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ON A CLASSIFICATION OF WEAK DISCONTINUITIES IN MICROPOLAR THERMOELASTICITY

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Abstract. The present study is devoted to problem of propagating surfaces of weak discontinuities of translational displacements, microrotations and temperature in type-I micropolar (MP) thermoelastic (TE) continuum. Geometrical and kinematical compatibility conditions due to Hadamard and Thomas are used to study possible wave surfaces of weak discontinuities in MPTE-I continua. Weak discontinuities are discriminated according to spatial orientations of the discontinuities polarization vectors (DPVs). It is shown that the surfaces of weak discontinuities can propagate exist without weak discontinuities of the temperature field.

1. Preliminary remarks

A notion of micropolar continua takes its origin from the classical E. & F. Cosserat paper [1]. Micropolar continuum theories include not only translational displacements but also additional degrees of freedom. These degrees of freedom are due to changes of a trihedron associated with microvolume. In contrary to conventional elasticity a continuum with microstucture is described by the asymmetric strain and stress tensors known from many previous discussions. Thus the asymmetric elasticity theory is characterized by a comparatively large number of constitutive elastic constants need to be determined from the experimental observations. There are several phenomena (for example, the anomalous piezoelectric effect in quartz, the dispersion of elastic waves, as well as a number of other experimentally observed elastic properties of the pure crystals) being beyond the scope of the conventional thermoelasticity (CTE) and piezoelectroelasticity. That is why a development of complex theories seems to be actual.

Type-I micropolar thermoelastic (MPTE-I) continuum may be described from viewpoint of the Green-Naghdi thermoelasticity (GN-theory). Now such mathematical frameworks of the thermoelastic behavior of solids are rapidly refined [2, 3]. They are based on different modifications of the classical Fourier law of heat conduction. The refinements aim at derivations of *hyperbolic* partial differential equations of coupled thermoelasticity. Those are to simultaneously fulfill the following conditions: 1.) Finiteness of the heat signal propagation velocity, and 2.) Spatial propagation of the thermoelastic waves without attenuation, and 3.) Existence of distortionless wave forms akin to the classical d'Alembert type waves.

In-depth study of plane harmonic type-I thermoelastic waves is given in [4]. It is shown that dispersion equation has exactly two complex wavenumbers for a given frequency.

Moreover their real and imaginary parts are strictly positive. In [4] the linear symmetrical thermoelasticity is employed.

2. Gaverning equations of the MPTE-I continuum

The system of coupled partial differential equations of motion and heat conduction for a linear isotropic type-I micropolar thermoelastic continuum in the absence of mass forces, moments, and heat sources can be written as [5]

$$\begin{cases} (\lambda+\mu-\eta)\nabla\nabla\cdot\mathbf{u}+(\mu+\eta)\nabla\cdot\nabla\mathbf{u}+2\eta\nabla\times\boldsymbol{\varphi}-\alpha\nabla\theta-\rho\ddot{\mathbf{u}}=\mathbf{0}, \\ (\beta+\gamma-\varepsilon)\nabla\nabla\cdot\boldsymbol{\varphi}+(\gamma+\varepsilon)\nabla\cdot\nabla\boldsymbol{\varphi}-4\eta\boldsymbol{\varphi}+2\eta\nabla\times\mathbf{u}-\varsigma\nabla\mathbf{u}-\Im\ddot{\boldsymbol{\varphi}}=\mathbf{0}, \\ \nabla^2\theta-\varsigma\Lambda_*^{-1}\dot{\theta}-\varsigma\Lambda_*^{-1}\nabla\cdot\dot{\mathbf{u}}-\varsigma\Lambda_*^{-1}\nabla\cdot\dot{\boldsymbol{\varphi}}=0. \end{cases} \tag{1}$$

Hereafter $\mathbf{u}$ is the translational displacements; $\boldsymbol{\varphi}$ – the microrotations; θ – the temperature increment over the referential temperature; ρ – the mass density; $\Im$ – the microinertia; ∇ – the three-dimensional Hamiltonian operator (the nabla symbol); dot over a symbol denotes partial differentiation with respect to time at fixed spatial coordinates; $\lambda,\mu,\eta,\beta,\gamma,\varepsilon$ are isothermal constitutive constants of MPTE-I continuum; α,ς are constitutive constants providing coupling of equations of motion and heat conduction; κ is the heat capacity (per unit volume) at constant (zero) strains; Λ_* is the thermal conductivity. Constants α,ς depend not only on the mechanical properties of the continuum, but also depend on the thermal properties.

3. Weak discontinuities in the MPTE-I continuum

System of partial differential equations (1) includes partial derivative of the order not higher than the second. Let a wave surface Σ of weak discontinuities translational displacements $\mathbf{u}$, microrotations $\boldsymbol{\varphi}$ and temperature θ be propagating with normal velocity G in three-dimensional space.

Kinematical and geometrical compatibility conditions of the second order due to Hadamard and Thomas read

$$\begin{aligned} &[\nabla\otimes\nabla\otimes\mathbf{u}]=\mathbf{n}\otimes\mathbf{n}\otimes\mathbf{A}, && [\nabla\otimes\nabla\otimes\boldsymbol{\varphi}]=\mathbf{n}\otimes\mathbf{n}\otimes\mathbf{S}, \\ &[\nabla\otimes\dot{\mathbf{u}}]=-G\mathbf{n}\otimes\mathbf{A}, && [\nabla\otimes\dot{\boldsymbol{\varphi}}]=-G\mathbf{n}\otimes\mathbf{S}, \\ &[\ddot{\mathbf{u}}]=G^2\mathbf{A}, && [\ddot{\boldsymbol{\varphi}}]=G^2\mathbf{S}, \\ &[\nabla\otimes\nabla\theta]=B\mathbf{n}\otimes\mathbf{n}, \end{aligned} \tag{2}$$

where square brackets denote jump across surface of weak discontinuities. B, $\mathbf{A}$, $\mathbf{S}$ are fields defined on this surface. $\mathbf{A}$ and $\mathbf{S}$ are the DPVs of translational displacements and microrotations respectively. The equalities $B=0$, $\mathbf{A}=\mathbf{0}$, $\mathbf{S}=\mathbf{0}$ cannot be satisfied simultaneously at any point of the surface, if the surface Σ in fact is the surface of weak discontinuities.

Equations (1) and (2) give the following relations between the DPVs:

$$\begin{cases} (\rho G^2-(\mu+\eta))\mathbf{A}-(\lambda+\mu-\eta)\mathbf{n}(\mathbf{n}\cdot\mathbf{A})=\mathbf{0}, \\ (\rho\Im^2-(\gamma+\varepsilon))\mathbf{S}-(\beta+\gamma-\varepsilon)\mathbf{n}(\mathbf{n}\cdot\mathbf{S})=\mathbf{0}, \\ B+\alpha G\Lambda_*^{-1}\mathbf{n}\cdot\mathbf{A}+\varsigma G\Lambda_*^{-1}\mathbf{n}\cdot\mathbf{S}=\mathbf{0}. \end{cases} \tag{3}$$

The DPVs $\mathbf{A}$, $\mathbf{S}$ can be decomposed into sums of projections onto the tangent plane and on the normal direction to the wave surface:

$$\mathbf{A} = A_{\perp}\boldsymbol{\tau} + A_{\parallel}\mathbf{n}, \quad \mathbf{S} = S_{\perp}\boldsymbol{\tau} + S_{\parallel}\mathbf{n},$$

$$A_{\perp} = \mathbf{A}\cdot\boldsymbol{\tau}, \quad A_{\parallel} = \mathbf{A}\cdot\mathbf{n}, \quad S_{\perp} = \mathbf{S}\cdot\boldsymbol{\tau}, \quad S_{\parallel} = \mathbf{S}\cdot\mathbf{n}, \tag{4}$$

where $\boldsymbol{\tau}$ is the tangential unit vector and $\mathbf{n}$ is the normal unit one respectively. Taking account of equations (4) the system (3) after rearrangements is transformed into

$$\begin{cases} (\rho G^2 - (\mu+\eta))A_{\perp} = 0, \\ (\rho G^2 - (\lambda+2\mu))A_{\parallel} = 0, \end{cases} \qquad \begin{cases} (\rho G^2 - (\gamma+\varepsilon))S_{\perp} = 0, \\ (\rho G^2 - (\beta+2\gamma))S_{\parallel} = 0. \end{cases} \tag{5}$$

4. A classification of weak discontinuities in the MPTE-I continuum

The 16 cases can be discriminated according to (5). These cases are gathered into the following Table 1. We proceed by considering the discriminated cases separately. For the case (I) $\mathbf{A} = \mathbf{0}$, $\mathbf{S} = \mathbf{0}$ and returning to (3) the scalar equation in (3) is satisfied identically, so the surface Σ is actually not a surface of weak discontinuities. In the case (II), the first equation of system (5) is valid only on a wave surface propagating with normal velocity $G = c_{\parallel} = \sqrt{(\lambda+2\mu)/\rho}$. In the case (III) a weak discontinuity of translational displacements exists only on the surface propagating with the velocity $G = c_{\perp}^{\mu} = \sqrt{(\mu+\eta)/\rho}$. The case (IV) implies existence of a weak discontinuity of microrotations. Then the fourth equation of system (5) is satisfied only on the surface of weak discontinuities propagating with normal velocity $G = c_{\perp}^{\mu\mu} = \sqrt{(\beta+2\gamma)/\rho}$. In the case (V), the third equation of the system (5) allows to compute the propagation velocity of a weak discontinuities of microrotations $G = c_{\perp}^{\mu\mu} = \sqrt{(\gamma+\varepsilon)/\rho}$.

As is it seen from the third equation in (3) a weak discontinuity of temperature is not associated with the tangential projections of polarization vectors of weak discontinuities $A_{\perp}$ and $S_{\perp}$.

In cases (VI), (XII), (XIV), (XVI) the weak discontinuities of temperature can be derived from third equation in (4). In these cases propagating longitudinal waves are possible, if normal projections of the DPVs $\mathbf{A}$ and $\mathbf{S}$ satisfy the equation

$$\alpha/\varsigma = -S_{\parallel}/A_{\parallel}$$

and simultaneously $B = 0$.

Table 1. Discriminated cases for DPVs and scalar intensity B.

No	Projections of DPV $\mathbf{A}$		Projections of DPV $\mathbf{S}$		Intensity B
I	$A_{\parallel} = 0$	$A_{\perp} = 0$	$S_{\parallel} = 0$	$S_{\perp} = 0$	$B = 0$
II	$A_{\parallel} = 0$	$A_{\perp} \neq 0$	$S_{\parallel} = 0$	$S_{\perp} = 0$	$B = 0$
III	$A_{\parallel} \neq 0$	$A_{\perp} = 0$	$S_{\parallel} = 0$	$S_{\perp} = 0$	$B = -\dfrac{\alpha G}{\Lambda_*}A_{\parallel}$
IV	$A_{\parallel} = 0$	$A_{\perp} = 0$	$S_{\parallel} = 0$	$S_{\perp} \neq 0$	$B = 0$
V	$A_{\parallel} = 0$	$A_{\perp} = 0$	$S_{\parallel} \neq 0$	$S_{\perp} = 0$	$B = -\dfrac{\varsigma G}{\Lambda_*}S_{\parallel}$
VI	$A_{\parallel} = 0$	$A_{\perp} \neq 0$	$S_{\parallel} = 0$	$S_{\perp} \neq 0$	$B = -\dfrac{\alpha G}{\Lambda_*}A_{\parallel} - \dfrac{\varsigma G}{\Lambda_*}S_{\parallel}$

VII	$A_{\parallel} \neq 0$	$A_{\perp} = 0$	$S_{\parallel} \neq 0$	$S_{\perp} = 0$	$B = 0$
VIII	$A_{\parallel} \neq 0$	$A_{\perp} \neq 0$	$S_{\parallel} = 0$	$S_{\perp} = 0$	$B = -\frac{\alpha G}{\Lambda_*} A_{\parallel}$
IX	$A_{\parallel} = 0$	$A_{\perp} = 0$	$S_{\parallel} \neq 0$	$S_{\perp} \neq 0$	$B = -\frac{\varsigma G}{\Lambda_*} S_{\parallel}$
X	$A_{\parallel} \neq 0$	$A_{\perp} = 0$	$S_{\parallel} = 0$	$S_{\perp} \neq 0$	$B = -\frac{\alpha G}{\Lambda_*} A_{\parallel}$
XI	$A_{\parallel} = 0$	$A_{\perp} \neq 0$	$S_{\parallel} = 0$	$S_{\perp} \neq 0$	$B = 0$
XII	$A_{\parallel} \neq 0$	$A_{\perp} = 0$	$S_{\parallel} \neq 0$	$S_{\perp} \neq 0$	$B = -\frac{\alpha G}{\Lambda_*} A_{\parallel} - \frac{\varsigma G}{\Lambda_*} S_{\parallel}$
XIII	$A_{\parallel} = 0$	$A_{\perp} \neq 0$	$S_{\parallel} \neq 0$	$S_{\perp} \neq 0$	$B = -\frac{\varsigma G}{\Lambda_*} S_{\parallel}$
XIV	$A_{\parallel} \neq 0$	$A_{\perp} \neq 0$	$S_{\parallel} \neq 0$	$S_{\perp} = 0$	$B = -\frac{\alpha G}{\Lambda_*} A_{\parallel} - \frac{\varsigma G}{\Lambda_*} S_{\parallel}$
XV	$A_{\parallel} \neq 0$	$A_{\perp} \neq 0$	$S_{\parallel} = 0$	$S_{\perp} \neq 0$	$B = -\frac{\alpha G}{\Lambda_*} A_{\parallel}$
XVI	$A_{\parallel} \neq 0$	$A_{\perp} \neq 0$	$S_{\parallel} \neq 0$	$S_{\perp} \neq 0$	$B = -\frac{\alpha G}{\Lambda_*} A_{\parallel} - \frac{\varsigma G}{\Lambda_*} S_{\parallel}$

In other cases propagating wave surfaces of weak discontinuities displacements, microrotations and temperature do not exist if the constitutive characteristics of the MPTE-I continuum do not satisfy the limitations as determined by (5).

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