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INFLUENCE OF POINT DEFECTS ON ULTRASONIC WAVES PROPAGATING IN THE THIN PLATE

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Abstract. In the linear formulation a two-dimensional self-consistent problem of the propagation of elastic (ultrasonic) waves in the plate, taking into account its interaction with point defects present in its material is provided. We study the effect of point defects on the dispersion laws of planar and bending elastic waves.

1. Introduction

Since the 80s of the last century, the effect of radiation (including laser) for materials is intensively studied. Theoretically and experimentally it has been shown that under the influence of the laser beam in the materials produced numerous point defects (vacancies, interstitials), created in the surface layer of the stress-strain state [1]. The surface layer is modeled by a thin elastic plate undergoing a planar or bending vibrations interacting with the point defects [2-6]. Two-dimensional equations of vibrations of plates obtained from the three-dimensional equations of elasticity theory by applying Kirchhoff hypotheses [7]. The question about how to obtain two-dimensional kinetic equations for point defects is usually passed over in silence.

For obtaining of two-dimensional kinetic equations, describing the behavior of point defects can use two ways. In the first case it can be assumed that the point defects changing along the thickness of the plate changes in a linear manner. However, experimentally confirm this hypothesis is impossible. Therefore, more correctly, in our opinion, proceed from the value of the number of point defects on the plate boundaries, which can be measured experimentally.

The aim of the paper is to pose and study the self-consistent problem of propagation of elastic waves in a plate with regard to their interaction with point defects present in its material.

2. Statement of the problem and the general equations

In three dimensions, the linear system of equations describing the above process is as follows:

$$\rho \frac{\partial^2 u_2}{\partial t^2} = \frac{\partial \sigma_{ik}}{\partial x_k}, \tag{1}$$

$$\frac{\partial n_1}{\partial t} = q_{01} + q_\varepsilon \varepsilon_{kk} + D_1 \Delta n_1 - \alpha_{11} n_1 - \alpha_{12} n_2 , \tag{2}$$

$$\frac{\partial n_2}{\partial t} = q_{02} + q_\varepsilon \varepsilon_{kk} + D_2 \Delta n_1 - \alpha_{11} n_1 - \alpha_{12} n_2 , \tag{3}$$

The definitions are conventional [1, 8], ε_{kk} – volumetric deformation, $\varepsilon_{kk} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$, ε_{ik} - deformations, q_{01} and q_{02} the rate of defects generation before the disturbances that without loss of generality, we can take $q_{01} = q_{02} = 0$.

Boundary conditions are the following:

$$z = h \qquad n_1^+ = n_1(x, y, h, t), \quad n_2^+ = n_2(x, y, h, t), \tag{4}$$

$$z = -h \qquad n_1^- = n_1(x, y, -h, t), \quad n_2^- = n_2(x, y, -h, t), \tag{5}$$

$$z = \pm h, \ \sigma_{32} = \sigma_{31} = \sigma_{33} = 0. \tag{6}$$

For the sake of simplicity, coordinates x_1, x_2, x_3 are changed to x, y, z respectively. By analogy with the thermoelasticity problems (for example [9]), for point defects we consider the following approximations:

$$n_1 = n_{11} + n_{12} z, \ n_2 = n_{21} + n_{22} z, \tag{7}$$

where $n_{11} = \frac{n_1^+ + n_1^-}{2}$, $n_{12} = \frac{n_1^+ - n_1^-}{2}$, $n_{21} = \frac{n_2^+ + n_2^-}{2}$, $n_{22} = \frac{n_2^+ - n_2^-}{2}$.

In Kirchhoff theory it is assumed that the movements of the plate have the following form:

$$u_1 = u - z\frac{\partial w}{\partial x}, \ u_2 = \mathrm{v} - z\frac{\partial w}{\partial y}, \ u_3 = w, \tag{8}$$

where functions u, v, w do not depend on z coordinate.

Similarly as it is done in the theory of plates and shells, we can obtain equations for planar vibrations of plates and shells. Equations for u, v и γ_i [10], have the following form:

$$\Delta_2 u + \theta \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - \alpha \frac{\partial \gamma_1}{\partial x} = C_1^{-2} \frac{\partial^2 u}{\partial t^2} \tag{9}$$

$$\Delta_2 \mathrm{v} + \theta \frac{\partial}{\partial xy}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - \alpha \frac{\partial \gamma_1}{\partial y} = C_1^{-2} \frac{\partial^2 \mathrm{v}}{\partial t^2} \tag{10}$$

For bending vibrations we have:

$$D\Delta^2 w + D\Delta_2 \gamma_2 + 2\rho h \frac{\partial^2 w}{\partial t^2} = 0 , \tag{11}$$

where

$$\Delta_2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad \theta = \frac{1+v}{1-v}, \ C_1^2 = \frac{E}{2(1+v)\rho}, \ \alpha = \frac{2\theta(1-2v)}{E}, \ \gamma_1 = d_1 n_{11} + d_2 n_{21}, \quad D = \frac{2Eh^3}{(1-v)^2},$$

$\gamma_2 = d_1 n_{12} + d_2 n_{22}$, $\Delta_2^2 \equiv \frac{\partial^4}{\partial x^4} + 2\frac{\partial^2}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}$, v is Poisson's ratio, E is Young's modulus.

Substitution of (18) to the first, third and fourth equations of system (16) lead us to the following system of algebraic equations with respect to arbitrary constants A, B, C

$$\begin{aligned}&\left(\omega^2-k^2C_l^2\right)A+i\alpha kC_l^2\left(d_1B+d_2C\right)=0,\\&\beta q_\varepsilon A+\left(\frac{\omega}{k}-iD_1k-i\frac{q_{11}}{k}q\right)B+i\frac{q_{12}}{k}C=0,\\&\beta q_\varepsilon A+i\frac{q_{21}}{k}B+\left(\frac{\omega}{k}-iD_2k-i\frac{q_{22}}{k}\right)=0.\end{aligned} \tag{19}$$

In (19) we have introduced the following definitions

$$\alpha=\frac{(1-\nu)(1-2\nu)}{E},\quad \beta=\frac{1-2\nu}{1-\nu}. \tag{20}$$

The condition that the determinant of the system (19) equals to zero leads us to the equations, which allows us to define the phase velocity of the wave:

$$\begin{aligned}&\frac{\omega^4}{k^4}-i\left(F_1+F_2\right)\frac{\omega^3}{k^4}-\left(C_l^2+\frac{F_1F_2}{k^2}-\frac{q_{12}q_{21}}{k^2}\right)\frac{\omega^2}{k^2}+iC_l^2\left[F_1+F_2-\alpha\beta q_\varepsilon\left(d_1+d_2\right)\right]\frac{\omega}{k^2}-\\&-\frac{C_l^2}{k^2}\left[q_{12}q_{21}-F_1F_2-\alpha\beta q_\varepsilon\left(q_{12}d_1+q_{21}d_2+d_2F_1+d_1F_2\right)\right]=0,\end{aligned} \tag{21}$$

where

$$F_1=D_1k^2+q_{11},\quad F_2=D_2k^2+q_{22}, \tag{22}$$

From the (21) follows, that elastic vibration and vacancy (internode) vibrations do not interact in three cases:

a) $\nu=0,5$ or b) $q_\varepsilon=0$ or c) $d_1=d_2=0.$ (23)

Taking into account (23) the equation (21) will look as follows

$$\left(\frac{\omega^2}{k^2}-C_l^2\right)\left[\omega^2-i\left(F_1+F_2\right)\omega-F_1F_2+q_{12}q_{21}\right]=0. \tag{24}$$

In case of the first two equalities in (23), when $q_\varepsilon=0$ or $\nu=0,5$ but $d_1\neq 0,\ d_2\neq 0$, means, that defects do not “feel” the wave and in a layer, as it follows from (24), propagate two waves. The first has velocity C_l, and the frequency of the other can be defined from the second multiplicand in (24). The second wave decays if

$$F_1F_2\geq q_{12}q_{21}. \tag{25}$$

In the inverse inequality (25) the oscillation amplitude will increase unlimitedly. In this case, the linear theory is not suitable, it is necessary to take into account the nonlinear terms in the equations (16).

When $d_1=d_2=0$, a $q_\varepsilon\neq 0$ и $\nu\neq 0,5$, then defect “feel” the wave, but the wave does not “feel” defects. In this case, the second multiplier in (24) describes the process Increasing or decreasing the number of all the aforementioned defects and remains valid

$$q_1-q>0. \tag{26}$$

Consider the general case when all the coefficients are different from zero, but the influence of defects on the elastic wave is weak.

For planar vibration plate we seek a solution in the form (18), we obtain the system of equations (19). The dispersion equation can be conveniently represented in the following form:

$$\omega^2 C_l^{-1} - k^2 = -k^2 \frac{\phi_1 + i\alpha\beta\omega\left(d_1 + d_2\right)}{\phi_2 + i\phi_3}, \tag{27}$$

where $\phi_1 = -\alpha\beta\left\lfloor \left(d_1 q_{12} + d_2 q_{21}\right) - F_1 d_2 - d_1 F_2 \right\rfloor$, $\phi_2 = -\omega_0^2 + D_1 D_2 k^2 + D_1 k^2 q_{22} - q_{11} k^2 D_2 + q_{11}\alpha_{22} + q_{12}\alpha_{21}$,

$\phi_3 = \omega\left\lfloor k^2\left(D_2 + D_1\right) + q_{11} + q_{22}\right\rfloor$.

The frequency is a complex value, i.e. $\omega = \omega_0 + \omega_1 + i\alpha_{11}$, ω_1 is the small disperse, α_1 is the small absorption coefficient, and ω_0 is the main frequency, $\omega_0 = kC_l$

$$\omega_1^2 = \omega_0^2\left\lfloor 1 - \frac{\phi_1\phi_2 - \alpha\beta\omega\phi_3\left(d_1 + d_2\right)}{\phi_2^2 + \phi_3^2}\right\rfloor, \quad \alpha_1 = -\frac{\alpha\beta\omega_0\phi_2\left(d_1 + d_2\right) - \phi_1\phi_3}{2\left(\phi_2^2 + \phi_3^2\right)}.$$

In the case when bending waves described by equations (17) propagate in the plate, the solution is sought in the form (18). Then the dispersion equation takes the form:

$$\frac{Dk^2}{2\rho h} - \omega^2 = -2k^4\beta D_* h\rho \frac{T_1 + i\omega\left(d_2 + d_1\right)}{T_2 + i\omega\left(F_1 + F_2\right)}. \tag{28}$$

If $q_{11} = q_{22} = D_1 = D_2 = q_{12} = q_{21} = d_1 = d_2 = 0$ then $\omega_0^2 = \dfrac{D}{2\rho h}k^2 = v_0 k^2$.

Here $T_1 = d_2 F_1 + d_1 F_2 - q_{21} d_2 - d_1 q_{12}$, $T_2 = F_1 F_2 - \alpha_0\omega^2 - q_{21} q_{12}$,

$$\omega_1^2 = \omega_0^2\left\lfloor 1 - \frac{k^2\beta D_*}{D}\frac{T_1 T_2 + \omega_0^2\left(d_1 + d_2\right)\left(F_1 + F_2\right)}{T_2^2 + \omega_0^2\left(F_1 + F_2\right)^2}\right\rfloor, \quad \alpha = \frac{k^4\beta D}{4\rho h}\frac{\left\lfloor T_2\left(d_1 + d_2\right) - T_1\left(F_1 + F_2\right)\right\rfloor}{\left[T_2^2 + \omega_0^2\left(F_1 + F_2\right)^2\right]}.$$

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