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FREE VIBRATION ANALYSIS OF ROTATING PIEZOELECTRIC BAR OF CIRCULAR CROSS SECTION IMMERSED IN FLUID

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Abstract. Free vibration analysis of rotating piezoelectric bar of circular cross-section immersed in fluid is discussed using three-dimensional theory of piezoelectricity. The equations of motion of the bar are formulated using the constitutive equations of a piezoelectric material. The equations of motion of the fluid are formulated using the constitutive equations of an inviscid fluid. Three displacement potential functions are introduced to uncouple the equations of motion, electric conduction. The perfect-slip boundary conditions are applied at the solid-fluid interfaces to obtain the frequency equation of the coupled systems. The frequency equations are obtained for longitudinal and flexural (symmetric and antisymmetric) modes of vibration and are studied numerically for PZT-4 material. The computed wave number and electro mechanical coupling is presented in the form of dispersion curves. The secant method is used to obtain the roots of the frequency equation.

1. Introduction

The development in piezoelectric sensors and actuators is important for the design and construction of light weighted and high performance smart structures. Piezoelectric polymers allow their use in a multitude of compositions and geometrical shapes for a large variety of applications from transducers in acoustics, ultrasonic's and hydrophone applications to resonators in band pass filters, power supplies, delay lines, medical scans and some industrial non-destructive testing instruments. Some of the applications of these polymers include Audio device-microphones, high frequency speakers, tone generators and acoustic modems; Pressure switches – position switches, accelerometers, impact detectors, flow meters and load cells; Actuators- electronic fans and high shutters. The rotating piezoelectric bar of circular cross section has gained importance in construction of gyroscope to measure the angular velocity of a rotating body.

Most of the studies in elastic wave propagation in cylindrical waveguides are concerned with isotropic cylinders. The wave propagation in elastic solid has been discussed extensively in details by Graff [1]. The propagation of compressional elastic waves along an anisotropic circular cylinder with hexagonal symmetry was first studied by Morse [2]. Theoretical studies on electroelastic wave propagation in anisotropic piezoceramic cylinders have also been pursued for many years. The approach usually applied for piezoelectric solids is the simplification of Maxwell's equations by neglecting magnetic effects, conduction, free charges, and displacement currents. Studies by Tiersten [3] should be mentioned among the early notable contributions to the topic of the mechanics of piezoelectric solids. Electroelastic governing equations of piezoelectric materials are presented by Parton and Kudryavtsev [4]. Shul'ga [5] studied the propagation of axisymmetric and non-axisymmetric waves in anisotropic piezoceramic cylinders with various prepolarization directions and boundary conditions.

Paul and Venkatesan [6-7] studied the wave propagation in infinite piezoelectric solid cylinders of arbitrary cross-section using Fourier expansion collocation method, formulated by Nagaya [8]. Rajapakse and Zhou [9] solved the coupled electroelastic equations for a long piezoceramic cylinder by applying Fourier integral transforms. Paper by Wang [10] should be mentioned among the studies of cylindrical shells with a piezoelectric coat. Ebenezer and Ramesh [11] analyzed axially polarized piezoelectric cylinders with arbitrary boundary conditions on the flat surfaces using the Bessel series. Berg et al. [12] assumed electric field not to be constant over the thickness of piezoceramic cylindrical shells. Later Botta and Cerri [13] extended this approach and compared their results with those in which the effect of variable electric potential was not considered. Kim and Lee [14] studied piezoelectric cylindrical transducers with radial polarization and compared their results with those obtained experimentally and numerically by the finite-element method.

Berliner and Solecki [15] have studied the wave propagation in a fluid loaded transversely isotropic cylinder. In that paper, Part I consists of the analytical formulation of the frequency equation of the coupled system consisting of the cylinder with inner and outer fluid and Part II gives the numerical results. Guo and Sun [16] discussed the propagation of Bleustein - Gulyaev wave in 6mm piezoelectric materials loaded with viscous liquid using the theory of continuum mechanics. Qian et al [17] analyzed the propagation of Bleustein-Gulyaev waves in 6mm piezoelectric materials loaded with a viscous liquid layer of finite thickness. Dayal [18] investigated the free vibrations of a fluid loaded transversely isotropic rod based on uncoupling the radial and axial wave equations by introducing scalar and vector potentials. Nagy [19] studied the propagation of longitudinal guided waves in fluid-loaded transversely isotropic rod based on the superposition of partial waves. Guided waves in a transversely isotropic cylinder immersed in a fluid were analyzed by Ahmad [20]. Ponnusamy and Selvamani [21, 22] have studied respectively, the three dimensional wave propagation of transversely isotropic magneto thermo elastic cylindrical panel and flexural vibration in a heat conducting cylindrical panel embedded in a Winkler elastic medium in the context of the linear theory of thermo elasticity. The dynamic response of a heat conducting solid bar of polygonal cross section subjected to moving heat source is discussed by Selvamani [23] using the Fourier expansion collocation method (FECM).

Zhang [24] investigated the parametric analysis of frequency of rotating laminated composite cylindrical shell using wave propagation approach. Body wave propagation in rotating thermo elastic media was investigated by Sharma and Grover [25]. The propagation of waves in conducting piezoelectric solid is studied for the case when the entire medium rotates with a uniform angular velocity by Wauer [26]. Roychoudhuri and Mukhopadhyay [27] studied the effect of rotation and relaxation times on plane waves in generalized thermo visco elasticity. Hua and Lam [28] has studied the frequency characteristics of a thin rotating cylindrical shell using general differential quadrature method. Sergiu et al. [29] studied the energy dissipation and critical speed of granular flow in a rotating cylinder and they found that the coefficient of friction have the greatest significance on the centrifuging speed.

The aim of the present article is to study the free wave propagation in a rotating piezoelectric bar of circular cross-section immersed in fluid. The frequency equations are obtained from the solid-fluid interfacial boundary conditions. The computed wave number and electromechanical coupling with respect to frequency are plotted in the form of dispersion curves for longitudinal and flexural modes of vibrations for the material PZT-4.

2. Governing field equations

The linear constitutive equations of coupled elastic and electric field in a piezoelectric medium are given by

$$\{\sigma\} = [C]\{e\} - [\eta]^T \{E\}, \qquad \{D\} = [\eta]\{e\} + [\varepsilon]^T \{E\}, \tag{1}$$

where the stress vector $\{\sigma\}$, the strain vector $\{e\}$, the electric field vector $\{E\}$ and the electric displacement vector $\{D\}$ are given in the cylindrical coordinate system (r,θ,z) (Fig. 1) by

$$\{\sigma\}=\left[\sigma_{rr},\sigma_{\theta\theta},\sigma_{zz},\sigma_{r\theta},\sigma_{\theta z},\sigma_{rz}\right]^T,\quad \{E\}=\left[E_r,E_\theta,E_z\right]^T,$$

$$\{e\}=\left[e_{rr},e_{\theta\theta},e_{zz},e_{r\theta},e_{\theta z},e_{rz}\right]^T,\quad \{D\}=\left[D_r,D_\theta,D_z\right]^T, \tag{2}$$

where $\lfloor C\rfloor$, $\lfloor \eta\rfloor$ and $\lfloor \varepsilon\rfloor$ denotes the matrices of elastic constants, piezoelectric constants and dielectric constants respectively.

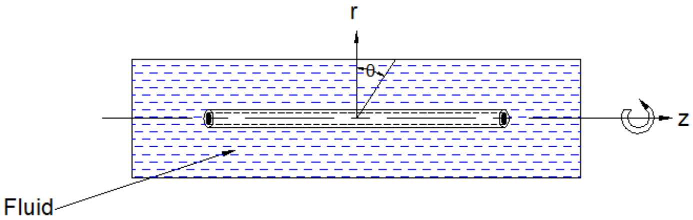

Fig. 1. Rotating bar immersed in fluid.

The matrices $\lfloor C\rfloor$, $\lfloor \eta\rfloor$ and $\lfloor \varepsilon\rfloor$ for the transversely isotropic material is given by

$$[C]=\begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0\\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0\\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & c_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & c_{44} & 0\\ 0 & 0 & 0 & 0 & 0 & c_{66}\end{bmatrix},\quad [\eta]=\begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0\\ 0 & 0 & 0 & e_{15} & 0 & 0\\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0\end{bmatrix},\quad [\varepsilon]=\begin{bmatrix}\varepsilon_{11} & 0 & 0\\ 0 & \varepsilon_{11} & 0\\ 0 & 0 & \varepsilon_{33}\end{bmatrix}. \tag{3}$$

By considering a homogeneous transversely isotropic piezoelectric rotating circular bar of infinite length immersed in fluid, the equations of motion in the absence of body force are

$$\frac{\partial}{\partial r}\sigma_{rr}+\frac{1}{r}\frac{\partial}{\partial\theta}\sigma_{r\theta}+\frac{\partial}{\partial z}\sigma_{rz}+\frac{(\sigma_{rr}-\sigma_{\theta\theta})}{r}+\rho(\vec{\Omega}\times(\vec{\Omega}\times\vec{u})+2(\overrightarrow{\Omega}\times\vec{u}_{,t}))=\rho\frac{\partial^2u_r}{\partial t^2},$$

$$\frac{\partial}{\partial r}\sigma_{r\theta}+\frac{1}{r}\frac{\partial}{\partial\theta}\sigma_{\theta\theta}+\frac{\partial}{\partial z}\sigma_{\theta z}+\frac{2\sigma_{r\theta}}{r}=\rho\frac{\partial^2u_\theta}{\partial t^2},$$

$$\frac{\partial}{\partial r}\sigma_{rz}+\frac{1}{r}\frac{\partial}{\partial\theta}\sigma_{\theta z}+\frac{\partial}{\partial z}\sigma_{zz}+\frac{\sigma_{rz}}{r}+\rho(\vec{\Omega}\times(\vec{\Omega}\times\vec{u})+2(\overrightarrow{\Omega}\times\vec{u}_{,t}))=\rho\frac{\partial^2u_z}{\partial t^2}. \tag{4}$$

The electric displacements D_r, D_θ and D_z satisfy the Gaussian equation is

$$\frac{1}{r}\frac{\partial}{\partial r}(rD_r)+\frac{1}{r}\frac{\partial D_\theta}{\partial\theta}+\frac{\partial D_z}{\partial r}=0. \tag{5}$$

The elastic, the piezoelectric, and dielectric matrices of the 6mm crystal class, the piezoelectric relations are

$$\sigma_{rr}=c_{11}e_{rr}+c_{12}e_{\theta\theta}+c_{13}e_{zz}-e_{31}E_z,\quad \sigma_{\theta\theta}=c_{12}e_{rr}+c_{11}e_{\theta\theta}+c_{13}e_{zz}-e_{31}E_z,$$

$$\sigma_{zz}=c_{13}e_{rr}+c_{13}e_{\theta\theta}+c_{33}e_{zz}-e_{33}E_z,\ \sigma_{r\theta}=c_{66}e_{r\theta},\ \sigma_{\theta z}=c_{44}e_{\theta z}-e_{15}E_\theta,\ \sigma_{rz}=2c_{44}e_{rz}-e_{15}E_r, \tag{6}$$

and

$$D_r = e_{15}e_{rz} + \varepsilon_{11}E_r\,,\quad D_\theta = e_{15}e_{\theta z} + \varepsilon_{11}E_\theta\,,\quad D_z = e_{31}\left(e_{rr} + e_{\theta\theta}\right) + e_{33}e_{zz} + \varepsilon_{33}E_z\,, \tag{7}$$

where are the stress components, $e_{rr}, e_{\theta\theta}, e_{zz}, e_{r\theta}, e_{\theta z}, e_{rz}$ are the strain components, $c_{11}, c_{12}, c_{13}, c_{33}, c_{44}$ and $c_{66} = \left(c_{11} - c_{12}\right)/2$ are the five elastic constants, e_{31}, e_{15}, e_{33} are the piezoelectric constants, $\varepsilon_{11}, \varepsilon_{33}$ are the dielectric constants, ρ is the mass density. The comma in the subscripts denotes the partial differentiation with respect to the variables. The displacement equation of motion has the additional terms with a time dependent centripetal acceleration $\vec{\Omega}\times(\vec{\Omega}\times\vec{u})$ and $2(\vec{\Omega}\times\vec{u}_{,t})$ where, $\vec{u} = (u, 0, w)$ is the displacement vector and $\vec{\Omega} = (0, \Omega, 0)$ is the angular velocity.

The strain e_{ij} are related to the displacements are given by

$$e_{rr} = u_{r,r}\,,\quad e_{\theta\theta} = r^{-1}\left(u_r + u_{\theta,\theta}\right),\quad e_{zz} = u_{z,z}\,, \tag{8a}$$

$$e_{r\theta} = u_{\theta,r} + r^{-1}\left(u_{r,\theta} - u_\theta\right),\quad e_{z\theta} = \left(u_{\theta,z} + r^{-1}u_{z,\theta}\right),\quad e_{rz} = u_{z,r} + u_{r,z}\,. \tag{8b}$$

The comma in the subscripts denotes the partial differentiation with respect to the variables.

Substituting the Eqs. (6), (7) and (8) in the Eqs. (4) and (5), results in the following three-dimensional equations of motion, electric conductions as follows:

$$\begin{aligned}&c_{11}\left(u_{rr,r} + r^{-1}u_{r,r} - r^{-2}u_r\right) - r^{-2}\left(c_{11} + c_{66}\right)u_{\theta,\theta} + r^{-2}c_{66}u_{r,\theta\theta} + c_{44}u_{r,zz} + \left(c_{44} + c_{13}\right)u_{z,rz} + r^{-1}\left(c_{66} + c_{12}\right)u_{\theta,r\theta}\\ &+\left(e_{31} + e_{15}\right)V_{,rz} + \rho\left(\Omega^2 u + 2\Omega w_{,t}\right) = \rho u_{r,tt}\,,\end{aligned} \tag{9a}$$

$$\begin{aligned}&r^{-1}\left(c_{12} + c_{66}\right)u_{r,r\theta} + r^{-2}\left(c_{66} + c_{11}\right)u_{r,\theta} + c_{66}\left(u_{\theta,rr} + r^{-1}u_{\theta,r} - r^{-2}u_\theta\right) + r^{-2}c_{11}u_{\theta,\theta\theta} + c_{44}u_{\theta,zz} + r^{-1}\left(c_{44} + c_{13}\right)u_{z,\theta z}\\ &+\left(e_{31} + e_{15}\right)V_{,\theta z} = \rho u_{\theta,tt}\,,\end{aligned} \tag{9b}$$

$$\begin{aligned}&c_{44}\left(u_{z,rr} + r^{-1}u_{z,r} + r^{-2}u_{z,\theta\theta}\right) + r^{-1}\left(c_{44} + c_{13}\right)\left(u_{r,z} + u_{\theta,\theta z}\right) + \left(c_{44} + c_{13}\right)u_{r,rz} + c_{33}u_{z,zz} + e_{33}V_{,zz}\\ &+e_{15}\left(V_{,rr} + r^{-1}V_{,r} + r^{-2}V_{,\theta\theta}\right) + \rho\left(\Omega^2 w + 2\Omega u_{,t}\right) = \rho u_{z,tt}\,,\end{aligned} \tag{9c}$$

$$\begin{aligned}&e_{15}\left(u_{z,rr} + r^{-1}u_{z,r} + r^{-2}u_{z,\theta\theta}\right) + \left(e_{31} + e_{15}\right)\left(u_{r,zr} + r^{-1}u_{r,z} + r^{-1}u_{\theta,z\theta}\right) + e_{33}u_{z,zz} - \varepsilon_{33}V_{,zz}\\ &-\varepsilon_{11}\left(V_{,rr} + r^{-1}V_{,r} + r^{-2}V_{,\theta\theta}\right) = 0\,.\end{aligned} \tag{9d}$$

3. Solutions of the field equation

To obtain the propagation of harmonic waves in piezoelectric circular solid bar, we assume the solutions of the displacement components to be expressed in terms of derivatives of potentials, which are taken from Paul [6]. Thus, we seek the solution of the Eqs. (9) in the form of Paul [6] are

$$u_r(r,\theta,z,t) = \left(\phi_{,r} + r^{-1}\psi_{,\theta}\right)e^{i(kz+\omega t)},\ u_\theta(r,\theta,z,t) = \left(r^{-1}\phi_{,\theta} - \psi_{,r}\right)e^{i(kz+\omega t)},\ u_z(r,\theta,z,t) = \left(\frac{i}{a}\right)We^{i(kz+\omega t)},$$

$$V(r,\theta,z,t) = iVe^{i(kz+\omega t)},\ E_r(r,\theta,z,t) = -E_{,r}e^{i(kz+\omega t)},\ E_\theta(r,\theta,z,t) = -r^{-1}E_{,\theta}e^{i(kz+\omega t)},$$

$$E_z\left(r,\theta,z,t\right) = E_{,z}e^{i(kz+\omega t)}\,, \tag{10}$$

where $i = \sqrt{-1}$, k is the wave number, ω is the angular frequency, $\phi(r,\theta), W(r,\theta)$,

$\psi(r,\theta)$ and $E(r,\theta)$ are the displacement potentials and $V(r,\theta)$ is the electric potentials and a is the geometrical parameter of the bar. By introducing the dimensionless quantities such as $x = r/a$, $\zeta = ka$, $\varpi^2 = \rho\omega^2 a^2/c_{44}$, $\bar{c}_{11} = c_{11}/c_{44}$, $\bar{c}_{13} = c_{13}/c_{44}$, $\bar{c}_{33} = c_{33}/c_{44}$, $\bar{c}_{66} = c_{66}/c_{44}$, $\Gamma = \dfrac{\rho\Omega^2 R^2}{2+\bar{\lambda}}$ and substituting Eq.(9) in Eqs.(10), we obtain

$$\left\lfloor \bar{c}_{11}\nabla^2 + \left(\varpi^2 + \Gamma - \zeta^2\right)\right\rfloor \phi - \zeta\left(1+\bar{c}_{13}\right)W - \zeta\left(\bar{e}_{15}+\bar{e}_{31}\right)V = 0,$$

$$\zeta\left(1+\bar{c}_{13}\right)\nabla^2\phi + \left\lfloor \nabla^2 + \left(\varpi^2 + \Gamma - \zeta^2\bar{c}_{33}\right)\right\rfloor W + \left(\bar{e}_{15}\nabla^2 - \zeta^2\right)V = 0,$$

$$\zeta\left(\bar{e}_{15}+\bar{e}_{31}\right)\nabla^2\phi + \left(\bar{e}_{15}\nabla^2 - \zeta^2\right)W + \left(\zeta^2\bar{\varepsilon}_{33} - \bar{\varepsilon}_{11}\nabla^2\right)V = 0, \tag{11}$$

and

$$\left(\bar{c}_{66}\nabla^2 + \left(\varpi^2 + \Gamma - \zeta^2\right)\right)\psi = 0, \tag{12}$$

where $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + x^{-1}\dfrac{\partial}{\partial x} + x^{-2}\dfrac{\partial^2}{\partial \theta^2}$.

The Eq. (11) can be written as

$$\begin{vmatrix} \bar{c}_{11}\nabla^2 + g_3 & -\zeta g_6 & -\zeta g_5 \\ \zeta g_6\nabla^2 & \nabla^2 + g_1 & \left(\bar{e}_{15}\nabla^2 - \zeta^2\right) \\ \zeta g_5\nabla^2 & \left(\bar{e}_{15}\nabla^2 - \zeta^2\right) & \left(\zeta^2\bar{\varepsilon}_{33} - \bar{\varepsilon}_{11}\nabla^2\right) \end{vmatrix} (\phi, W, V) = 0, \tag{13}$$

where $g_1 = \varpi^2 + \Gamma - \zeta^2\bar{c}_{33}$, $g_3 = \varpi^2 + \Gamma - \zeta^2$, $g_4 = \bar{\varepsilon}_{11}^2 + \bar{e}_{15}^2$, $g_5 = \bar{e}_{31} + \bar{e}_{15}$ and $g_6 = 1 + \bar{c}_{13}$.
Evaluating the determinant given in Eq. (13), we obtain a partial differential equation of the form

$$\left(A\nabla^6 + B\nabla^4 + C\nabla^2 + D\right)(\phi, W, V) = 0, \tag{14}$$

where $A = c_{11}\left(\bar{e}_{15}^2 + \varepsilon_{11}\right)$,

$B = \left[\left(1+\bar{c}_{11}\right)\bar{\varepsilon}_{11} + \bar{e}_{15}^2\right]\varpi^2 + \left\{2\left(\bar{e}_{31}+\bar{e}_{15}\right)\bar{c}_{13}\bar{e}_{15} - \left(1+\bar{\varepsilon}_{11}\bar{c}_{33}\right)\bar{c}_{11} + \bar{c}_{13}^2\bar{\varepsilon}_{11} + 2\bar{c}_{13}\bar{\varepsilon}_{11} - 2\bar{e}_{15}\bar{c}_{11} + 2\bar{e}_{13}^2\right\}\varsigma^2$,

$C = \bar{\varepsilon}_{11}\varpi^4 - \left[\left(1+\bar{c}_{13}\right)\bar{\varepsilon}_{11} + \left(1+\bar{c}_{11}\right) + \left(\bar{e}_{31}+\bar{e}_{15}\right) + 2\bar{e}_{15}\right]\varsigma^2\varpi^2 + \{\bar{c}_{11}\left(1+\bar{c}_{33}\bar{\varepsilon}_{33}\right) - \left\lfloor\left(\bar{e}_{31}+\bar{e}_{15}\right)^2 + \bar{\varepsilon}_{11}\right\rfloor$
$-2\bar{e}_{31}\left(1+\bar{c}_{13}\right) - \bar{c}_{13}\bar{\varepsilon}_{33}\left(\bar{c}_{33}+\bar{c}_{13}\right) + 2\bar{e}_{15}\}\varsigma^4$, $D = -\left\{\left(1+\bar{c}_{33}\right)\varsigma^6 - \left[2\left(1+\bar{c}_{33}\right)\bar{\varepsilon}_{33} + 1\right]\varsigma^4\varpi^2 + \bar{\varepsilon}_{33}\varsigma^2\varpi^4\right\}$.

Solving the Eq. (14), we get solutions for a circular bar as

$$\phi = \sum_{i=1}^{3} A_i J_n\left(\alpha_i a x\right)\cos n\theta, \quad W = \sum_{i=1}^{3} a_i A_i J_n\left(\alpha_i a x\right)\cos n\theta, \quad V = \sum_{i=1}^{3} b_i A_i J_n\left(\alpha_i a x\right)\cos n\theta. \tag{15}$$

Here $\left(\alpha_i a\right)^2 > 0, \quad (i = 1,2,3)$ are the roots of the algebraic equation

$$A(\alpha a)^6 - B(\alpha a)^4 + C(\alpha a)^2 + D = 0. \tag{16}$$

The solutions corresponding to the root $\left(\alpha_i a\right)^2 = 0$ is not considered here, since $J_n(0)$ is zero, except for $n = 0$. The Bessel function J_n is used when the roots $\left(\alpha_i a\right)^2, (i = 1,2,3)$ are real or complex and the modified Bessel function I_n is used when the roots $(\alpha_i a)^2, (i = 1, 2, 3)$ are imaginary.

The constants a_i, b_i defined in the Eq. (15) can be calculated from the equations

$$(1+\bar{c}_{13})\varsigma a_i + (\bar{e}_{31}+\bar{e}_{15})\varsigma b_i = -\left(\bar{c}_{11}(\alpha_i a)^2 - \varpi^2 + \varsigma^2\right),$$

$$\left((\alpha_i a)^2 - \varpi^2 + \varsigma^2 \bar{c}_{33}\right)a_i + \left(\bar{e}_{15}(\alpha_i a)^2 + \varsigma^2\right)b_i = -(\bar{c}_{13}+1)\varsigma(\alpha_i a)^2. \tag{17}$$

Solving the Eq. (12), we obtain

$$\psi = A_4 J_n(\alpha_4 a x)\sin n\theta, \tag{18}$$

where $(\alpha_4 a)^2 = \varpi^2 - \zeta^2$. If $(\alpha_4 a)^2 < 0$, the Bessel function J_n is replaced by the modified Bessel function I_n.

4. Equations of motion of the fluid

In cylindrical polar coordinates r, θ and z the acoustic pressure and radial displacement equation of motion for an in viscid fluid are of the form Achenbach [30]

$$p^f = -B^f\left(u^f_{r,r} + r^{-1}\left(u^f_r + u^f_{\theta,\theta}\right) + u^f_{z,z}\right) \tag{19}$$

and

$$\bar{c}_f^2 u^f_{r,tt} = \Delta_{,r}, \tag{20}$$

respectively, where B^f is the adiabatic bulk modulus, ρ^f is the density, $c_f = \sqrt{B^f/\rho^f}$ is the acoustic phase velocity in the fluid, and

$$\Delta = \left(u^f_{r,r} + r^{-1}\left(u^f_r + u^f_{\theta,\theta}\right) + u^f_{z,z}\right). \tag{21}$$

Substituting

$$u^f_r = \phi^f_{,r},\ u^f_\theta = r^{-1}\phi^f_{,\theta} \text{ and } u^f_z = \phi^f_{,z} \tag{22}$$

and seeking the solution of Eq.(19) in the form

$$\phi^f(r,\theta,z,t) = \sum_{n=0}^{\infty}\varepsilon_n \phi^f(r)\cos n\theta e^{i(kz+\omega t)}. \tag{23}$$

The fluid that represents the oscillatory wave propagating away is given as

$$\phi^f = A_5 H_n^{(1)}(\alpha_5 a x), \tag{24}$$

where $(\alpha_5 a)^2 = \varpi^2/\bar{\rho}\bar{B}^f - \zeta^2$, in which $\bar{\rho} = \rho/\rho^f$, $\bar{B}^f = B^f/c_{44}$, $H_n^{(1)}$ is the Hankel function of first kind. If $(\alpha_5 a)^2 < 0$, then the Hankel function of first kind is to be replaced by K_n, where K_n is the modified Bessel function of the second kind. By substituting Eq. (23) in Eq. (19) along with Eq. (24), the acoustic pressure for the fluid can be expressed as

$$p^f = \sum_{n=0}^{\infty} A_5 \Omega^2 \bar{\rho} H_n^{(1)}(\alpha_5 a x)\cos n\theta e^{i(\zeta z + \Omega T_a)}, \tag{25}$$

4.1. Electro mechanical coupling. The electromechanical coupling (ε^2) for a cylindrical bar is important for alteration of structural responses through applied electric fields in the design of sensors and surface acoustic damping wave filters. By Teston et al. [31], the electro mechanical coupling is defined as

$$\varepsilon^2 = \left| \frac{V_e - V_f}{V_e} \right|, \tag{26}$$

where V_e and V_f are the Phase velocities of the wave under electrically shorted and charge free boundary conditions at the surface of the bar.

5. Boundary conditions and frequency equations

In this problem, the free vibration of transversely isotropic rotating piezoelectric solid bar of circular cross-section immersed in fluid is considered. In the solid-fluid interface problems, the normal stress of the bar is equal to the negative of the pressure exerted by the fluid and the displacement component in the normal direction of the lateral surface of the cylinder is equal to the displacement of the fluid in the same direction. These conditions are due to the continuity of the stresses and displacements of the solid and fluid boundaries. Since the inviscid fluid cannot sustain shear stress, the shear stress of the outer fluid is equal to zero.

For the solid-fluid problems, the continuity conditions require that the displacement components, the surface stress components and electric potential must be equal. The boundary conditions can be written as

$$\left(\sigma_{rr}, \sigma_{r\theta}, \sigma_{rz}, V, u_r\right)_i = \left(-p^f, 0, 0, 0, u_r^f\right), \quad r = a\,. \tag{27}$$

Substituting the solutions given in the Eqs. (15), (18) and (26) in the boundary condition in the Eq. (27), we obtain a system of five linear algebraic equations as follows:

$$[A]\{X\} = \{0\}, \tag{28}$$

where $\lfloor A \rfloor$ is a 5×5 matrix of unknown wave amplitudes, and $\{X\}$ is an 5×1 column vector of the unknown amplitude coefficients A_1, A_2, A_3, A_4, A_5. The components of $\lfloor A \rfloor$ are defined in the Appendix A. The solution of Eq. (28) is nontrivial when the determinant of the coefficient of the wave amplitudes $\{X\}$ vanishes, that is

$$|A| = 0\,. \tag{29}$$

Eq. (29) is the frequency equation of the coupled system consisting of a transversely isotropic rotating piezoelectric solid circular bar immersed in inviscid fluid.

6. Numerical results and discussion

The frequency equation given in Eq. (28) is transcendental in nature with unknown frequency and wave number. The material chosen for the numerical calculation is PZT-4. The material properties of PZT-4 is taken from Berlincourt et al. [32] are used for the numerical calculation is given below: $c_{11} = 13.9\times10^{10}\,Nm^{-2}$, $c_{12} = 7.78\times10^{10}\,Nm^{-2}$, $c_{13} = 7.43\times10^{10}\,Nm^{-2}$, $c_{33} = 11.5\times10^{10}\,Nm^{-2}$, $c_{44} = 2.56\times10^{10}\,Nm^{-2}$, $c_{66} = 3.06\times10^{10}\,Nm^{-2}$, $e_{31} = -5.2Cm^{-2}$, $e_{33} = 15.1Cm^{-2}$, $e_{15} = 12.7Cm^{-2}$, $\varepsilon_{11} = 6.46\times10^{-9}C^2N^{-1}m^{-2}$, $\varepsilon_{33} = 5.62\times10^{-9}C^2N^{-1}m^{-2}$, $\rho = 7500\,Kgm^{-2}$ and for fluid the density $\rho^f = 1000Kgm^{-3}$, phase velocity $c = 1500m\sec^{-1}$ and used for the numerical calculations.

In this problem, there are two kinds of basic independent modes of wave propagation have been considered, namely, the longitudinal and flexural modes. By choosing respectively n=0 and n=1, we can obtain the non-dimensional frequencies of longitudinal and flexural modes of vibrations.

The dispersion curves are drawn in Figs. 2-4 for wave number versus the frequency for longitudinal and flexural (symmetric and anti symmetric) modes of piezoelectric circular bar immersed in fluid. From the Figs. 2-4, it is observed that the wave numbers are increased with respect to its frequencies. The increase in angular velocities in all the modes is significant and the flexural modes are getting dispersed compared with longitudinal mode.

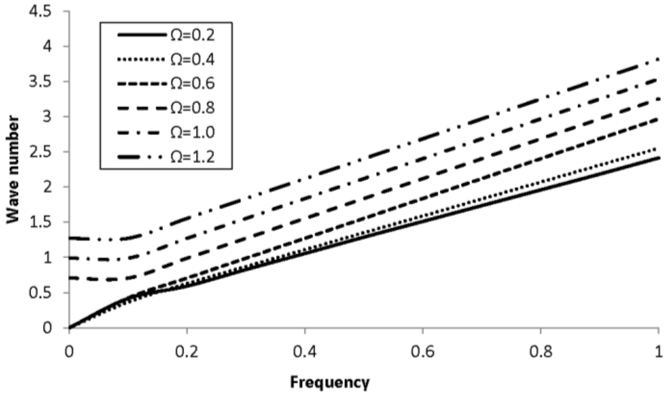

Fig. 2. Dispersion of wave number with frequency for longitudinal modes of piezoelectric rotating bar immersed in fluid.

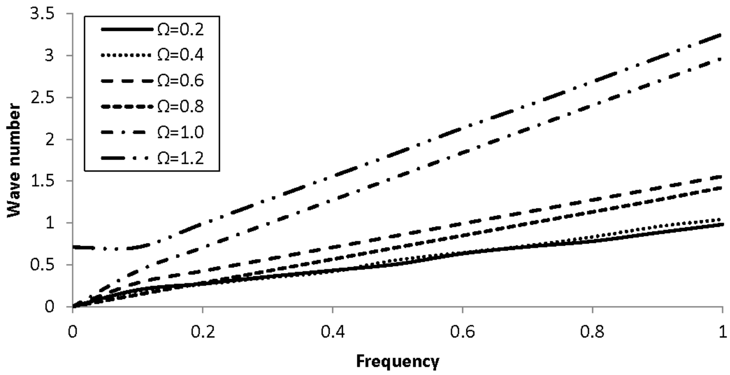

Fig. 3. Dispersion of wave number with frequency for flexural symmetric modes of piezoelectric rotating bar immersed in fluid.

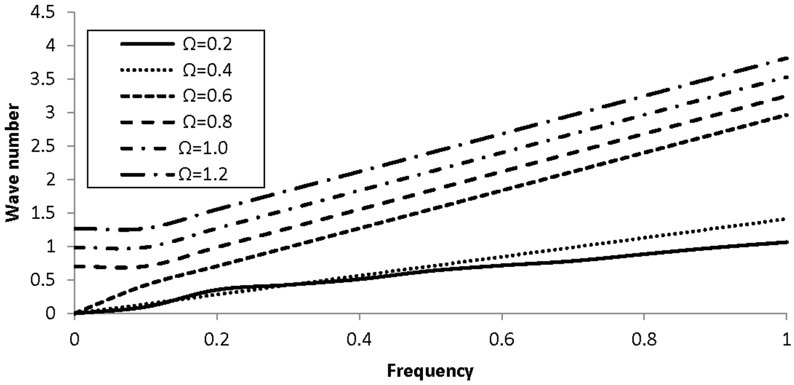

Fig. 4. Dispersion of wave number with frequency for flexural antisymmetric modes of piezoelectric rotating bar immersed in fluid.

A comparison is made among the electro mechanical coupling with respect to frequency for longitudinal and flexural (symmetric and antisymmetric) modes of vibration in the Figs. 5-7, respectively. From the Figs. 5-7, it is clear that, the modes of electro mechanical coupling are merges for a particular period of frequency after that, it starts increase and decreases.

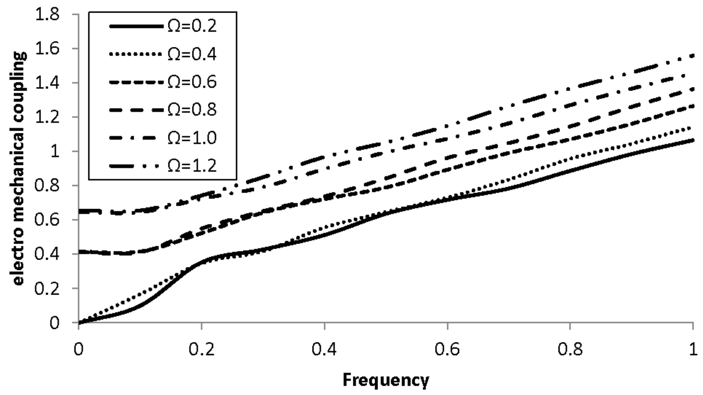

Fig. 5. Dispersion of electro mechanical coupling with frequency for longitudinal modes of piezoelectric rotating bar immersed in fluid.

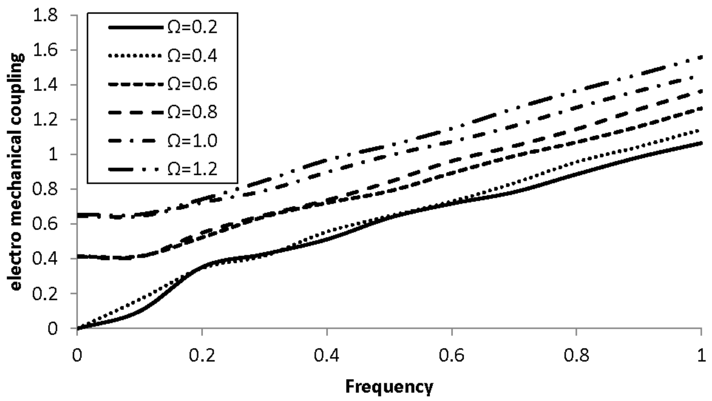

Fig. 6. Dispersion of electro mechanical coupling with frequency for flexural symmetric modes of piezoelectric rotating bar immersed in fluid.

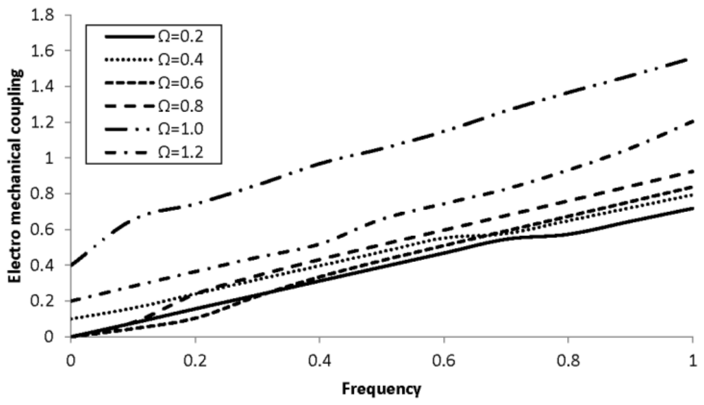

Fig. 7. Dispersion of electro mechanical coupling with frequency for flexural antisymmetric modes of piezoelectric rotating bar immersed in fluid.

The cross-over points between the modes of electro mechanical coupling with respect to the increasing angular velocities shows that, there is energy transfer between the modes of vibrations due to the extra added force of rotating and hosting fluid.

7. Conclusion

The propagation of waves in a piezoelectric solid bar of circular cross-section immersed in fluid is discussed using three-dimensional theory of piezoelectricity. Three displacement potential functions are introduced to uncouple the equations of motion, electric conduction. The frequency equation of the coupled system consisting of bar and fluid is developed under the assumption of perfect-slip boundary conditions at the fluid-solid interfaces. The frequency equations are obtained for longitudinal and flexural modes of vibration and are studied numerically for PZT-4 material bar immersed in fluid. The effect of rotation and the hosting fluid is pronounced in the dispersion of wave number and electromechanical coupling.

Appendix A

$$a_{1i}=2\bar{c}_{66}\left\{n(n-1)-\bar{c}_{11}(\alpha_i a)^2-\varsigma\left(\bar{c}_{13}a_i+\bar{e}_{31}b_i\right)\right\}J_n(\alpha_i a)+2\bar{c}_{66}(\alpha_i a)J_{n+1}(\alpha_i a),\quad i=1,2,3,$$

$$a_{14}=2\bar{c}_{66}n\left\{(n-1)J_n(\alpha_4 a)-(\alpha_4 a)J_{n+1}(\alpha_4 a)\right\},\quad a_{15}=\rho^f\varpi^2 H_n^{(1)}(\alpha_5 a),$$

$$a_{2i}=2n\left\{(n-1)J_n(\alpha_i a)+(\alpha_i a)J_{n+1}(\alpha_i a)\right\},\ i=1,2,3,$$

$$a_{24}=\left\{\left[(\alpha_4 a)^2-2n(n-1)\right]J_n(\alpha_4 a)-2(\alpha_4 a)J_{n+1}(\alpha_4 a)\right\},\quad a_{25}=0,$$

$$a_{3i}=\left((\varsigma+a_i)+\bar{e}_{15}b_i\right)\left\{nJ_n(\alpha_i a)-(\alpha_i a)J_{n+1}(\alpha_i a)\right\},\quad i=1,2,3,\quad a_{34}=n\varsigma J_n(\alpha_4 a),\quad a_{35}=0,$$

$$a_{4i}=b_iJ_n(\alpha_i a),\quad i=1,2,3,\quad a_{44}=0,\quad a_{45}=0,\quad a_{5i}=\left\{nH_n^{(1)}(\alpha_i a)-(\alpha_i a)H_{n+1}^{(1)}(\alpha_i a)\right\},\quad i=1,2,3,$$

$$a_{54}=nJ_n(\alpha_4 a),\quad a_{55}=-\left\{nH_n^{(1)}(\alpha_5 a)-(\alpha_5 a)H_{n+1}^{(1)}(\alpha_5 a)\right\}.$$

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