

STATE SPACE APPROACH TO PLANE DEFORMATION IN ELASTIC MATERIAL WITH DOUBLE POROSITY

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Abstract. The present paper is to focus on a boundary value problem in a homogeneous, isotropic, elastic body with double porosity. After developing mathematical formulation, a state space approach has been applied to investigate the problem. As an application of the approach, normal force has been taken to illustrate the utility of the approach. The expressions for the components of normal stress and equilibrated stress are obtained in the frequency domain and computed numerically. Numerical simulation is prepared for these quantities and simulated results for these quantities are depicted graphically for a particular model. A particular case of interest is also deduced from the present investigation.

1. Introduction

Porous media theories play an important role in many branches of engineering including material science, the petroleum industry, chemical engineering, biomechanics and other such fields of engineering. Representation of a fluid saturated porous medium as a single phase material has been virtually discarded. The material with the pore spaces such as concrete can be treated easily because all concrete ingredients have the same motion if the concrete body is deformed. However the situation is more complicated if the pores are filled with liquid and in that case the solid and liquid phases have different motions. Due to these different motions, the different material properties and the complicated geometry of pore structures, the mechanical behavior of a fluid saturated porous thermoelastic medium becomes very difficult. So researchers from time to time, have tried to overcome this difficulty and we see many porous media in the literature. A brief historical background of these theories is given by de Boer [1, 2].

As far as modern era is concerned, Biot [3] proposed a general theory of three-dimensional deformation of fluid saturated porous salts. Biot theory is based on the assumption of compressible constituents and till recently, some of his results have been taken as standard references and basis for subsequent analysis in acoustic, geophysics and other such fields. Another interesting theory is given by Bowen [4], de Boer and Ehlers [5] in which all the constituents of a porous medium are assumed to be incompressible. The fluid saturated porous material is modeled as a two phase system composed of an incompressible solid phase and incompressible fluid phase, thus meeting the many problems in engineering practice, e.g. in soil mechanics. One important generalization of Biot's theory of poroelasticity that has been studied extensively started with the works by Barenblatt et al. [6], where the double porosity model was first proposed to express the fluid flow in hydrocarbon reservoirs and aquifers.

The double porosity model represents a new possibility for the study of important

problems concerning the civil engineering. It is well-known that, under super-saturation conditions due to water or other fluid effects, the so-called neutral pressures generate unbearable stress states on the solid matrix and on the fracture faces, with severe (sometimes disastrous) instability effects like landslides, rock fall or soil fluidization (typical phenomenon connected with propagation of seismic waves). In such a context it seems possible, acting suitably on the boundary pressure state, to regulate the internal pressures in order to deactivate the noxious effects related to neutral pressures; finally, a further but connected positive effect could be lightening of the solid matrix/fluid system.

Wilson and Aifantis [7] presented the theory of consolidation with the double porosity. Khaled, Beskos, and Aifantis [8] employed a finite element method to consider the numerical solutions of the differential equation of the theory of consolidation with double porosity developed by Aifantis [7]. Wilson and Aifantis [9] discussed the propagation of acoustic waves in a fluid-saturated porous medium. The propagation of acoustic waves in a fluid-saturated porous medium containing a continuously distributed system of fractures is discussed. The porous medium is assumed to consist of two degrees of porosity and the resulting model thus yields three types of longitudinal waves, one associated with the elastic properties of the matrix material and one each for the fluids in the pore space and the fracture space.

Beskos and Aifantis [10] presented the theory of consolidation with double porosity-II and obtained the analytical solutions to two boundary value problems. Khalili and Valliappan [11] studied the unified theory of flow and deformation in double porous media. Aifantis introduced a multi-porous system and studied the mechanics of diffusion in solids [12-16]. Moutsopoulos et al. [17] obtained the numerical simulation of transport phenomena by using the double porosity / diffusivity continuum model. Pride and Berryman [18] studied the linear dynamics of double-porosity dual-permeability materials. Straughan [19] studied the stability and uniqueness in double porous elastic media. Svanadze [20-23] investigated some problems on elastic solids and viscoelastic solids with double porosity. Iesan and Quintanilla [39] presented the theory of thermoelastic materials with double porosity structure.

In recent years the state space description of linear systems has been used extensively in various areas of engineering, such as the analysis of control systems. The state space approach offers an attractive way to avoid the difficulties of the traditional linear model approach. The state-space representation is a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations. To abstract from the number of inputs, outputs and states, the variables are expressed as vectors. If the dynamical system is linear and time invariant, the differential and algebraic equations may be written in matrix form. The state-space representation provides a convenient and compact way to model and analyze systems with multiple inputs and outputs.

Bahar and Hetnarski investigated a good number of problems in thermoelasticity by using the state space approach [24-29]. Sharma [30] studied the one-dimensional problems in generalized theories of thermoelasticity subjected to heat source and body forces by using the state space approach. Othman et al. [31] established the model of the two-dimensional generalized thermo-viscoelasticity with two relaxation times and used normal mode analysis to obtain the exact expressions for the temperature distribution, thermal stresses and the displacement components. Ezzat et al. [32] applied the state space approach to generalized thermo-viscoelasticity with two relaxation times. Maghraby et al. [33] formulated the state space approach to the one-dimensional problem of thermoelasticity with two relaxation times. Youssef and Al-Lehaibi [34] considered a half-space filled with an elastic material and the state space approach is used to obtain the general solution for any set of boundary conditions. Youssef and Harby [35] considered an infinite elastic body with a spherical cavity and

$$x' = \frac{\omega}{c_1} x, u' = \frac{\omega}{c_1} u, \varphi' = \frac{k_1 \omega^2}{\alpha_1} \varphi, \psi' = \frac{k_1 \omega^2}{\alpha_1},$$

$$\sigma_1' = \frac{c_1}{\alpha \omega} \sigma_1, \tau_1' = \frac{c_1}{\alpha \omega} \tau_1, t' = \omega t, t'_{ij} = \frac{t_{ij}}{\lambda}, \quad (8)$$

where $c_1^2 = \frac{\lambda+2\mu}{\rho}$, $c = \rho c^*$, $\omega = \frac{\lambda}{k_1}$. Here ω is the constant having the dimension of frequency and c_1 is the velocity in the medium.

Making use of dimensionless quantities given in (8) in equations (4)-(6) and with the aid of (7), we get

$$\frac{\partial^2 u}{\partial x^2} + \delta_1 \frac{\partial \varphi}{\partial x} + \delta_2 \frac{\partial \psi}{\partial x} = \frac{\partial^2 u}{\partial t^2}, \quad (9)$$

$$\delta_3 \frac{\partial^2 \varphi}{\partial x^2} + \delta_4 \frac{\partial^2 \psi}{\partial x^2} - \delta_5 \frac{\partial u}{\partial x} - \delta_6 \varphi - \delta_7 \psi = \frac{\partial^2 \varphi}{\partial t^2}, \quad (10)$$

$$\delta_8 \frac{\partial^2 \varphi}{\partial x^2} + \delta_9 \frac{\partial^2 \psi}{\partial x^2} - \delta_{10} \frac{\partial u}{\partial x} - \delta_{11} \varphi - \delta_{12} \psi = \frac{\partial^2 \psi}{\partial t^2}, \quad (11)$$

where

$$\delta_1 = \frac{b\alpha_1}{\rho c_1^2 k_1 \omega^2}, \quad \delta_2 = \frac{d\alpha_1}{\rho c_1^2 k_1 \omega^2}, \quad \delta_3 = \frac{\alpha}{c_1^2 k_1}, \quad \delta_4 = \frac{b_1}{c_1^2 k_1}, \quad \delta_5 = \frac{b}{\alpha_1}, \quad \delta_6 = \frac{\alpha_1}{k_1 \omega^2}, \quad \delta_7 = \frac{\alpha_3}{k_1 \omega^2},$$

$$\delta_8 = \frac{b_1}{c_1^2 k_2}, \quad \delta_9 = \frac{\gamma}{c_1^2 k_2}, \quad \delta_{10} = \frac{dk_1}{\alpha_1 k_2}, \quad \delta_{11} = \frac{\alpha_3}{k_2 \omega^2}, \quad \delta_{12} = \frac{\alpha_2}{k_2 \omega^2}. \quad (12)$$

Assuming the time harmonic solution of the equations (9)-(11) as,

$$(u(x, t), \varphi(x, t), \psi(x, t)) = (\bar{u}, \bar{\varphi}, \bar{\psi}) e^{-i\omega t}, \quad (13)$$

where ω is the frequency.

Equations (9)-(11) with the aid of equation (13) yield,

$$\bar{u}_{,11} = N_1 \bar{u} + N_2 \bar{\varphi}_{,1} + N_3 \bar{\psi}_{,1}, \quad (14)$$

$$\bar{\varphi}_{,11} = M_1 \bar{\psi}_{,11} + M_2 \bar{u}_{,1} + M_3 \bar{\varphi} + M_4 \bar{\psi}, \quad (15)$$

$$\bar{\psi}_{,11} = M_5 \bar{\varphi}_{,11} + M_6 \bar{u}_{,1} + M_7 \bar{\varphi} + M_8 \bar{\psi}, \quad (16)$$

After using (16) in (15) and rewriting equation (14) and the resulting equations, we get

$$\bar{u}_{,11} = N_1 \bar{u} + N_2 \bar{\varphi}_{,1} + N_3 \bar{\psi}_{,1}, \quad (17)$$

$$\bar{\varphi}_{,11} = N_4 \bar{u}_{,1} + N_5 \bar{\varphi} + N_6 \bar{\psi}, \quad (18)$$

$$\bar{\psi}_{,11} = N_7 \bar{u}_{,1} + N_8 \bar{\varphi} + N_9 \bar{\psi}, \quad (19)$$

where

$$N_1 = -\omega^2, \quad N_2 = -\delta_1, \quad N_3 = -\delta_2, \quad M_1 = \frac{-\delta_4}{\delta_3}, \quad M_2 = \frac{\delta_5}{\delta_3}, \quad M_3 = \frac{\delta_6 - \omega^2}{\delta_3}, \quad M_4 = \frac{\delta_7}{\delta_3},$$

$$M_5 = \frac{-\delta_8}{\delta_9}, \quad M_6 = \frac{\delta_{10}}{\delta_9}, \quad M_7 = \frac{\delta_{11}}{\delta_9}, \quad M_8 = \frac{\delta_{12} - \omega^2}{\delta_9}, \quad M_9 = 1 - M_1 M_5, \quad N_4 = \frac{M_1 M_6 + M_2}{M_9},$$

$$N_5 = \frac{M_1 M_7 + M_3}{M_9}, \quad N_6 = \frac{M_1 M_8 + M_4}{M_9}, \quad N_7 = M_5 N_4 + M_6, \quad N_8 = M_5 N_5 + M_7, \quad N_9 = M_5 N_6 + M_8 \quad (20)$$

4. State - space formulation

Choosing as a state variable displacement \bar{u} , volume fraction $\bar{\varphi}$ and $\bar{\psi}$ in the x - direction,

5. Boundary conditions

A homogeneous isotropic elastic solid with double porosity structure occupying the region $0 \leq x < \infty$ is considered. The bounding plane $x = 0$ subjected to normal force. Mathematically these can be written as,

$$(i) \quad t_{11} = -F_1 \exp[-i\omega t], \quad (32)$$

$$(ii) \quad \sigma_1 = -F_2 \exp[-i\omega t], \quad (33)$$

$$(iii) \quad \tau_1 = -F_3 \exp[-i\omega t], \quad (34)$$

where $F_i; i = 1,2,3$ is the magnitude of the force applied on the boundary.

Substituting the values of \bar{u} , $\bar{\varphi}$, $\bar{\psi}$ from the equation (31) in the boundary condition (32)-(34) and with the aid of equations (1)-(3), (7), (8) and (13), we obtain

$$\begin{bmatrix} Q_1 & Q_2 & Q_3 \\ Q_4 & Q_5 & Q_6 \\ Q_7 & Q_8 & Q_9 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} -F_1 \\ -F_2 \\ -F_3 \end{bmatrix}. \quad (35)$$

The values of Q_1, Q_2, \dots, Q_9 are given in the appendix I.

Solving (35) for A_1, A_2, A_3 and substituting the resulting values in equation (30) yield the value of normal stress and the components of equilibrated stress,

$$t_{11} = (S_1 \frac{\Gamma_1}{\Gamma} + S_2 \frac{\Gamma_2}{\Gamma} + S_3 \frac{\Gamma_3}{\Gamma}) e^{-i\omega t}, \quad (36)$$

$$\sigma_1 = (S_4 \frac{\Gamma_1}{\Gamma} + S_5 \frac{\Gamma_2}{\Gamma} + S_6 \frac{\Gamma_3}{\Gamma}) e^{-i\omega t}, \quad (37)$$

$$\tau_1 = (S_7 \frac{\Gamma_1}{\Gamma} + S_8 \frac{\Gamma_2}{\Gamma} + S_9 \frac{\Gamma_3}{\Gamma}) e^{-i\omega t}. \quad (38)$$

6. Particular case

If $F_3 = 0$ in equation (36)-(38), yields the corresponding expressions for elastic with primary porous material.

7. Numerical results and discussion

With the view of illustrating theoretical results obtained in the preceding sections and to compare these in the context of above cases, we now represent some numerical results for copper material (elastic solid), the physical data for which is given below (2005):

$$\lambda = 7.76 \times 10^{10} Nm^{-2}, \quad \mu = 3.86 \times 10^{10} Nm^{-2}, \quad \alpha = 1.78 \times 10^{-5} K^{-1}, \quad \omega = 1 \times 10^{11} s^{-1}, \\ t = 0.1s, \quad \rho = 8.954 \times 10^3 Kgm^{-3}.$$

The double porous parameters are taken as

$$\alpha_2 = 1.96 \times 10^{10} Nm^{-2}, \quad \alpha_3 = 1.86 \times 10^{10} Nm^{-2}, \quad \gamma = 0.19 \times 10^{-5} N, \quad b_1 = 0.12 \times 10^{-5} N, \\ d = 0.49 \times 10^{10} Nm^{-2}, \quad k_1 = 0.1456 \times 10^{-12} Nm^{-2} s^2, \quad b = 0.4 \times 10^{10} Nm^{-2}, \\ \alpha_1 = 1.65 \times 10^{10} Nm^{-2}, \quad k_2 = 0.1546 \times 10^{-12} Nm^{-2} s^2.$$

The software Matlab R2010a has been used to determine the values of normal stress and equilibrated stresses. The variation of these values with respect to distance x have been shown in Figs. (1)-(3) respectively. In Figures 1 and 2, the comparisons for elastic with double porous material (DPM) and elastic with primary porous material (SPM) are represented graphically. Figure 3 depicts the variation of equilibrated stress τ_1 with distance x for porous parameter b_1 in case of elastic with double porous material.

Appendix I

$$A(x, w) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ N_1 & 0 & 0 & 0 & N_2 & N_3 \\ 0 & N_5 & N_6 & N_4 & 0 & 0 \\ 0 & N_8 & N_9 & N_7 & 0 & 0 \end{bmatrix}, V(x, w) = \begin{bmatrix} \bar{u}(x, w) \\ \bar{\varphi}(x, w) \\ \bar{\psi}(x, w) \\ (\bar{u}(x, w))_{,1} \\ (\bar{\varphi}(x, w))_{,1} \\ (\bar{\psi}(x, w))_{,1} \end{bmatrix}, V(0, w) = \begin{bmatrix} \bar{u}(0w) \\ \bar{\varphi}(0, w) \\ \bar{\psi}(0, w) \\ (\bar{u}(0, w))_{,1} \\ (\bar{\varphi}(0, w))_{,1} \\ (\bar{\psi}(0, w))_{,1} \end{bmatrix},$$

$$a_0 = e^{-\lambda_1 x} \left[\frac{\lambda_2 \lambda_3}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)} \right] + e^{-\lambda_2 x} \left[\frac{\lambda_1 \lambda_3}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)} \right] + e^{-\lambda_3 x} \left[\frac{\lambda_1 \lambda_2}{(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)} \right],$$

$$a_1 = e^{-\lambda_1 x} \left[\frac{(\lambda_2 + \lambda_3)}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)} \right] + e^{-\lambda_2 x} \left[\frac{(\lambda_1 + \lambda_3)}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)} \right] + e^{-\lambda_3 x} \left[\frac{(\lambda_1 + \lambda_2)}{(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)} \right],$$

$$a_2 = e^{-\lambda_1 x} \left[\frac{1}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)} \right] + e^{-\lambda_2 x} \left[\frac{1}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)} \right] + e^{-\lambda_3 x} \left[\frac{1}{(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)} \right],$$

$$Q_1 = P_1(Z_1 + N_1 Z_3), Q_2 = P_2(a_0^0 + a_2^0 N_5) + P_3(a_2^0 N_8), Q_3 = P_2(a_2^0 N_6) + P_3(a_0^0 + a_2^0 N_9),$$

$$Q_4 = 0, Q_5 = P_4(Z_1 + N_5 Z_3) + P_5 N_8 Z_3, Q_6 = P_4 N_6 Z_3 + P_5(Z_1 + N_9 Z_3),$$

$$Q_7 = 0, Q_8 = P_5(Z_1 + N_5 Z_3) + P_6 N_8 Z_3, Q_9 = P_5 N_6 Z_3 + P_6(Z_1 + N_9 Z_3),$$

where

$$P_1 = \frac{\lambda + 2\mu}{\lambda}, P_2 = \frac{b\alpha_1}{\lambda k_1 \omega^2}, P_3 = \frac{d\alpha_1}{\lambda k_1 \omega^2}, P_4 = \frac{\alpha_1}{k_1 \omega^2}, P_5 = \frac{b_1 \alpha_1}{\alpha k_1 \omega^2}, P_6 = \frac{\gamma \alpha_1}{\alpha k_1 \omega^2},$$

$$Z_1 = -\lambda_1 D_{11} - \lambda_2 D_{12} - \lambda_3 D_{13}, Z_2 = -\lambda_1 D_{21} - \lambda_2 D_{22} - \lambda_3 D_{23}, Z_3 = -\lambda_1 D_{31} - \lambda_2 D_{32} - \lambda_3 D_{33},$$

$$Y_1 = -\lambda_1 D_{11} e^{-\lambda_1 x} - \lambda_2 D_{12} e^{-\lambda_2 x} - \lambda_3 D_{13} e^{-\lambda_3 x}, Y_2 = -\lambda_1 D_{11} e^{-\lambda_1 x} - \lambda_2 D_{12} e^{-\lambda_2 x} - \lambda_3 D_{13} e^{-\lambda_3 x},$$

$$Y_3 = -\lambda_1 D_{21} e^{-\lambda_1 x} - \lambda_2 D_{22} e^{-\lambda_2 x} - \lambda_3 D_{23} e^{-\lambda_3 x}, S_1 = P_1(Y_1 + N_1 Y_3), S_2 = 0, S_3 = 0,$$

$$S_4 = 0, S_5 = P_4(Y_1 + N_5 Y_3) + P_5 N_8 Y_3, S_6 = P_4 N_6 Y_3 + P_5(Y_1 + N_9 Y_3), S_7 = 0,$$

$$S_8 = P_5(Y_1 + N_5 Y_3) + P_6 N_8 Y_3, S_9 = P_5 N_6 Y_3 + P_6(Y_1 + N_9 Y_3),$$

$$D_{11} = \frac{\lambda_2 \lambda_3}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)}, D_{12} = \frac{\lambda_1 \lambda_3}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)}, D_{13} = \frac{\lambda_1 \lambda_2}{(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)},$$

$$D_{31} = \frac{1}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)}, D_{32} = \frac{1}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)}, D_{33} = -\frac{1}{(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)},$$

$$\Gamma = \begin{bmatrix} Q_1 & Q_2 & Q_3 \\ Q_4 & Q_5 & Q_6 \\ Q_7 & Q_8 & Q_9 \end{bmatrix}, \Gamma_1 = \begin{bmatrix} -F_1 & Q_2 & Q_3 \\ -F_2 & Q_5 & Q_6 \\ -F_3 & Q_8 & Q_9 \end{bmatrix}, \Gamma_2 = \begin{bmatrix} Q_1 & -F_1 & Q_3 \\ Q_4 & -F_2 & Q_6 \\ Q_7 & -F_3 & Q_9 \end{bmatrix}, \Gamma_3 = \begin{bmatrix} Q_1 & Q_2 & -F_1 \\ Q_4 & Q_5 & -F_2 \\ Q_7 & Q_8 & -F_3 \end{bmatrix},$$

$$\text{and } a_0^0 = a_0, a_2^0 = a_2, \text{ at } x = 0, A_1 = \frac{\Gamma_1}{\Gamma}, A_2 = \frac{\Gamma_2}{\Gamma}, A_3 = \frac{\Gamma_3}{\Gamma}.$$

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