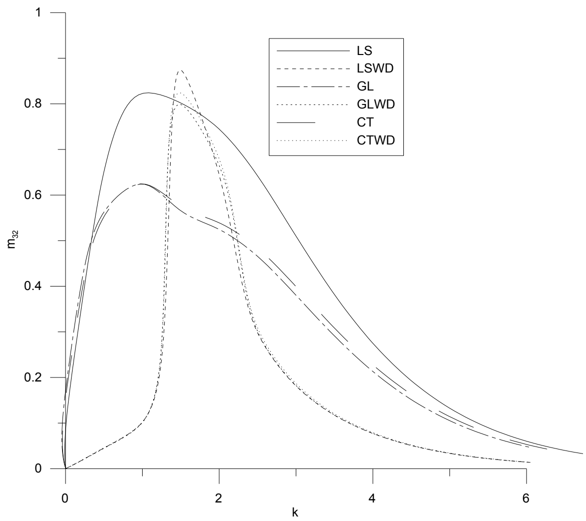

Fig. 3. Tangential couple stress m_{32} as a function of wave number *k*.

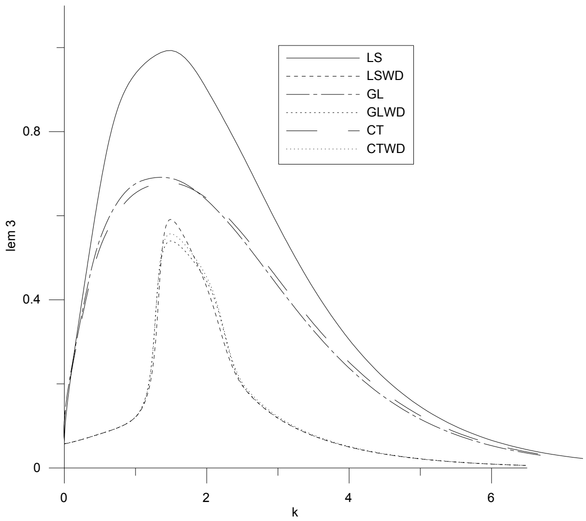

Fig. 4. The micro stress λ_3 as a function of wave number *k*.

Throughout all the Figs. LSWD stands for LS theory for microstretch thermoelastic medium without mass diffusion and LSWM stands for LS theory for micropolar thermoelastic medium with mass diffusion. GLWD stands for GL theory for microstretch thermoelastic medium without mass diffusion and GLWM stands for GL theory for micropolar thermoelastic medium with mass diffusion. CTWD stands for CT theory for microstretch thermoelastic medium without mass diffusion and CTWM stands for CT theory for micropolar thermoelastic medium with mass diffusion.

It is observed from the Fig. 1 that the value of normal stress t_{33} for LS, GL and CT theories of thermoelasticity for microstretch thermoelastic medium with mass diffusion decreases with the increase of the parameter k under the effect of mechanical source. The normal stress for LS, GL and CT theory for microstretch thermoelastic medium without mass diffusion increases near the source and then uniformly decreases and converges to zero.

In Figures 2 the variation of tangential stress t_{31} with wave number k is shown. The diffusion effect is shown for LS, GL and CT theories.

In Figures 3 the variation of tangential couple stress (m_{32}) is shown. Here the tangential couple stress increases up to value $k = 2$ then decreases regularly. The same trend is followed after neglecting the mass concentration effect.

In Figures 4 the variation of micro stress (λ_3) is shown. For microstretch thermoelastic solid with mass diffusion with LS theory, the micro stress first increase until k reaches value around 2 and then decreases. The same behavior is shown in case of GL theory and the CT theory.

It is observed from the Fig. 5 that the value of normal stress t_{33} for LS, GL and CT theory of thermoelasticity for microstretch thermoelastic medium with mass diffusion decreases with the increase of the parameter k under the effect of mechanical source. The normal stress for LS, GL and CT theory for micropolar thermoelastic medium without mass diffusion increases near the source and then uniformly decreases and converges to zero.

In Figures 6 the variation of tangential stress t_{31} with wave number k is shown. The microstretch effect is shown for LS, GL and CT theories.

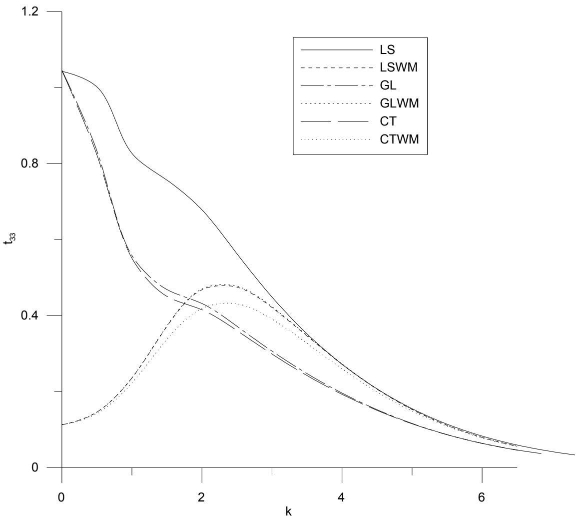

Fig. 5. Normal stress t_{33} as a function of wave number k.

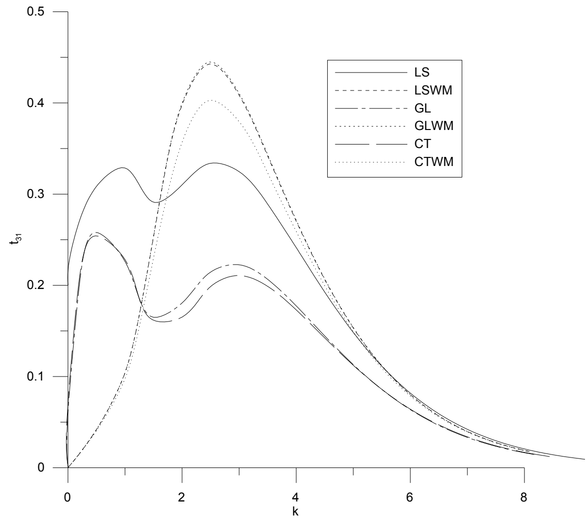

Fig. 6. Tangential stress t_{31} as a function of wave number k.

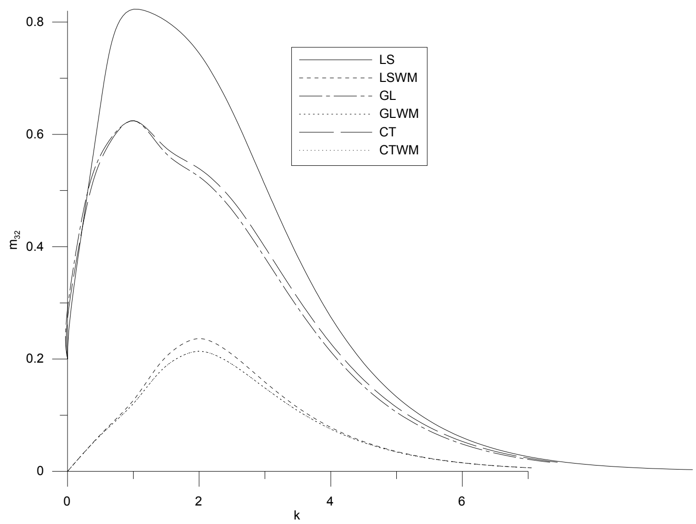

Fig. 7. Tangential couple stress m_{32} as a function of wave number k.

In Figure 7 the variation of tangential couple stress (m_{32}) is shown. Here the tangential couple stress increases up to value $k = 2$ then decreases regularly. The same trend is followed after neglecting the microstretch effect.

5. Conclusions

Such dynamical loading may produce shear deformation and temperature rise in a thin zone near the half space surface and thereby cause excessive wear and even cracking near the contact zone. It is therefore useful to analyze this class of problems by using a formulation that is as exact as possible and provide to the result for surface field quantities (displacement stress, micro-rotation and temperature change) that may be required for designing problems.

The problem is useful for geophysical mechanics where the interest is the phenomenon in earth quake and measuring of displacement in certain sources. The results of the problem may be applied to a wide class of geophysical problems involving temperature change. The deformation at any point of the medium at any point is useful to analyze the deformation field around mining tremors and drilling into the crust of the earth.

Finally we conclude:

(i) The normal mode analysis technique is used to derive the components of normal stress, shear stress, couple stress, microstress, temperature distribution and the mass concentration.

(ii) Values of displacement components, stress components are close to each other due to LS, GL and CT theories.

(iii) Behavior of variation of stress components is shown in figures.

(iv) The stress components show a similar trend under different theories.

(v) The effect of mass concentration and microstretch are shown in figures for three theories i.e. LS, GL and CT theories.

References

[1] A.C. Eringen // *Indiana University Mathematics Journal* **15** (1966) 909.

[2] A.C. Eringen, *Micropolar elastic solids with stretch*, ed. by M.I. Anisina (Ari Kitabevi Matbassi, Istanbul, Turkey, 1971).

[3] A.C. Eringen // *International Journal of Engineering Science* **28(12)** (1990) 1291.

[4] A.C. Eringen, *Microcontinuum Field Theories I: Foundations and Solids* (Springer-Verleg, New York, 1999).

[5] W. Nowacki // *Bulletin de l'Acad´emie Polonaise des Sciences. S´erie des Sciences* Techniques **22** (1974) 55.

[6] W. Nowacki // *Bulletin de l'Acad´emie Polonaise des Sciences. S´erie des Sciences Techniques* **22** (1974) 129.

[7] W. Nowacki // *Bulletin de l'Acad´emie Polonaise des Sciences. S´erie des Sciences Techniques* **22** (1974) 275.

[8] W. Nowacki // *Engineering Fracture Mechanics* **8** (1976) 261.

[9] W. Dudziak, S.J. Kowalski // *International Journal of Heat and Mass Transfer* **32(11)** (1989) 2005.

[10] Z.S. Olesiak, Y.A. Pyryev // *International Journal of Engineering Science* **33(6)** (1995) 773.

[11] S.D. Cicco // *International Journal of Engineering Sciences* **41** (2003) 187.

[12] M.A. Ezzat, E.S. Awad // *Journal of Thermal Stresses* **33(3)** (2010) 226.

[13] R. Kumar, A. Miglani, S. Kumar // *Multidiscipline Modeling in Materials and Structures* 7(1) (2011) 29.

[14] R. Kumar, T. Kansal // *ISRN Applied Mathematics* (2011) Article ID 764632.

[15] H.H. Sherief, F. Hamza, H. Saleh // *International Journal of Engineering Sciences* **42** (2004) 591.