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STATE SPACE APPROACH TO PLANE DEFORMATION IN ELASTIC MATERIAL WITH DOUBLE POROSITY

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Abstract. The present paper is to focus on a boundary value problem in a homogeneous, isotropic, elastic body with double porosity. After developing mathematical formulation, a state space approach has been applied to investigate the problem. As an application of the approach, normal force has been taken to illustrate the utility of the approach. The expressions for the components of normal stress and equilibrated stress are obtained in the frequency domain and computed numerically. Numerical simulation is prepared for these quantities and simulated results for these quantities are depicted graphically for a particular model. A particular case of interest is also deduced from the present investigation.

1. Introduction

Porous media theories play an important role in many branches of engineering including material science, the petroleum industry, chemical engineering, biomechanics and other such fields of engineering. Representation of a fluid saturated porous medium as a single phase material has been virtually discarded. The material with the pore spaces such as concrete can be treated easily because all concrete ingredients have the same motion if the concrete body is deformed. However the situation is more complicated if the pores are filled with liquid and in that case the solid and liquid phases have different motions. Due to these different motions, the different material properties and the complicated geometry of pore structures, the mechanical behavior of a fluid saturated porous thermoelastic medium becomes very difficult. So researchers from time to time, have tried to overcome this difficulty and we see many porous media in the literature. A brief historical background of these theories is given by de Boer [1, 2].

As far as modern era is concerned, Biot [3] proposed a general theory of three-dimensional deformation of fluid saturated porous salts. Biot theory is based on the assumption of compressible constituents and till recently, some of his results have been taken as standard references and basis for subsequent analysis in acoustic, geophysics and other such fields. Another interesting theory is given by Bowen [4], de Boer and Ehlers [5] in which all the constituents of a porous medium are assumed to be incompressible. The fluid saturated porous material is modeled as a two phase system composed of an incompressible solid phase and incompressible fluid phase, thus meeting the many problems in engineering practice, e.g. in soil mechanics. One important generalization of Biot's theory of poroelasticity that has been studied extensively started with the works by Barenblatt et al. [6], where the double porosity model was first proposed to express the fluid flow in hydrocarbon reservoirs and aquifers.

The double porosity model represents a new possibility for the study of important problems concerning the civil engineering. It is well-known that, under super- saturation conditions due to water of other fluid effects, the so called neutral pressures generate unbearable stress states on the solid matrix and on the fracture faces, with severe (sometimes disastrous) instability effects like landslides, rock fall or soil fluidization (typical phenomenon connected with propagation of seismic waves). In such a context it seems possible, acting suitably on the boundary pressure state, to regulate the internal pressures in order to deactivate the noxious effects related to neutral pressures; finally, a further but connected positive effect could be lightening of the solid matrix/fluid system.

Wilson and Aifanits [7] presented the theory of consolidation with the double porosity. Khaled, Beskos, and Aifantis [8] employed a finite element method to consider the numerical solutions of the differential equation of the theory of consolidation with double porosity developed by Aifantis [7]. Wilson and Aifantis [9] discussed the propagation of acoustics waves in a fluid fluid saturated porous medium. The propagation of acoustic waves in a fluid-saturated porous medium containing a continuously distributed system of fractures is discussed. The porous medium is assumed to consist of two degrees of porosity and the resulting model thus yields three types of longitudinal waves, one associated with the elastic properties of the matrix material and one each for the fluids in the pore space and the fracture space.

Beskos and Aifantis [10] presented the theory of consolidation with double porosity-II and obtained the analytical solutions to two boundary value problems. Khalili and Valliappan [11] studied the unified theory of flow and deformation in double porous media. Aifantis introduced a multi-porous system and studied the mechanics of diffusion in solids [12-16]. Moutsopoulos et al. [17] obtained the numerical simulation of transport phenomena by using the double porosity / diffusivity continuum model. Pride and Berryman [18] studied the linear dynamics of double –porosity dual-permeability materials. Straughan [19] studied the stability and uniqueness in double porous elastic media. Svanadze [20-23] investigated some problems on elastic solids and viscoelastic solids with double porosity. Iesan and Quintanilla [39] presented the theory of thermoelastic materials with double porosity structure.

In recent years the state space description of linear systems has been used extensively in various areas of engineering, such as the analysis of control systems. The state space approach offers an attractive way to avoid the difficulties of the traditional linear model approach. The state –space representation is a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations. To abstract from the number of inputs, outputs and states, the variables are expressed as vectors. If the dynamical system is linear and time invariant, the differential and algebraic equations may be written in matrix form. The state-space representation provides a convenient and compact way to model and analyze systems with multiple inputs and outputs.

Bahar and Hetnarski investigated good number of problems in thermoelasticity by using state space approach [24-29]. Sharma [30] studied the one dimensional problems in generalized theories of thermoelasticity subjected to heat source and body forces by using state space approach. Othman et al. [31] established the model of the two-dimensional generalized thermo-viscoelasticity with two relaxation times and used normal mode analysis to obtain the exact expressions for the temperature distribution, thermal stresses and the displacement components. Ezzat et al. [32] applied state space approach to generalized thermo-viscoelasticity with two relaxation times. Maghraby et al. [33] formulated the state space approach to the one-dimensional problem of thermoelasticity with two relaxation times. Youssef and Al-Lehaibi [34] considered a half-space filled with an elastic material and state space approach are used to obtain the general solution for any set of boundary conditions. Youssef and Harby [35] considered an infinite elastic body with a spherical cavity and constant elastic parameters and used state space techniques to obtain the general solution for any set of boundary conditions. Othman [36] studied a two-dimensional equation of generalized thermoelasticity with one relaxation time in an isotropic elastic medium with the elastic modulus dependent on temperature and with an internal heat source. He used State space approach to find the expressions for displacements, temperature and stresses. Elisbai and Youseff [37] investigated a generalized solution for the vibration of gold nano-beam problem by using State space approach induced by ramp type heating without energy dissipation in femtoseconds scale. Sherief and El-sayed [38] applied the state space approach to two–dimensional generalized micropolar thermoelasticity.

In the present paper, we shall formulate the state space approach to boundary value problem for elastic material with double porosity. The expressions for normal stress and equilibrated stresses are obtained in closed form, computed numerically and represented graphically for normal force . The comparisons are made in case of elastic with double porous material and elastic with primary porous material.

2. Basic equations

Following Iesan and Quintanilla [39], the constitutive relations and field equations for homogeneous elastic material with double porosity structure without body force and extrinsic equilibrated body forces can be written as:

Constitutive Relations:

$$t_{ij} = \lambda e_{rr}\delta_{ij} + 2\mu e_{ij} + b\delta_{ij}\varphi + d\delta_{ij}\psi, \tag{1}$$

$$\sigma_i = \alpha\varphi_{,i} + b_1\psi_{,i}\ , \tag{2}$$

$$\tau_i = b_1\varphi_{,i} + \gamma\psi_{,i}\ ; \tag{3}$$

Equation of motion:

$$\mu\Delta u_i + (\lambda+\mu)u_{j,ji} + b\varphi_{,i} + d\psi_{,i} = \rho\ddot{u}_i\ , \tag{4}$$

Equilibrated Stress Equations of motion:

$$\alpha\Delta\varphi + b_1\Delta\psi - bu_{r,r} - \alpha_1\varphi - \alpha_3\psi = \kappa_1\ddot{\varphi}\ , \tag{5}$$

$$b_1\Delta\varphi + \gamma\Delta\psi - du_{r,r} - \alpha_3\varphi - \alpha_2\psi = \kappa_2\ddot{\psi}\ , \tag{6}$$

where λ and μ are Lame's constants, ρ is the mass density; u_i (i=1,2,3) is the displacement components; t_{ij} is the stress tensor; $\kappa_1$and κ_2 are coefficients of equilibrated inertia; ν_1 is the volume fraction field corresponding to pores and ν_2 is the volume fraction field corresponding to fissures φ and ψ are the volume fraction fields corresponding to ν_1 and ν_2 respectively; σ_1 is the equilibrated stress corresponding to ν_1; τ_1 is the equilibrated stress corresponding to ν_2; and $b, d, b_1, \alpha, \gamma$ are constitutive coefficients; δ_{ij} is the Kronecker's delta; Δ is the Laplacian operator; a superposed dot represents differentiation with respect to time variable t.

3. Formulation of the problem

Now, we consider a homogeneous, isotropic elastic solid with double porosity structure occupying the region $0 \le x < \infty$ whose state variable depend only on the space variables distance x and time t, for which the displacement component u_i, volume fraction φ and ψ will be taken as,

$$u_i = u(x,t), \quad \varphi(x,t), \quad \psi(x,t)\ . \tag{7}$$

We define the dimensionless quantities

$$x' = \frac{\omega}{c_1}x, u' = \frac{\omega}{c_1}u\ ,\ \varphi' = \frac{k_1\omega^2}{\alpha_1}\varphi, \psi' = \frac{k_1\omega^2}{\alpha_1},$$

$$\sigma_1' = \frac{c_1}{\alpha\omega}\sigma_1,\quad \tau_1' = \frac{c_1}{\alpha\omega}\tau_1,\quad t' = \omega t,\quad t_{ij}' = \frac{t_{ij}}{\lambda}, \tag{8}$$

where $c_1^2 = \frac{\lambda+2\mu}{\rho},\ c = \rho c^*,\ \omega = \frac{\lambda}{k_1}$. Here ω is the constant having the dimension of frequency and c_1 is the velocity in the medium.
Making use of dimensionless quantities given in (8) in equations (4)-(6) and with the aid of (7), we get

$$\frac{\partial^2 u}{\partial x^2} + \delta_1\frac{\partial\varphi}{\partial x} + \delta_2\frac{\partial\psi}{\partial x} = \frac{\partial^2 u}{\partial t^2}, \tag{9}$$

$$\delta_3\frac{\partial^2\varphi}{\partial x^2} + \delta_4\frac{\partial^2\psi}{\partial x^2} - \delta_5\frac{\partial u}{\partial x} - \delta_6\varphi - \delta_7\psi = \frac{\partial^2\varphi}{\partial t^2}, \tag{10}$$

$$\delta_8\frac{\partial^2\varphi}{\partial x^2} + \delta_9\frac{\partial^2\psi}{\partial x^2} - \delta_{10}\frac{\partial u}{\partial x} - \delta_{11}\varphi - \delta_{12}\psi = \frac{\partial^2\psi}{\partial t^2}, \tag{11}$$

where

$$\delta_1 = \frac{\mathrm{b}\alpha_1}{\rho C_1^2 k_1\omega^2},\quad \delta_2 = \frac{\mathrm{d}\alpha_1}{\rho C_1^2 k_1\omega^2},\quad \delta_3 = \frac{\alpha}{C_1^2 k_1},\ \delta_4 = \frac{b_1}{C_1^2 k_1},\ \delta_5 = \frac{\mathrm{b}}{\alpha_1},\ \delta_6 = \frac{\alpha_1}{k_1\omega^2},\ \delta_7 = \frac{\alpha_3}{k_1\omega^2},$$

$$\delta_8 = \frac{b_1}{C_1^2 k_2},\ \delta_9 = \frac{\gamma}{C_1^2 k_2},\ \delta_{10} = \frac{dk_1}{\alpha_1 k_2},\ \delta_{11} = \frac{\alpha_3}{k_2\omega^2},\ \delta_{12} = \frac{\alpha_2}{k_2\omega^2}\,. \tag{12}$$

Assuming the time harmonic solution of the equations (9)-(11) as,

$$\big(u(x,t), \varphi(x,t), \psi(x,t)\big) = (\bar{u}, \bar{\varphi}, \bar{\psi})e^{-i\omega t}, \tag{13}$$

where ω is the frequency.
Equations (9)-(11) with the aid of equation (13) yield,

$$\bar{u}_{,11} = N_1\bar{u} + N_2\bar{\phi}_{,1} + N_3\bar{\psi}_{,1}, \tag{14}$$

$$\bar{\phi}_{,11} = M_1\bar{\psi}_{,11} + M_2\bar{u}_{,1} + M_3\bar{\phi} + M_4\bar{\psi}, \tag{15}$$

$$\bar{\psi}_{,11} = M_5\bar{\phi}_{,11} + M_6\bar{u}_{,1} + M_7\bar{\phi} + M_8\bar{\psi}, \tag{16}$$

After using (16) in (15) and rewriting equation (14) and the resulting equations, we get

$$\bar{u}_{,11} = N_1\bar{u} + N_2\bar{\phi}_{,1} + N_3\bar{\psi}_{,1}, \tag{17}$$

$$\bar{\phi}_{,11} = N_4\bar{u}_{,1} + N_5\bar{\phi} + N_6\bar{\psi}, \tag{18}$$

$$\bar{\psi}_{,11} = N_7\bar{u}_{,1} + N_8\bar{\phi} + N_9\bar{\psi}, \tag{19}$$

where

$$N_1 = -\omega^2,\ N_2 = -\delta_1\ ,\ N_3 = -\delta_2\ ,\ M_1 = \frac{-\delta_4}{\delta_3}\ ,\ M_2 = \frac{\delta_5}{\delta_3}\ ,\ M_3 = \frac{\delta_6-\omega^2}{\delta_3}\ ,\ M_4 = \frac{\delta_7}{\delta_3},$$

$$M_5 = \frac{-\delta_8}{\delta_9},\ M_6 = \frac{\delta_{10}}{\delta_9},\ M_7 = \frac{\delta_{11}}{\delta_9},\ M_8 = \frac{\delta_{12}-\omega^2}{\delta_9},\ M_9 = 1 - M_1M_5,\ N_4 = \frac{M_1M_6+M_2}{M_9},$$

$$N_5 = \frac{M_1M_7+M_3}{M_9},\ N_6 = \frac{M_1M_8+M_4}{M_9},\ N_7 = M_5N_4 + M_6,\ N_8 = M_5N_5 + M_7,\ N_9 = M_5N_6 + M_8 \tag{20}$$

4. State - space formulation

Choosing as a state variable displacement $\bar{u}$, volume fraction $\bar{\varphi}$ and $\bar{\psi}$ in the x- direction, then the equations can be written in the matrix form as

$$\frac{dV(x,\omega)}{dx} = A(\omega)V(x,\omega), \tag{21}$$

and the values of $A(\omega), V(x,\omega)$ are given in the appendix I.
The formal solution of system (21) can be written in the form

$$V(x,\omega) = exp[A(\omega)x]V(0,\omega). \tag{22}$$

Value of $V(0,\omega)$ is given in the appendix I.
We shall use the well-known Cayley-Hamilton theorem to find the form of the matrix $exp[A(\omega)x]$. The characteristics equation of the matrix $A(\omega)$ can be written as

$$\lambda^6 + D_1\lambda^4 + D_2\lambda^2 + D_3 = 0, \tag{23}$$

where

$$D_1 = -N_1 - N_5 - N_9 - N_2N_4 - N_3N_7,$$

$$D_2 = N_1N_5 + N_1N_9 + N_5N_9 - N_6N_8 + N_2N_4N_9 - N_2N_6N_7 + N_3N_5N_7 - N_3N_4N_8,$$

$$D_3 = N_1N_6N_8 - N_1N_5N_9. \tag{24}$$

Equation (23) is cubic in λ^2, yield three roots says $\lambda_1, \lambda_2, \lambda_3$.
Now the Taylor series expansion for matrix exponential in equation (25) is given by

$$exp[A(\omega)x] = \sum_{n=0}^{\infty}\left\{\frac{[A(\omega)x]^n}{n!}\right\}. \tag{25}$$

Using Cayley-Hamilton theorem, this infinite series can be truncated as

$$exp[A(\omega)x] = a_0I + a_1A + a_2A^2, \tag{26}$$

where $a_0,\ a_1,\ a_2$ are parameters depending on x and ω.
According to Cayley-Hamilton theorem the characteristic roots $-\lambda_1, -\lambda_2, -\lambda_3$ of the matrix A must satisfy equation (26). Therefore, we get,

$$exp[-\lambda_1x] = a_0I - a_1\lambda_1 + a_2{\lambda_1}^2, \quad exp[-\lambda_2x] = a_0I - a_1\lambda_2 + a_2{\lambda_2}^2,$$

$$exp[-\lambda_3x] = a_0I - a_1\lambda_3 + a_2{\lambda_3}^2. \tag{27}$$

Solving the above system of equations, we obtain the value of parameters a_0, a_1, a_2 and these values are given in appendix I.
Therefore, we have

$$exp[A(\omega)x] = L(x,\omega), \tag{28}$$

where $L(x,w)$ is a 6×6 matrix with the components

$$l_{11} = a_0 + a_2N_1, l_{12} = 0, l_{13} = 0, \quad l_{21} = 0, l_{22} = a_0 + a_2N_5, l_{23} = a_2N_6,$$

$$l_{31} = 0, l_{32} = a_2N_8, l_{33} = a_0 + a_2N_9, \tag{29}$$

Rewriting the equation (22) with the aid of equation (28) yield,

$$V(x,\omega) = L(x,\omega)V(0,\omega). \tag{30}$$

Therefore, we obtain

$$\begin{bmatrix}\bar{u}\\ \bar{\varphi}\\ \bar{\psi}\end{bmatrix} = \begin{bmatrix}l_{11} & l_{12} & l_{13}\\ l_{21} & l_{22} & l_{23}\\ l_{31} & l_{32} & l_{33}\end{bmatrix}\begin{bmatrix}A_1\\ A_2\\ A_3\end{bmatrix}. \tag{31}$$

5. Boundary conditions

A homogeneous isotropic elastic solid with double porosity structure occupying the region $0 \leq x < \infty$ is considered. The bounding plane $x = 0$ subjected to normal force. Mathematically these can be written as,

(i) $t_{11} = -F_1 \exp[-i\omega t]$, (32)

(ii) $\sigma_1 = -F_2 \exp[-i\omega t]$, (33)

(iii) $\tau_1 = -F_3 \exp[-i\omega t]$, (34)

where $F_i; i = 1,2,3$ is the magnitude of the force applied on the boundary.
Substituting the values of $\bar{u}$, $\bar{\varphi}$, $\bar{\psi}$ from the equation (31) in the boundary condition (32)-(34) and with the aid of equations (1)-(3), (7), (8) and (13), we obtain

$$\begin{bmatrix} Q_1 & Q_2 & Q_3 \\ Q_4 & Q_5 & Q_6 \\ Q_7 & Q_8 & Q_9 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} -F_1 \\ -F_2 \\ -F_3 \end{bmatrix}. \quad (35)$$

The values of $Q_1, Q_2, \dots \dots \dots \dots, Q_9$ are given in the appendix I.
Solving (35) for A_1, A_2, A_3 and substituting the resulting values in equation (30) yield the value of normal stress and the components of equilibrated stress,

$$t_{11} = (S_1 \frac{\Gamma_1}{\Gamma} + S_2 \frac{\Gamma_2}{\Gamma} + S_3 \frac{\Gamma_3}{\Gamma}) e^{-i\omega t}, \quad (36)$$

$$\sigma_1 = (S_4 \frac{\Gamma_1}{\Gamma} + S_5 \frac{\Gamma_2}{\Gamma} + S_6 \frac{\Gamma_3}{\Gamma}) e^{-i\omega t}, \quad (37)$$

$$\tau_1 = (S_7 \frac{\Gamma_1}{\Gamma} + S_8 \frac{\Gamma_2}{\Gamma} + S_9 \frac{\Gamma_3}{\Gamma}) e^{-i\omega t}. \quad (38)$$

6. Particular case

If $F_3 = 0$ in equation (36)-(38), yields the corresponding expressions for elastic with primary porous material.

7. Numerical results and discussion

With the view of illustrating theoretical results obtained in the preceding sections and to compare these in the context of above cases, we now represent some numerical results for copper material (elastic solid), the physical data for which is given below (2005):

$\lambda = 7.76 \times 10^{10} Nm^{-2}$, $\mu = 3.86 \times 10^{10} Nm^{-2}$, $\alpha = 1.78 \times 10^{-5} K^{-1}$, $\omega = 1 \times 10^{11} s^{-1}$,

$t = 0.1s, \rho = 8.954 \times 10^3 Kgm^{-3}$.

The double porous parameters are taken as

$\alpha_2 = 1.96 \times 10^{10} Nm^{-2}, \alpha_3 = 1.86 \times 10^{10} Nm^{-2}$, $\gamma = 0.19 \times 10^{-5} N$, $b_1 = 0.12 \times 10^{-5}\ N$,

$d = 0.49 \times 10^{10} Nm^{-2}$, $k_1 = 0.1456 \times 10^{-12} Nm^{-2} s^2$, $b = 0.4 \times 10^{10} Nm^{-2}$,

$\alpha_1 = 1.65 \times 10^{10} Nm^{-2}, k_2 = 0.1546 \times 10^{-12} Nm^{-2} s^2$.

The software Matlab R2010a has been used to determine the values of normal stress and equilibrated stresses.The variation of these values with respect to distance x have been shown in Figs. (1)-(3) respectively. In Figures 1 and 2, the comparisons for elastic with double porous material (DPM) and elastic with primary porous material (SPM) are represented graphically. Figure 3 depicts the variation of equilibrated stress τ_1 with distance x for porous parameter b_1 in case of elastic with double porous material.

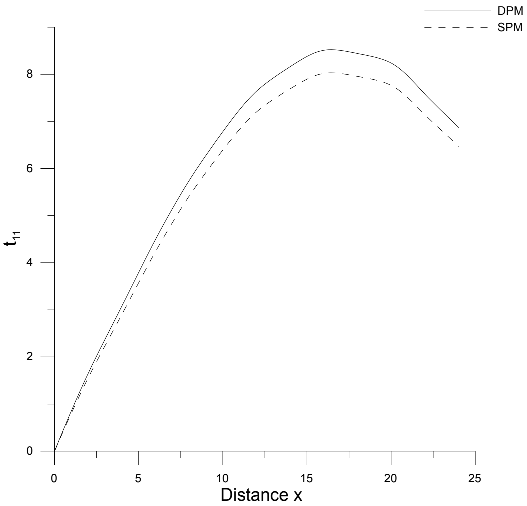

Fig. 1. Variation of normal stress t_{11} with respect to distance x.

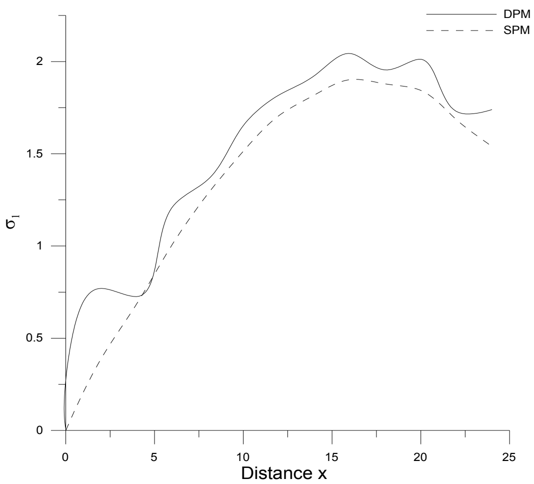

Fig. 2. Variation of equilibrated stress σ_1 with respect to distance x.

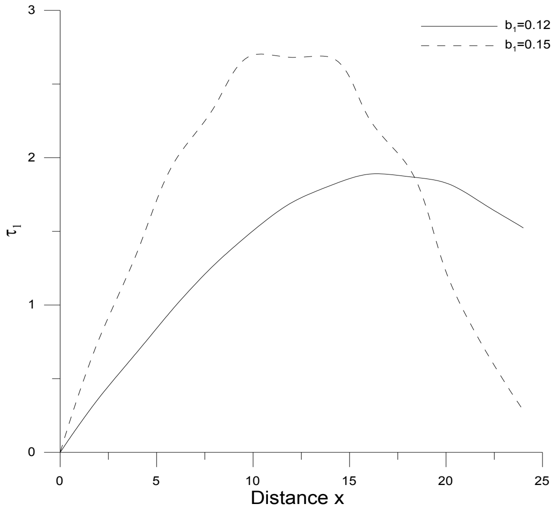

Fig. 3. Variation of equilibrated stress τ_1 with respect to x.

Figure 1 shows the variation of t_{11} with respect to distance x. The value of t_{11} increases monotonically and then decreases for both the materials but the magnitude value is more for DPM in comparison to SPM.

Figure 2 shows the variation of σ_1 with respect to distance x. The value of σ_1 for DPM is oscillatory while for SPM it increases monotonically and then starts decreasing again.

Figure 3 shows the variation of τ_1 with respect to distance x for porous parameter b_1. For $b_1 = 0.12$, the value of τ_1 increases for $0 < x \leq 17.5$ and then starts decreasing again. For $b_1 = 0.15$, the value of τ_1 increases for $0 < x \leq 10$, then becomes almost constant for $10 < x \leq 15$ and then decreases for all x.

8. Conclusion

The behavior of normal stress and equilibrated stresses in an isotropic homogeneous elastic material with double porosity has been investigated for elastic with double porous material and elastic with primary porous medium by using state space approach. It is observed that double porosity increases the value of t_{11} and σ_1. The magnitude of τ_1 also increases with the increase in value of porous parameter b_1 .This type of study is useful due to its application in geophysics and rock mechanics.

Appendix I

$$A(x,w)=\begin{bmatrix}0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\\N_1&0&0&0&N_2&N_3\\0&N_5&N_6&N_4&0&0\\0&N_8&N_9&N_7&0&0\end{bmatrix},V(x,w)=\begin{bmatrix}\bar{u}(x,w)\\\bar{\varphi}(x,w)\\\bar{\psi}(x,w)\\(\bar{u}(x,w))_{,1}\\(\bar{\varphi}(x,w))_{,1}\\(\bar{\psi}(x,w))_{,1}\end{bmatrix},V(0,w)=\begin{bmatrix}\bar{u}(0w)\\\bar{\varphi}(0,w)\\\bar{\psi}(0,w)\\(\bar{u}(0,w))_{,1}\\(\bar{\varphi}(0,w))_{,1}\\(\bar{\psi}(0,w))_{,1}\end{bmatrix},$$

$$a_0=e^{-\lambda_1 x}[\frac{\lambda_2\lambda_3}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}]+e^{-\lambda_2 x}[\frac{\lambda_1\lambda_3}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)}]+e^{-\lambda_3 x}[\frac{\lambda_1\lambda_2}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)}],$$

$$a_1=e^{-\lambda_1 x}[\frac{(\lambda_2+\lambda_3)}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}]+e^{-\lambda_2 x}[\frac{(\lambda_1+\lambda_3)}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)}]+e^{-\lambda_3 x}[\frac{(\lambda_1+\lambda_2)}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)}],$$

$$a_2=e^{-\lambda_1 x}[\frac{1}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}]+e^{-\lambda_2 x}[\frac{1}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)}]+e^{-\lambda_3 x}[\frac{1}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)}],$$

$$Q_1=P_1(Z_1+N_1Z_3),Q_2=P_2(a_0^0+a_2^0N_5)+P_3(a_2^0N_8),Q_3=P_2(a_2^0N_6)+P_3(a_0^0+a_2^0N_9),$$

$$Q_4=0\,,Q_5=P_4(Z_1+N_5Z_3)+P_5N_8Z_3,Q_6=P_4N_6Z_3+P_5(Z_1+N_9Z_3),$$

$$Q_7=0,Q_8=P_5(Z_1+N_5Z_3)+P_6N_8Z_3,Q_9=P_5N_6Z_3+P_6(Z_1+N_9Z_3),$$

where

$$P_1=\frac{\lambda+2\mu}{\lambda},\ P_2=\frac{b\alpha_1}{\lambda k_1\omega^2},\ P_3=\frac{d\alpha_1}{\lambda k_1\omega^2},\ P_4=\frac{\alpha_1}{k_1\omega^2},\ P_5=\frac{b_1\alpha_1}{\alpha k_1\omega^2},\ P_6=\frac{\gamma\alpha_1}{\alpha k_1\omega^2},$$

$$Z_1=-\lambda_1D_{11}-\lambda_2D_{12}-\lambda_3D_{13},Z_2=-\lambda_1D_{21}-\lambda_2D_{22}-\lambda_3D_{23},\ Z_3=-\lambda_1D_{31}-\lambda_2D_{32}-\lambda_3D_{33},$$

$$Y_1=-\lambda_1D_{11}e^{-\lambda_1x}-\lambda_2D_{12}e^{-\lambda_2x}-\lambda_3D_{13}e^{-\lambda_3x},Y_2=-\lambda_1D_{11}e^{-\lambda_1x}-\lambda_2D_{12}e^{-\lambda_2x}-\lambda_3D_{13}e^{-\lambda_3x},$$

$$Y_3=-\lambda_1D_{21}e^{-\lambda_1x}-\lambda_2D_{22}e^{-\lambda_2x}-\lambda_3D_{23}e^{-\lambda_3x},\ S_1=P_1(Y_1+N_1Y_3),\ S_2=0,S_3=0,$$

$$S_4=0,S_5=P_4(Y_1+N_5Y_3)+P_5N_8Y_3,S_6=P_4N_6Y_3+P_5(Y_1+N_9Y_3),\ S_7=0,$$

$$S_8=P_5(Y_1+N_5Y_3)+P_6N_8Y_3,\ S_9=P_5N_6Y_3+P_6(Y_1+N_9Y_3),$$

$$D_{11}=\frac{\lambda_2\lambda_3}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)},\ D_{12}=\frac{\lambda_1\lambda_3}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)},\ D_{13}=\frac{\lambda_1\lambda_2}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)},$$

$$D_{31}=\frac{1}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)},D_{32}=\frac{1}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)},\ D_{33}=-\frac{1}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)},$$

$$\Gamma=\begin{vmatrix}Q_1&Q_2&Q_3\\Q_4&Q_5&Q_6\\Q_7&Q_8&Q_9\end{vmatrix},\ \Gamma_1=\begin{vmatrix}-F_1&Q_2&Q_3\\-F_2&Q_5&Q_6\\-F_3&Q_8&Q_9\end{vmatrix},\ \Gamma_2=\begin{vmatrix}Q_1&-F_1&Q_3\\Q_4&-F_2&Q_6\\Q_7&-F_3&Q_9\end{vmatrix},\ \Gamma_3=\begin{vmatrix}Q_1&Q_2&-F_1\\Q_4&Q_5&-F_2\\Q_7&Q_8&-F_3\end{vmatrix},$$

and $a_0^0=a_0,\ a_2^0=a_2,\ at\ x=0,\ A_1=\frac{\Gamma_1}{\Gamma},\ A_2=\frac{\Gamma_2}{\Gamma},\ A_3=\frac{\Gamma_3}{\Gamma}.$

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