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RESEARCH ON PIEZOELECTRIC QUARTZ UNDER MULTIDIMENSIONAL FORCES

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Abstract. Considering that the traditional sensors can't work well in the field with spacial requirements, such as wind tunnel experiment, a research of small sensor to measure multidimensional aerodynamic load is carried out. Based on anisotropic elastic theory, the polarization field inside the square Y0-cut quartz wafer under multidimensional/six dimensional (6-D) forces is studied. With the method of splitting electrode, the mapping relationships between forces and the induced charges on each area are deduced. And then by using the piezoelectric effect models of ANSYS, simulation analysis about the quartz wafer under 6-D forces is made, obtaining the distribution of electric potential and electric field. The results of the theoretical and simulated analysis show that it exists a quantitative and mathematical relationship between the density of induced charges on each area and the tangential force and torque.The other stresses can be acquired by overlaying the wafer group. The study provides a new ideal for the production of small-size piezoelectric system to measure multidimensional forces.

1. Introduction

In some fields, essential information is often acquired by measuring the force applied on the object. And with the improvement of property requirements for products, the demand of force measurement changes from static to dynamic and from macro to micro. But the traditional sensor for multidimensional forces has a large size, which limits the application in some narrow spaces, such as 6-D forces dynamic measurement of the micro robot joint and some hypersonic wind tunnel experiments.

At present, the multidimensional forces sensor usually consists of three-component sensors or single-component sensors. Professor Makoto, Kyushu University of Technology in Japan, designed the 6-D forces sensor through the ingenious combination of the two three-component sensors; Kistler, a company of Switzerland, built the 6-D forces platform by using multiple piezoelectric sensors; Professor Yongsheng Zhao in Yanshan University presented the overall preloaded sensor to measure 6-D forces based on the Stewart platform structure; References [1] and [2] realized the measurement of 6-D forces by optimizing the

Stewart platform structure of sensor. Reference [3] proposed a parallel structure model of 6-D forces sensors with the method of laying out quartz wafer in space. In the fields with special space requirements, the above traditional sensors have too large sizes to work well.

In the paper, the stress model of square Y0-cut quartz wafer under 6-D forces is established. Based on anisotropic elastic theory and Maxwell's electromagnetic theory, the distribution of polarization field is acquired. And the "many-to-many" mapping relationships between multidimensional forces and coupling induced charges are deduced. By using ANSYS, the simulation analysis of quartz wafer is conducted with ideal results, providing a new thought of overlaying the wafer group to measure multidimensional forces.

2. The stress analysis of quartz wafer

This paper regards a square with a hole Y0-cut α-dextrorotation quartz wafer as the study object. The inner hole radius is R = 2.5 mm, length of side is a=8 mm, and thickness is t=1 mm. Fix one end face and the other end face is acted on 6-D forces.

For the convenience of calculation, crystal coordinate O'x'y'z' is transformed to calculation coordinate Oxyz [4], as shown in Fig. 1.

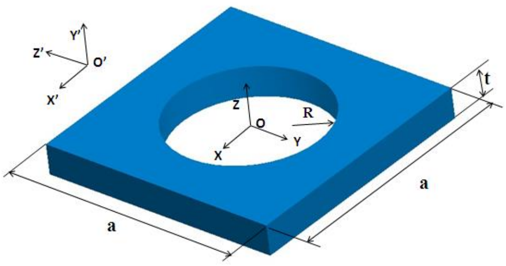

Fig. 1. Crystal coordinate and calculation coordinate.

The stress model of quartz wafer under 6-D forces. The elastic symmetry plane in the quartz wafer is perpendicular to the x axis of crystal coordinate, so the plane does not overlap with the cross section under 6-D forces. Namely this is a case of special bending-torsion of the anisotropic elastic body. And the corresponding stress components [5] can be obtained from equation (1).

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} + \overline{U}, \quad \sigma_y = \frac{\partial^2 F}{\partial x^2} + \overline{U},$$

$$\sigma_z = \left(\frac{s_{34}}{2s_{33}} M_z - M_y \right) \frac{x}{I_2} + \left(-\frac{s_{35}}{2s_{33}} M_z + M_x \right) \frac{y}{I_1} - \frac{1}{s_{33}} (s_{13}\sigma_x + s_{23}\sigma_y + s_{34}\tau_{yz} + s_{35}\tau_{xz} + s_{36}\tau_{xy}) + \frac{F_z}{A},$$

$$\tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y}, \quad \tau_{xz} = \frac{\partial \varphi}{\partial y}, \quad \tau_{yz} = -\frac{\partial \varphi}{\partial x}, \tag{1}$$

where $\bar{U}$ represents the external force (volume and surface force); s_{ij} represents elastic compliance coefficient. I_1 and I_2 are the principal moments of inertia of wafer for the x and y axis respectively; F_x, F_y, F_z are three normal forces; M_x, M_y are the torques for x and y axis respectively; M_z is the torque for z axis.

Stress results under 6-D forces. When three moments are acted on the wafer, no other surface or volume forces exist on the xy plane, $\bar{U} = 0$. So in the numerical analysis of electric potential, it's enough to apply only the three moments on the wafer according to anisotropic elastic theory [6]. Then based on the independence and the superposition of stresses, the final stress mathematical model of quartz wafer under 6-D forces is acquired.

The wafer acted on three moments is anisotropic elastic's space rotation and satisfies the differential equation (2) and the boundary condition equation (3) [7].

$$\begin{cases} L_4 F + L_3 \phi = 0, \\ L_3 F + L_2 \phi = -2\vartheta + \dfrac{M_z}{2 s_{33}} \left(\dfrac{s_{34}^2}{I_2} + \dfrac{s_{35}^2}{I_1} \right) - M_x \dfrac{s_{35}}{I_1} - M_y \dfrac{s_{34}}{I_2}, \end{cases} \tag{2}$$

$$\frac{\partial F}{\partial x} = 0,\ \frac{\partial F}{\partial y} = 0,\ \phi |_{\Gamma} = 0 \cdot \tag{3}$$

Eventually the torsion stress function of the wafer under three moments is obtained [8]. After determining the stress of the wafer under three moments, the stress under 6-D forces is calculated on the basis of the the independence and the superposition of stresses, as the equation (4).

$$\begin{cases} \sigma_x = 0 \\ \sigma_y = 0 \\ \sigma_z = \left(\dfrac{s_{34}}{2 s_{33}} M_z - M_y \right) \dfrac{x}{I_2} + M_x \bullet \dfrac{y}{I_1} - \dfrac{s_{34} \tau_{yz}}{s_{33}} + \dfrac{F_z}{a^2 - \pi R^2} \\ \tau_{yz} = -1.0136 \times 10^7 M_z \displaystyle\sum_{m=1,3,5\ldots}^{\infty} \dfrac{1}{m^2} \left(1 - \dfrac{ch \dfrac{2.5m}{a} y}{ch 1.25m} \right) \cos \dfrac{m\pi (2x+a)}{2a} + \dfrac{F_x}{a^2 - \pi R^2} \\ \tau_{xz} = 0.8063 \times 10^7 M_z \displaystyle\sum_{m=1,3,5\ldots}^{\infty} \dfrac{1}{m^2} \left(\dfrac{sh \dfrac{2.5m}{a} y}{ch 1.25m} \right) \sin \dfrac{m\pi (2x+a)}{2a} + \dfrac{F_y}{a^2 - \pi R^2} \\ \tau_{xy} = 0 \end{cases} \tag{4}$$

As can be seen from the above result, for the square wafer with a hole under 6-D forces, only the three stress components are existed.

3. Bound charge calculation

The experiment indicates that when the stress of piezoelectric quartz material is less, the value of the polarizing effect of each dipole is proportional to the local stress δ_{ij} [9]. By applying coordinate transformation method, the new piezoelectric coefficient d in Oxyz is calculated from the piezoelectric coefficient d' in Ox'y'z', as the equation (5).

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} -d_{11} & 0 & d_{11} & -d_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{35} & d_{14} \end{bmatrix} \begin{vmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{vmatrix}, \tag{5}$$

where $d_{11} = 2.31\times10^{-12} m^2/N$, $d_{14} = -0.727\times10^{-12} m^2/N$, $d_{35} = 4.62\times10^{-12} m^2/N$.

When extra electric field and mechanical load are applied, the density of bound charge on equivalent surface produced by bending torsional η_{z+}, η_{z-} (the density of induced charge on the top "z+" and bottom "z-" of the wafer in the *z*-direction respectively) is listed as the equation (6) and (7).

$$\eta_{z+} = \boldsymbol{P}\cdot\boldsymbol{n} = P_z \cdot \boldsymbol{e_z} = d_{35}(0.8063\times10^7 M_z \sum_{m=1,3,5\ldots}^{\infty} \frac{1}{m^2} \left(\frac{sh\frac{2.5m}{a}y}{ch1.25m} \right) \sin\frac{m\pi(2x+a)}{2a} + \frac{F_y}{a^2 - \pi R^2}), \tag{6}$$

$$\eta_{z-} = \boldsymbol{P}\cdot\boldsymbol{n} = -P_z \cdot \boldsymbol{e_z} = -d_{35}(0.8063\times10^7 M_z \sum_{m=1,3,5\ldots}^{\infty} \frac{1}{m^2} \left(\frac{sh\frac{2.5m}{a}y}{ch1.25m} \right) \sin\frac{m\pi(2x+a)}{2a} + \frac{F_y}{a^2 - \pi R^2}), \tag{7}$$

where unit vector of outer normal direction is $\boldsymbol{n} = \{0,\ 0,\ e_z\}$, and $\boldsymbol{e_z}$ is the unit normal vector.

According to the equation (5), $P_y = 0$ is acquired. It means that the wafer has not the phenomenon of polarization in the y direction. Two densities of the charge on the upper cross-section (perpendicular to the *z* axis) extracting charge are symmetry distribution based on the x axis, equal and opposite polarity. If the split electrode method is used, two electrode slices are bounded by the x axis, and the quantity of electric charge Q_1 and Q_2 are respectively extracted from area 1 and area 2, as shown in Fig. 2.

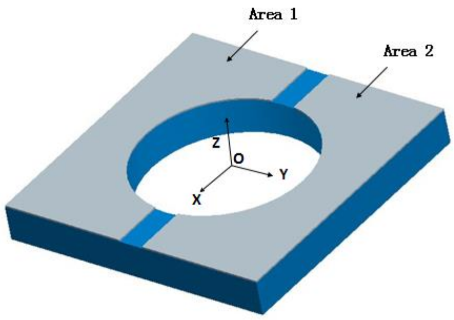

Fig. 2. Method of split electrode to extract the charge.

$$M_z = 9.867(Q_1 - Q_2)\times 10^9 (N\cdot m),$$
$$F_y = 6.935(Q_1 + Q_2)\times 10^{11} (N). \qquad (8)$$

Observing the theoretical results above, it is clear that the quantity of Q_1 and Q_2 respectively has a significant linear correlation with F_y and M_z. The many-to-many mapping relationships are built between multidimensional forces and induced charges on two areas.

In the same way, the other stresses can be acquired through placing the rotated *X0*-cut or *Y0*-cut quartz wafer (such as being rotated 90 degrees). So the 6-D forces are obtained by overlaying the wafer group to realize small size and real-time measurement.

4. Simulation research on the quartz wafer by ANSYS

In this paper, the Y0-cut α-dextrorotation quartz wafer with certain size under 6-D forces is analyzed, acquiring the corresponding electric potential and electric field intensity.

First of all, the model of the wafer is built in calculation coordinate Oxyz. The element type Solid5 should be chosen. The meshing is mapped by using the combination of manual control and free mesh [10]. Then, the loading step and solving step are followed. When setting constraints, node degrees of freedom on the end face of the wafer are all 0. To simulate the ideal load of concentrated force/torque, the load is applied by establishing key points on another end face of wafer and on z axis. So the key points and the upper surface form a rigid zone on the wafer.

From the above theoretical analysis, only applying tangential force and torque on the wafer is allowed in simulation. $F_y = 30\,kN$ and $M_z = 300\,N\cdot mm$ are respectively imposed on the wafer, getting the distribution of electric potential and electric field intensity, as shown in Fig. 3 and Fig. 4.

It is obvious that when only applying F_y on the wafer, potential values are equal and the same direction on area 1 and area 2. When only M_z, potential values are equal but in the opposite direction on area 1 and area 2.

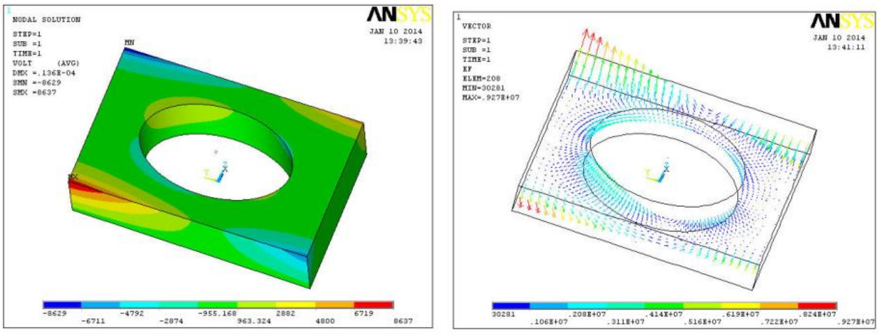

Fig. 3. Electric potential and electric field intensity (F_y = 30 *kN*).

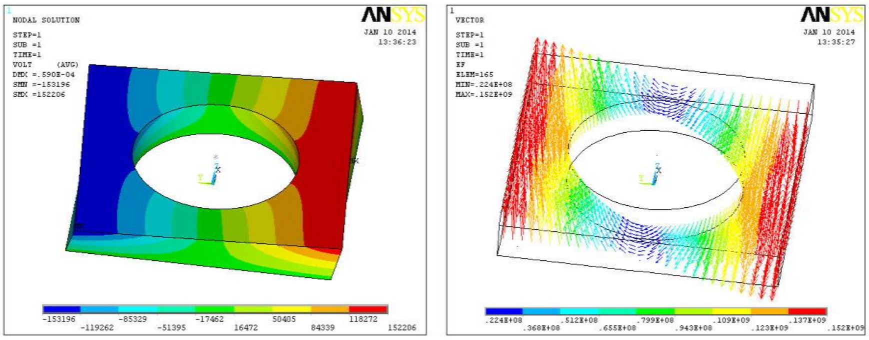

Fig. 4. Electric potential and electric field intensity (M_z = 300 *N·mm*).

Finally, the specific process of loading is: apply F_y and M_z on the quartz wafer. The numerical values of the tangential force F_y are 10, 15, 20, 25 kN in sequence and that of torque M_z is 100, 150, 200, 250 N·mm in sequence. When applying F_y and M_z, theoretical potential value at node 1 (0, 0.003, 0.001) and node 2 (0, 0.003, 0.003) of z plane of the wafer is calculated by using the above equations. At the same time, the simulation analysis value of potential is acquired. Comparison between theoretical calculation and simulation analysis is shown as Fig. 5 and Fig. 6.

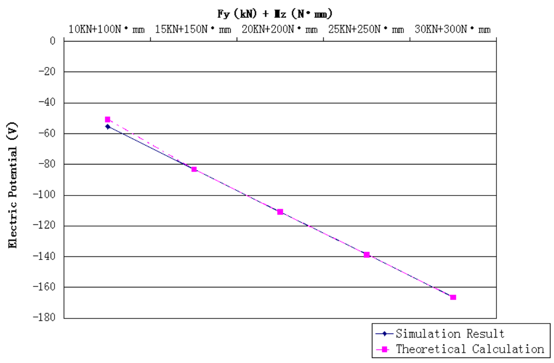

Fig. 5. Potential contrast figure at node 1.

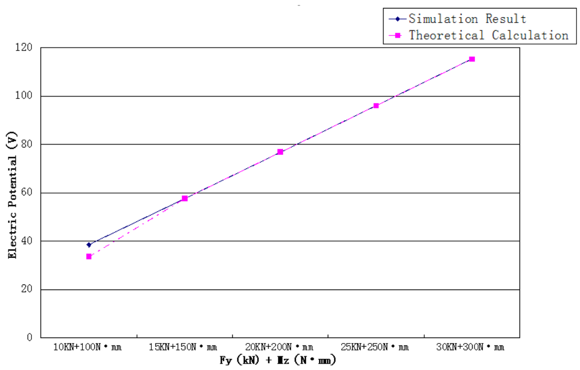

Fig. 6. Potential contrast figure at node 2.

As is shown in Fig. 5 and Fig. 6, it is obvious that for every node, when tangential force and torque act on the quartz wafer separately, the potential value is equal to the sum of value when acting tangential force and torque at the same time. Potential value has a linear correlation with the load, which is consistent with the result of theoretical analysis.

4. Conclusion

The polarization field of the Y0-cut wafer under 6-D forces is acquired by the building and the solution of stress model. At the same time, the mapping relationships between multidimensional forces and induced charges on each area are deduced. Then by using ANSYS, the quartz wafer is analyzed to get the distribution of electric potential and electric field under 6-D forces.

The results of the theoretical analysis and simulation show that it exists a quantitative and mathematical relationship between the density of induced charges on the surface of the wafer and the tangential force and torque, making possibility design the quartz wafer for the integrated measurement of tangential force and torque. The other stresses can be acquired through placing the rotated X0-cut or Y0-cut quartz wafer (such as being rotated 90 degrees). The study provides theory basis to the application of piezoelectric measurement system of multidimensional forces in some narrow space and greatly expands its application fields.

Acknowledgements

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