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PLANE WAVES AND FUNDAMENTAL SOLUTION IN A MODIFIED COUPLE STRESS GENERALIZED THERMOELASTIC WITH MASS DIFFUSION

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Abstract. The present investigation deals with the study of plane waves and fundamental solution in a modified couple stress generalized thermoelastic solid with mass diffusion. It is found that for two-dimensional model, there exists three longitudinal waves namely Longitudinal wave (P-wave), thermal wave (T- wave), mass diffusion wave (MD-wave) and a set of coupled transverse waves (SV1 and SV2 waves). The phase velocity, attenuation coefficient, specific loss and penetration depth are computed numerically and presented graphically. The fundamental solution for the system of differential equations for steady oscillations in terms of elementary functions has been constructed. Some properties of fundamental solution are also established. Various particular cases of interest are also deduced from the present investigations and compared with the known results.

1. Introduction

Classical first gradient approaches in continuum mechanics do not address the size dependency that is observed in smaller scales. Consequently, a number of theories that include higher gradients of deformation have been proposed to capture, at least partially, size-effects at the nano-scale. Additionally, consideration of the second gradient of deformation leads naturally to the introduction of the concept of couple-stresses. Thus, in the current form of these theories, the material continuum may respond to body and surface couples, as well as spin inertia for dynamical problems.

The existence of couple-stress in materials was originally postulated by Voigt (1887) [1]. However, Cosserat and Cosserat (1909) [2] were the first to develop a mathematical model to analyze materials with couple stresses. Lacking an internal material length scale parameter, classical elasticity and plasticity cannot be used to interpret the size effect observed in numerous tests at micron and nanometer scales. However, higher-order (non-local) continuum theories contain material length scale parameters and are capable of explaining microstructure related size (and other effects). Couple stress theories represent one class of such higher-order theories. The classical couple stress elasticity theory, proposed by, e.g., Mindlin and Tiersten (1962), Toupin (1962), Mindlin (1964), and Koiter (1964) [3-6], contains four material constants two classical and two additional for isotropic elastic materials. The couple stress theory can be viewed as a special format of strain gradient theory which uses rotation as a variable to describe curvature, while the strain gradient theory uses strain as variable to describe curvature.

Couple-stress theory is an extended continuum theory that includes the effects of a couple per unit area on a material volume, in addition to the classical direct and shear forces per unit area. This immediately admits the possibility of asymmetric stress tensor, since shear stress no longer have to be conjugate in order to ensure rotational equilibrium. The two additional constants are related to the underlying microstructure of the material and are inherently difficult to determine (e.g., Yang and Lakes (1982) [7], Lam et al. (2003) [8]). Every physical theory possesses a certain domain of applicability outside which it fails to predict the physical phenomena with reasonable accuracy. Hence, there has been a need to develop higher-order theories involving only one additional material length scale parameter.

Recently, Yang et al. (2002) [9] developed a modified couple-stress model, in which the couple stress tensor is symmetrical and only one material length parameter is needed to capture the size effect which is caused by micro-structure.

Park and Gao (2006) [10] studied the Bernoulli- Euler beam model based on a modified couple stress theory. Variational formulation of a modified couple stress theory and its application to a simple shear problem was studied by Ma et al. (2008) [11]. Ke and Wang (2010) [12] investigated the size effect on dynamic stability of functionally graded microbeams based on a modified couple stress theory. Chen et al. (2011) [13] presented a modified couple stress model for bending analysis of composite laminated beams with first order shear deformation. Asghari (2012) [14] studied the geometrically nonlinear micro-plate formulation based on the modified couple stress theory. Simsek and Reddy (2013) [15] investigated the bending and vibration of fuctionally graded microbeams using a new higher order beam theory and the modified couple stress theory. Recently, the size dependent buckling analysis of microbeams based on modified couple stresss theory with high order theories and general boundary conditions have been studied by Mohammad-Abadi and Daneshmehr (2014) [16]. Shaat et al. (2014) [17] studied the size-dependent bending analysis of Kirchhoff nano-plates based on a modified couple-stress theory including surface effects.

Thermodiffusion in an elastic solid is due to the coupling of the fields of temperature, mass diffusion and strain. Heat and mass exchange with the environment during the process of the thermodiffusion in an elastic solid. The concept of thermodiffusion is used to describe the processes of thermomechanical treatment of metals (carboning, nitriding steel, etc.) and these processes are thermally activated, and their diffusing substances being, e.g. nitrogen, carbon etc. They are accompanied by deformations of the solid. Nowacki (1974a, 1974b, 1974c, 1976) [18-21] and Podstrigach (1965) [22] developed the theory of thermoelastic with mass diffusion. In this theory, the coupled thermoelastic model is used. This implies infinite speeds of propagation of thermoelastic waves. Sherief et al. (2004) [23] developed the theory of generalized thermoelastic diffusion that predicts finite speeds of propagation for thermoelastic and diffusive waves. Sherief and Saleh (2005) [24] worked on a problem of a thermoelastic half space with a permeating substance in contact with the bounding plane in the context of the theory of generalized thermoelastic diffusion with one relaxation time. Recently, Kumar et al. (2008) [25] derived the basic equations in generalized thermoelastic diffusion for Green Lindsay (GL-model) theory and discussed the Lamb waves.

It is necessary to construct a fundamental solution of systems of partial differential equations to investigate the boundary value problems of the theory of elasticity and thermoelasticity. The fundamental solutions in the classical theory of coupled thermoelasticity were studied by Hetnarski (Hetnarski 1964a; Hetnarski 1964b) [26, 27]. The fundamental solutions in the microcontinuum fields theories have been constructed by Svanadze (Svanadze 1988; Svanadze 1996; Svanadze 2004a; Svanadze 2004b) [28-31]. Kumar and Kansal (2012) [32] studied the plane waves and fundamental solution in the generalized theories of thermoelastic diffusion. Fundamental solution in the theory of micropolar thermoelastic diffusion with voids was studied by Kumar and Kansal (2012) [33]. Kumar et al. (2014) [34] investigated the plane wave and fundamental solution in a couple stress generalized thermoelastic solids. Plane wave propagation in microstretch thermoelastic medium with microtemperatures studied by Kumar et al. (2014) [35]. Kumar et al. (2015) [36] discussed the fundamental solution in micropolar viscothermoelastic solids with void.

In this article, propagation of plane waves in an isotropic modified couple stress generalized thermoelastic with mass diffusion solid has been investigated. The phase velocity, attenuation coefficient, specific loss and penetration depth of plane waves are computed and presented graphically for different values of frequency. The representation of the fundamental solution of system of equations in the case of steady oscillations is considered in terms of elementary functions. Some basic properties of the fundamental solution are also established.

2. Basic equations

Let $\boldsymbol{x} = (x_1, x_2, x_3)$ will be the point of Euclidean three-dimensional space $E^3 = |\boldsymbol{x}| = (x_1^2 + x_2^2 + x_3^2)^{\frac{1}{2}}$ and let t denotes the time variable.

Following Yang et al. (2002), Sherief (2004), and Kumar et al. (2008) [9, 23, 25] the governing equations in a modified couple-stress generalized thermoelastic elastic with mass diffusion in absence of body forces, body couples, heat and mass diffusion sources are given by

$$(\lambda + \mu) grad\ div\, u_i + \mu\, \Delta\, u_i + \frac{\alpha}{4}\Delta\big(e_{ijk}e_{kpq} grad\ div\ \ u_i\ \big) - \beta_1\left(1 + \tau_1\frac{\partial}{\partial t}\right) grad\ T$$

$$-\beta_2\left(1 + \tau^1\frac{\partial}{\partial t}\right) grad\ C = \rho\, \ddot{u}_i, \tag{1}$$

$$K^*\Delta T - \rho c_e\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)T - aT_0\left(\frac{\partial}{\partial t} + \gamma\frac{\partial^2}{\partial t^2}\right)C = T_0\beta_1\left(\frac{\partial}{\partial t} + \tau_0\eta_0\frac{\partial^2}{\partial t^2}\right) div\ u_i, \tag{2}$$

$$D\beta_2\Delta\ div\ u_i + Da\left(1 + \tau_1\frac{\partial}{\partial t}\right)\Delta T + \left(\frac{\partial}{\partial t} + \tau^0\eta_0\frac{\partial^2}{\partial t^2}\right)C - Db\Delta\left(1 + \tau^1\frac{\partial}{\partial t}\right)C = 0, \tag{3}$$

where u_i are the displacement components; T is the temperature change; T_0 is the reference temperature assumed to be such that $|T/T_0| \ll 1$; C is the concentration of the diffusion material in the elastic body; $\lambda,\ \mu,\ \gamma$ are material constants; α is the couple stress parameter; K^* is the coefficient of the thermal conductivity; ρ is the density; c_e is the specific heat at constant strain; D is the thermoelastic diffusion constant; a is the coefficient describing the measure of thermo diffusion; b is the coefficient describing the measure of mass diffusion effects; Δ is the Laplacian operator; $\beta_1 = (3\lambda + 2\mu)\alpha_t$, $\beta_2 = (3\lambda + 2\mu)\alpha_c$. Here α_t , α_c are the coefficients of linear thermal expansion and diffusion expansion, respectively. Here τ^0, τ^1 are the diffusion relaxation times with $\tau^1 \geq \tau^0 \geq 0$ and $\tau_0,\ \tau_1$ are thermal relaxation times with $\tau_1 \geq \tau_0 \geq 0$. Here $\tau_1 = \ \tau^1 = 0,\ \eta_0 = 1, \gamma = \tau_0$, for Lord-Shulman (L-S) model and $\eta_0 = 0,\ \gamma = \tau^0$ for Green Lindsay (G-L) model.

We define the dimensionless quantities:

$$x_i' = \frac{\omega^*}{c_1}x_i,\quad u_i' = \frac{\omega^*}{c_1}u_i,\ t' = \omega^* t,\ \gamma' = \omega^*\gamma,\ \tau_1' = \omega^*\tau_1,$$

$$\tau_0' = \omega^*\tau_0,\ \tau^{0'} = \omega^*\,\tau^0,\ \tau^{1\prime} = \omega^*\tau^1,\ T' = \frac{\beta_1 T}{\rho c_1^2}\ ,\ C' = \frac{\beta_2 C}{\rho c_1^2}, c_1^2 = \frac{\lambda+2\mu}{\rho},\ \omega^{*2} = \frac{\lambda}{(\mu t^2 + \rho\alpha)}. \tag{4}$$

Here ω^* and c_1 are characteristic frequency and longitudinal wave velocity in the media, respectively.

Using the dimensionless quantities given by equation (4) in equations (1) - (3), after suppressing the primes, we obtain

$$a_1\frac{\partial e}{\partial x_i} + a_2\Delta u_i + a_3\Delta\frac{\partial e}{\partial x_i} - a_3\Delta^2 u_i - \tau_t\frac{\partial T}{\partial x_i} - \tau_t^1\frac{\partial C}{\partial x_i} = \frac{\partial^2 u_i}{\partial t^2}, \tag{5}$$

$$\Delta T - a_4\tau_t^0 T - a_5\tau_\gamma^0 C = a_6\tau_{\eta_0}^0, \tag{6}$$

$$\Delta \mathrm{e} + a_7\tau_t\Delta T + a_8\tau_t^{10}\mathrm{C} - a_9\tau_t^1\Delta C = 0, \tag{7}$$

where

$$a_1 = \frac{(\lambda+\mu)}{\rho c_1^2}, a_2 = \frac{\mu}{\rho c_1^2}, a_3 = \frac{\alpha\omega^{*2}}{4\rho c_1^4}, a_4 = \frac{\rho c_e c_1^2}{K^*\omega^*}, a_5 = \frac{aT_0\beta_1 c_1^2}{\beta_2 K^*\omega^*}, a_6 = \frac{T_0\beta_1^2}{\rho K^*\omega^*}, \ a_7 = \frac{a\rho c_1^2}{\beta_1\beta_2},$$

$$a_8 = \frac{\rho c_1^4}{\beta_2^2 D\omega^*}, a_9 = \frac{b\rho c_1^2}{\beta_2^2}, \tau_t = \left(1 + \tau_1\frac{\partial}{\partial t}\right), \tau_t^1 = \left(1 + \tau^1\frac{\partial}{\partial t}\right), \tau_{\eta_0}^0 = \left(\frac{\partial}{\partial t} + \eta_0\tau_0\frac{\partial^2}{\partial t^2}\right),$$

$$\tau_t^0 = \left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right), \tau_\gamma^0 = \left(\frac{\partial}{\partial t} + \gamma\frac{\partial^2}{\partial t^2}\right), \tau_t^{10} = \left(\frac{\partial}{\partial t} + \eta_0\tau^0\frac{\partial^2}{\partial t^2}\right), e = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3},$$

$$\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}.$$

3. Plane waves

We consider a plane wave propagation in a homogeneous isotropic modified couple stress generalized thermoelastic elastic with mass diffusion.

For two-dimensional problem, we take

$$u_i = \big(u_1(x_1, x_3, t), 0, u_3(x_1, x_3, t)\big), T(x_1, x_3, t), C(x_1, x_3, t). \tag{8}$$

The relation connecting the displacement components to the potential functions in dimensional form is taken as

$$u_1 = \frac{\partial\phi}{\partial x_1} - \frac{\partial\psi}{\partial x_3}, u_3 = \frac{\partial\phi}{\partial x_3} + \frac{\partial\psi}{\partial x_1}. \tag{9}$$

Equations (5)-(7) with the aid of (8) and (9) take the form

$$\left[(a_1 + a_2)\nabla^2 - \frac{\partial^2}{\partial t^2}\right]\phi - \tau_t T - \tau_t^1 C = 0, \tag{10}$$

$$\left[a_2\nabla^2 - a_3\nabla^4 - \frac{\partial^2}{\partial t^2}\right]\psi = 0, \tag{11}$$

$$\left(-a_6\tau_{\eta_0}^0\nabla^2\right)\phi + (\nabla^2 - a_4\tau_t^0)T - a_5\tau_\gamma^0 C = 0, \tag{12}$$

$$\nabla^4\phi + a_7\tau_t\nabla^2 T + (a_8\tau_t^{10} - a_9\tau_t^1\nabla^2)C = 0, \tag{13}$$

where $\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}$, $e = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}$.

We assume the solution of the form

$$(\phi, T, C, \psi) = \left(\tilde{\phi}, \tilde{T}, \tilde{C}, \tilde{\psi}\right) e^{[i(\xi(l_1x_1 + l_3x_3) - \omega t)]}, \tag{14}$$

where $\omega(= \xi c)$ is the frequency; ξ is the wave number and c is the phase velocity; $\tilde{\phi}, \tilde{T}, \tilde{C}, \tilde{\psi}$ are undetermined amplitudes, that are dependent on time t and coordinates $x_m (m = 1, 3)$; l_1 and l_3 are the direction cosines of the wave normal to the x_1x_3–plane with the property $l_1^2 + l_3^2 = 1$.

Using eq. (14) in (10)-(13), we obtain

$$[-(a_1 + a_2)\xi^2 + \omega^2]\tilde{\phi} - \tau_t^{11}\tilde{T} - \tau_t^{21}\tilde{C} = 0, \tag{15}$$

$$(a_6\tau_t^{31}\xi^2)\tilde{\phi} - (\xi^2 + a_4\tau_t^{41})\tilde{T} - a_5\,\tau_t^{51}\tilde{C} = 0, \tag{16}$$

$$\xi^4\tilde{\phi} - a_7\tau_t^{11}\xi^2\,\tilde{T} + (a_8\tau_t^{61} + a_9\tau_t^{21}\xi^2)\tilde{C} = 0, \tag{17}$$

$$[a_3\xi^4 + a_2\xi^2 - \omega^2]\tilde{\psi} = 0, \tag{18}$$

where $\tau_t^{11} = \left(1 + \tau_1(-i\omega)\right), \tau_t^{21} = \left(1 + \tau^1(-i\omega)\right), \tau_t^{\,31} = i\omega(1 - \eta_0\tau_0 i\omega)$, $\tau_t^{41} = i\omega(1 - \tau_0 i\omega)$, $\tau_t^{51} = i\omega(-1 + i\omega\gamma)$.

For the non-trivial solution of the system of equations (15)-(17) yields the following polynomial characteristic equation in ξ as:

$$F_1\,\xi^6 + F_2\xi^4 + F_3\,\xi^2 + F_4 = 0\,. \tag{19}$$

where $F_1 = (a_9(a_1 + a_2) - 1)\tau_t^{21}$,

$F_2 = (a_1 + a_2)\{a_8\tau_t^{61} + a_4a_9\tau_t^{21}\tau_t^{41} + a_5a_7\tau_t^{11}\,\tau_t^{51}\} + a_6\tau_t^{31}\tau_t^{11}\tau_t^{21}(a_9 + a_7) -$

$\tau_t^{21}(a_9\,\omega^2 + a_4\tau_t^{41}) + a_5\,\tau_t^{51}\tau_t^{11}$, $F_3 = a_8\tau_t^{61}\left(-\omega^2 + a_6\tau_t^{31}\tau_t^{11} + a_4\tau_t^{41}(a_1 + a_2)\right) -$

$\omega^2(a_5a_7\tau_t^{11}\,\tau_t^{51} + a_4a_9\tau_t^{21}\tau_t^{41})$, $F_4 = -a_4a_8\tau_t^{41}\tau_t^{61}\omega^2$.

Solving equation (19), we obtain six roots of ξ, in which three roots ξ_1, ξ_2 and ξ_3 correspond to positive x_3 – direction and other three roots $-\xi_1, -\xi_2$ and $-\xi_3$ correspond to negative x_3 – direction. Corresponding to roots ξ_1, ξ_2 and ξ_3 there exist three longitudinal waves in descending order of their velocities, namely longitudinal wave (P-wave), thermal wave (T-wave) and mass diffusive wave (MD-wave). From equation (18), we obtain four roots of ξ, in which two roots ξ_1 and ξ_2 correspond to positive x_3 – direction and other three roots $-\xi_1$ and $-\xi_2$ correspond to negative x_3 – direction. Corresponding to roots ξ_1 and ξ_2 there exist two waves in descending order of their velocities, namely a set of coupled transverse waves (SV1 and SV2) waves. It is noticed that these two values are not affected by thermal and diffusion properly of the medium.

We now derive the expressions of phase velocity, attenuation coefficient, specific loss and penetration depth of these types of waves as

(i) Phase velocity

The Phase velocities is given by

$$V_i = \frac{\omega}{|\mathrm{Re}(\xi_i)|}, \quad i = 1, 2, 3, 4, 5, \tag{20}$$

where V_1, V_2, V_3, V_4, V_5 are the phase velocities of P, T, MD, SV1 and SV2 waves respectively.

(ii) Attenuation coefficient

The attenuation coefficient are defined as

$$Q_i = \mathrm{Im}(\xi_i), \quad i = 1, 2, 3 \tag{21}$$

where Q_1, Q_2, Q_3 are the attenuation coefficients of P-wave, T-wave, and MD-wave, respectively.

(iii) Specific loss

The specific loss is the ratio of energy (ΔW) dissipated in taking a specien through a stress cycle, to the elastic energy (W) stored in the specimen when the strain is a maximum. The specific loss is the most direct method of defining internal friction for a material. For a sinusoidal plane wave of small amplitude, Kolsky (1963) [37] shows that the specific loss $\Delta W/W$ equals 4π times of the absolute values of the imaginary part of ξ to the real part of ξ, i.e.

$$R_i = \left(\frac{\Delta W}{W}\right)_i = 4\pi\left|\frac{\mathrm{Im}(\xi_i)}{\mathrm{Re}(\xi_i)}\right|, \quad i = 1, 2, 3 \tag{22}$$

(iv) Penetration depth

The penetration depths is defined by

$$S_i = \frac{1}{|\mathrm{Im}(\xi_i)|}, \quad i = 1, 2, 3. \tag{23}$$

4. Steady oscillations

For the case of steady oscillations, we assume the displacement vector, temperature change and mass concentration as

$$\big(\boldsymbol{u}(\boldsymbol{x},t), T(\boldsymbol{x},t), C(\boldsymbol{x},t)\big) = Re\,\big[(\bar{\boldsymbol{u}}, \bar{T}, \bar{C},)e^{-i\omega t}\big], \tag{24}$$

where ω is the oscillation frequency and $\omega > 0$.
Make use (24) in equations (5) - (7), we obtain the system of equations for steady oscillations as

$$(a_2\Delta + \omega^2)\boldsymbol{u} + (a_1 + a_3\Delta)grad\ div\ \boldsymbol{u} - a_3\Delta^2\boldsymbol{u} - \delta_t^1 grad\ T - \delta_t^2 grad\ C = 0, \tag{25}$$

$$\Delta T + a_4\delta_t^3 T + a_5\delta_t^4 C + a_6\delta_t^5 div\ \boldsymbol{u} = 0, \tag{26}$$

$$\Delta\ div\ \boldsymbol{u} + a_7\delta_t^1\Delta T - (a_8\delta_t^6 + a_9\delta_t^2\Delta)C = 0, \tag{27}$$

where
$\delta_t^1 = \big(1 + \tau_1(-i\omega)\big)$, $\delta_t^2 = \big(1 - \tau^1(-i\omega)\big)$, $\delta_t^3 = i\omega(1 - i\tau_0\omega)$, $\delta_t^4 = i\omega(1 - i\gamma\omega)$, $\delta_t^5 = i\omega(1 - \eta_0\tau_0 i\omega)$, $\delta_t^6 = i\omega\big(1 - \eta_0\tau^0(i\omega)\big)$.
Introducing the matrix differential operator as

$$\boldsymbol{F}(\boldsymbol{D_x}) = \|F_{gh}(\boldsymbol{D_x})\|_{5\times5}, \tag{28}$$

where $F_{mn}(\boldsymbol{D_x}) = (a_2\Delta + \omega^2 - a_3\Delta^2)\delta_{mn} + (a_1 + a_3\Delta)\frac{\partial^2}{\partial x_m\partial x_n}$, $F_{45}(\boldsymbol{D_x}) = a_5\delta_t^4$,

$F_{4n}(\boldsymbol{D_x}) = a_6\delta_t^5\frac{\partial}{\partial x_n}$, $F_{44}(\boldsymbol{D_x}) = (\Delta + a_4\delta_t^3)$, $F_{54}(\boldsymbol{D_x}) = a_7\delta_t^1\Delta$, $F_{m4}(\boldsymbol{D_x}) = -\delta_t^1\frac{\partial}{\partial x_m}$,

$$F_{m5}(\boldsymbol{D_x}) = -\delta_t^2\frac{\partial}{\partial x_m},\ F_{55}(\boldsymbol{D_x}) = -(a_8\delta_t^6 + a_9\delta_t^2\Delta),\ F_{5n}(\boldsymbol{D_x}) = \Delta\frac{\partial}{\partial x_n}, m, n = 1, 2, 3. \tag{29}$$

Here δ_{mn} is the Kronecker delta function.
The system of equations (25) - (27) can be written as

$$\boldsymbol{F}(\boldsymbol{D_x})\boldsymbol{U}(\boldsymbol{x}) = 0, \tag{30}$$

where $\boldsymbol{U} = (\boldsymbol{u}, T, C)$ is a five component vector function on E^3.
We assume that

$$a_3^3 a_9\delta_t^2 \neq 0. \tag{31}$$

If the condition (31) is satisfied, then $\boldsymbol{F}$ is an elliptic differential operator (Hörmander, 1963) [38].
Definition: The fundamental solution of the system of equations (25) - (27) (the fundamental matrix of operator $\boldsymbol{F}$) is the matrix $G(\boldsymbol{x}) = \|G_{gh}(\boldsymbol{x})\|_{5\times5}$ satisfying the condition (Hörmander, 1963) [38]:

$$\boldsymbol{F}(\boldsymbol{D_x})\boldsymbol{G}(\boldsymbol{x}) = \delta(\boldsymbol{x})\boldsymbol{I}(\boldsymbol{x}), \tag{32}$$

where $\delta(\ )$ is the Dirac delta, $\boldsymbol{I} = \|\delta_{gh}\|_{5\times5}$ is the unit matrix and $\boldsymbol{x} \in E^3$.
Now we construct $\boldsymbol{G}(\boldsymbol{x})$ in terms of the elementary functions.

5. Fundamental solution of system of equations of steady oscillations

We consider the system of equations

$$(a_2\Delta + \omega^2 - a_3\Delta^2)\boldsymbol{u} + (a_1 + a_3\Delta)grad\ div\ \boldsymbol{u} + a_6\delta_t^5 grad\ T + \Delta grad\ C = \boldsymbol{H}, \tag{33}$$

$$-\delta_t^1 div\ \boldsymbol{u} + (\Delta + a_4\delta_t^3)T + (a_7\delta_t^1\Delta)C = L, \tag{34}$$

$$-\delta_t^2\, div\, \boldsymbol{u} + a_5\delta_t^4\, T - (a_8\delta_t^6 + a_9\delta_t^2\Delta)C = M, \tag{35}$$

where $\boldsymbol{H}$ is the three-component vector is function on E^3 and L and M are scalar functions on E^3.
The system of equations (33), (34) and (35) may be written in the form

$$\boldsymbol{F}^{tr}(\boldsymbol{D_x})\boldsymbol{U}(\boldsymbol{x}) = \boldsymbol{Q}(\boldsymbol{x}), \tag{36}$$

where $\boldsymbol{F}^{tr}$ is the transpose of matrix $\boldsymbol{F}, \boldsymbol{Q} = (\boldsymbol{H}, L, M)$, and $\boldsymbol{x} \in E^3$.
Applying the div operator on equation (33), we obtain

$$[(a_1 + a_2)\Delta + \omega^2] div\, \boldsymbol{u} + a_6\delta_t^5\Delta\, T + \Delta^2\, C = div\, \boldsymbol{H}, \tag{37}$$

$$-\delta_t^1\, div\, \boldsymbol{u} + (\Delta + a_4\delta_t^3)T + (a_7\delta_t^1\Delta)C = L, \tag{38}$$

$$-\delta_t^2\, div\, \boldsymbol{u} + a_5\delta_t^4 T - (a_8\delta_t^6 + a_9\delta_t^2\Delta)C = M, \tag{39}$$

Equations (37)-(39) can be written as

$$\Gamma_1(\Delta)\boldsymbol{S} = \boldsymbol{\psi}, \tag{40}$$

where $\Gamma_1(\Delta) = e^* \det N(\Delta)$, $\boldsymbol{S} = (div\, \boldsymbol{u}, T, C)$, $\boldsymbol{\psi} = (\psi_1, \psi_2, \psi_3)$ and

$$N(\Delta) = \|N_{mn}\|_{3\times3} = \left\| \begin{matrix} (a_1 + a_2)\Delta + \omega^2 & a_6\delta_t^5\Delta & \Delta^2 \\ -\delta_t^1 & (\Delta + a_4\delta_t^3) & (a_7\delta_t^1\Delta) \\ -\delta_t^2 & a_5\delta_t^4 & -(a_8\delta_t^6 + a_9\delta_t^2\Delta) \end{matrix} \right\|_{3\times3},$$

$$e^* = -\frac{1}{(a_1+a_2)a_9\delta_t^2}, \psi_n = e^* \textstyle\sum_{m=1}^3 N_{mn}^*\, d_m, (d_1,\ d_2,\ d_3) = (div\, \boldsymbol{H},\ L, M), n = 1, 2, 3. \tag{41}$$

and N_{mn}^* is the cofactor of the elements N_{mn} of the matrix $\boldsymbol{N}$.
From (41) we notice that

$$\Gamma_1(\Delta) = \textstyle\prod_{m=1}^3(\Delta + \lambda_m^2), \tag{42}$$

where λ_m^2 , $m = 1, 2, 3$ are the roots of the equation $\Gamma_1(-\kappa) = 0$ (with respect to κ).
Applying the operator $\Gamma_1(\Delta)$ on equation (33), we obtain

$$\Gamma_1(\Delta)(a_2\Delta + \omega^2 - a_3\Delta^2)\boldsymbol{u} = [-(a_1 + a_3\Delta) grad\, \psi_1 - (a_6\delta_t^5) grad\, \psi_2 - \Delta\, grad\, \psi_3 + \Gamma_1(\Delta)\boldsymbol{H}], \tag{43}$$

The above equation (43) can be written as

$$\Gamma_1(\Delta)\Gamma_2(\Delta)\boldsymbol{u} = \widetilde{\boldsymbol{\psi}}, \tag{44}$$

where

$$\Gamma_2(\Delta) = f^* \det \left\| \begin{matrix} -a_3\Delta + a_2 & -\omega \\ \omega & \Delta \end{matrix} \right\|_{2\times2}, f^* = -\frac{1}{a_3} \tag{45}$$

and

$$\widetilde{\boldsymbol{\psi}} = f^*[-(a_1 + a_3\Delta) grad\, \psi_1 + \{ \Gamma_1(\Delta)\boldsymbol{H} - (a_6\delta_t^5)\, grad\, \psi_2 - \Delta\, grad\, \psi_3\}]. \tag{46}$$

It is evident that

$$\Gamma_2(\Delta) = (\Delta + \lambda_3^2)(\Delta + \lambda_4^2), \tag{47}$$

where λ_m^2 , $m = 4, 5$ are the roots of the equation $\Gamma_1(-\kappa) = 0$ (with respect to κ).
From (40) and (44), we get

$$\Theta(\Delta)\, U(\boldsymbol{x}) = \widehat{\boldsymbol{\psi}}(\boldsymbol{x}), \tag{48}$$

where

$$\widehat{\boldsymbol{\psi}}(\boldsymbol{x}) = \left(\widetilde{\boldsymbol{\psi}}, \psi_2, \psi_3\right), \Theta(\Delta) = \left\|\Theta_{gh}(\Delta)\right\|_{5\times5}, \Theta_{mm}(\Delta) = \Gamma_1(\Delta)\Gamma_2(\Delta) = \prod_{q=1}^{5}\left(\Delta + \lambda_q^2\right),$$

$$\Theta_{gh}(\Delta) = 0,\ \Theta_{44}(\Delta) = \Gamma_1(\Delta), \Theta_{55}(\Delta) = \Gamma_1(\Delta), m = 1,2,3 \quad g,h = 1,2,3,4,5. \quad g \neq h. \tag{49}$$

Equations (40) and (44) can be rewritten in the form

$$\widetilde{\boldsymbol{\psi}} = [f^*\, \Gamma_1(\Delta)\boldsymbol{J} + q_{11}(\Delta) grad\ div]\boldsymbol{H} + q_{21}(\Delta)\ grad\ L\ + q_{31}(\Delta)\ grad\ M,$$

$$\psi_2 = q_{12}(\Delta)\ div\ \boldsymbol{H} + q_{22}\ (\Delta)L +\ q_{32}\ (\Delta)M,$$

$$\psi_3 = q_{13}(\Delta)\ div\ \boldsymbol{H} + q_{23}(\Delta)L +\ q_{33}(\Delta)\ M, \tag{50}$$

where $\boldsymbol{J} = \left\|\delta_{gh}\right\|_{3\times3}$ is the unit matrix. In equation (50) we use the following notations:

$$q_{m1}(\Delta) = -f^*e^*[(a_1 + a_3\Delta)N_{m1}^* + (a_6\delta_t^5)N_{m2}^* + \Delta\, N_{m3}^*],\ q_{m2}(\Delta) = e^*N_{m2}^*\,,$$

$$q_{m3}(\Delta) = e^*N_{m3}^*\,,\ m = 1,2,3.$$

Now from (50) we have that

$$\widehat{\boldsymbol{\psi}} = \boldsymbol{R}^{tr}(\boldsymbol{D_x})\boldsymbol{Q}(\boldsymbol{x}), \tag{51}$$

where $\boldsymbol{R}^{tr}$ is the transpose of matrix $\boldsymbol{R}$ and

$$\boldsymbol{R} = \left\|R_{gh}\right\|_{5\times5}, R_{mn}(\boldsymbol{D_x}) = f^*\,\Gamma_1(\Delta)\delta_{mn} + q_{11}(\Delta)\frac{\partial^2}{\partial x_m \partial x_n}\,,$$

$$R_{14}(\boldsymbol{D_x}) = q_{12}(\Delta)\frac{\partial}{\partial x_m},\ R_{15}(\boldsymbol{D_x}) = q_{13}(\Delta)\frac{\partial}{\partial x_m}, R_{41}(\boldsymbol{D_x}) = q_{21}(\Delta)\frac{\partial}{\partial x_m},$$

$$R_{44}(\boldsymbol{D_x}) = q_{22}\,(\Delta),\ R_{45}(\boldsymbol{D_x}) = q_{23}(\Delta)\,,\ R_{5n}(\boldsymbol{D_x}) = q_{31}(\Delta)\frac{\partial}{\partial x_m}\,, R_{54}(\boldsymbol{D_x}) = q_{32}(\Delta)\,,$$

$$R_{55}(\boldsymbol{D_x}) = q_{33}(\Delta). \qquad m,n = 1,2,3. \tag{52}$$

Now from equations (36), (48) and (51), we obtain

$$\boldsymbol{\Theta U} = \boldsymbol{R}^{tr}\boldsymbol{F}^{tr}\boldsymbol{U}, \tag{53}$$

It implies that $\boldsymbol{R}^{tr}\boldsymbol{F}^{tr} = \boldsymbol{\Theta}$

$$\boldsymbol{\Theta}(\boldsymbol{\Delta}) = \boldsymbol{R}(\boldsymbol{D_x})\boldsymbol{F}(\boldsymbol{D_x}). \tag{54}$$

We assume that

$$\lambda_m^2 \neq \lambda_n^2 \neq 0, \qquad m,n = 1,2,3,4,5 \quad m \neq n. \tag{55}$$

Let

$$Y(\boldsymbol{x}) = \|Y_{rs}(\boldsymbol{x})\|_{5\times5}, Y_{mm}(\boldsymbol{x}) = \sum_{n=1}^{5} r_{1n}\,\zeta_n(\boldsymbol{x}),$$

$$Y_{vw}(\boldsymbol{x}) = 0;\ m = 1,2,3,4;\ v,w = 1,2,3,4,5; \quad v \neq w, \tag{56}$$

where

$$\zeta_n(\boldsymbol{x}) = \frac{-1}{4\pi\bar{x}}\exp(i\,\lambda_n|\boldsymbol{x}|), \tag{57}$$

$$\mathrm{r}_{1n} = \prod_{m=1,m\neq n}^{4}(\lambda_m^2 - \lambda_n^2)^{-1}\,, \quad n = 1,2,3,4,5. \tag{58}$$

We will prove the following lemma.

Lemma: The matrix $\boldsymbol{Y}$ defined above is the fundamental matrix of operator $\Theta(\Delta)$, is that

$$\boldsymbol{\Theta}(\Delta)\boldsymbol{Y}(\boldsymbol{x}) = \delta(\boldsymbol{x})\boldsymbol{I}(\boldsymbol{x}) \quad (59)$$

Proof: To prove the lemma, it is sufficient to prove that

$$\Gamma_1(\Delta)\Gamma_2(\Delta)Y_{11}(\boldsymbol{x}) = \delta(\boldsymbol{x}). \quad (60)$$

We find that

$$\mathrm{r}_{11} + \mathrm{r}_{12} + \mathrm{r}_{13} + \mathrm{r}_{14} + \mathrm{r}_{15} = 0,$$

$$\mathrm{r}_{12}(\lambda_1^2 - \lambda_2^2) + \mathrm{r}_{13}(\lambda_1^2 - \lambda_3^2) + \mathrm{r}_{14}(\lambda_1^2 - \lambda_4^2) + \mathrm{r}_{15}(\lambda_1^2 - \lambda_5^2) = 0,$$

$$\mathrm{r}_{13}(\lambda_1^2 - \lambda_3^2)(\lambda_2^2 - \lambda_3^2) + \mathrm{r}_{14}(\lambda_1^2 - \lambda_4^2)(\lambda_2^2 - \lambda_4^2) + \mathrm{r}_{15}(\lambda_1^2 - \lambda_5^2)(\lambda_2^2 - \lambda_5^2) = 0,$$

$$\mathrm{r}_{14}(\lambda_1^2 - \lambda_4^2)(\lambda_2^2 - \lambda_4^2)(\lambda_3^2 - \lambda_4^2) + r_{15}(\lambda_1^2 - \lambda_5^2)(\lambda_2^2 - \lambda_5^2)(\lambda_3^2 - \lambda_5^2) = 0,$$

$$r_{15}(\lambda_1^2 - \lambda_5^2)(\lambda_2^2 - \lambda_5^2)(\lambda_3^2 - \lambda_5^2)(\lambda_4^2 - \lambda_5^2) = 1,$$

$$(\Delta + \lambda_m^2)\zeta_n(\boldsymbol{x}) = \delta(\boldsymbol{x}) + (\lambda_m^2 - \lambda_n^2)\zeta_n(\boldsymbol{x}), \qquad m,n = 1,2,3,4,5. \quad (61)$$

Now we consider

$$\Gamma_1(\Delta)\Gamma_2(\Delta)Y_{11}(\boldsymbol{x}) = (\Delta + \lambda_2^2)(\Delta + \lambda_3^2)(\Delta + \lambda_4^2)(\Delta + \lambda_5^2)\sum_{n=1}^{5}\left[r_{1n}\left(\delta + \left((\lambda_1^2 - \lambda_n^2)\right)\right)\zeta_n\right]$$

$$= (\Delta + \lambda_2^2)(\Delta + \lambda_3^2)(\Delta + \lambda_4^2)(\Delta + \lambda_5^2)\sum_{n=2}^{5}\left[r_{1n}(\lambda_1^2 - \lambda_n^2)\zeta_n\right]$$

$$= (\Delta + \lambda_3^2)(\Delta + \lambda_4^2)(\Delta + \lambda_5^2)\sum_{n=2}^{5}\left[r_{1n}(\lambda_1^2 - \lambda_n^2)\left(\delta + (\lambda_2^2 - \lambda_n^2)\right)\zeta_n\right]$$

$$= (\Delta + \lambda_3^2)(\Delta + \lambda_4^2)(\Delta + \lambda_5^2)\sum_{n=3}^{5}\left[r_{1n}(\lambda_1^2 - \lambda_n^2)\left((\lambda_2^2 - \lambda_n^2)\right)\zeta_n\right]$$

$$= (\Delta + \lambda_4^2)(\Delta + \lambda_5^2)\sum_{n=3}^{5}\left[r_{1n}(\lambda_1^2 - \lambda_n^2)(\lambda_2^2 - \lambda_n^2)\left(\delta + (\lambda_3^2 - \lambda_n^2)\right)\zeta_n\right]$$

$$= (\Delta + \lambda_4^2)(\Delta + \lambda_5^2)\sum_{n=4}^{5}\left[r_{1n}(\lambda_1^2 - \lambda_n^2)(\lambda_2^2 - \lambda_n^2)\left((\lambda_3^2 - \lambda_n^2)\right)\zeta_n\right]$$

$$= (\Delta + \lambda_5^2)\sum_{n=4}^{5}\begin{bmatrix} r_{1n}(\lambda_1^2 - \lambda_n^2)(\lambda_2^2 - \lambda_n^2)(\lambda_3^2 - \lambda_n^2)(\lambda_4^2 - \lambda_n^2) \\ \left(\delta + (\lambda_4^2 - \lambda_n^2)\right)\zeta_n \end{bmatrix} = (\Delta + \lambda_5^2)\zeta_n = \delta. \quad (62)$$

We introduce the matrix

$$\boldsymbol{G}(\boldsymbol{x}) = \boldsymbol{R}(\boldsymbol{D_x})\boldsymbol{Y}(\boldsymbol{x}). \quad (63)$$

From equations (50), (56) and (60), we obtain

$$\boldsymbol{F}(\boldsymbol{D_x})\boldsymbol{G}(\boldsymbol{x}) = \boldsymbol{F}(\boldsymbol{D_x})\boldsymbol{R}(\boldsymbol{D_x})\boldsymbol{Y}(\boldsymbol{x}) = \boldsymbol{\Theta}(\boldsymbol{\Delta})\boldsymbol{Y}(\boldsymbol{x}) = \boldsymbol{\delta}(\boldsymbol{x})\boldsymbol{I}(\boldsymbol{x}). \quad (64)$$

Hence $\boldsymbol{G}(\boldsymbol{x})$ is a solution of (32).
Therefore, we have proved the following theorem
Theorem: The matrix $\boldsymbol{G}(\boldsymbol{x})$ defined by (64) is the fundamental solution of the system of equations (25)-(27).

6. Basic properties of the matrix $\boldsymbol{G}(\boldsymbol{x})$

Property 1: Each column of the matrix $\boldsymbol{G}(\boldsymbol{x})$ is the solution of the system of equations (25)-(27) at every point $\boldsymbol{x} \in E^3$ except the origin.
Property 2: The matrix $\boldsymbol{G}(\boldsymbol{x})$ can be written in the form

$$\boldsymbol{G} = \left\|G_{gh}\right\|_{5\times5}, \boldsymbol{G}_{mn}(\boldsymbol{x}) = \boldsymbol{R}_{mn}(\boldsymbol{D_x})\boldsymbol{Y}_{11}(\boldsymbol{x}),$$

$$\boldsymbol{G}_{m5}(\boldsymbol{x}) = \boldsymbol{R}_{m5}(\boldsymbol{D_x})\boldsymbol{Y}_{55}(\boldsymbol{x}), \ \ m,n = 1,2,3,4,5. \quad (65)$$

7. Particular cases

(i) If $\tau_1 = \tau^1 = 0, \eta_0 = 1, \gamma = \tau_0$ in equation (19), we obtain the corresponding results for modified couple stress thermoelastic with mass diffusion material for Lord-Shulman (L-S) model.
(ii) If $\eta_0 = 0, \gamma = \tau^0$ in equation (19), we obtain the corresponding results for modified couple stress thermoelastic with mass diffusion material for Green-Lindsay (G-L) model.
(iii) If $\alpha = 0$ in equation (18) and (19), we obtain the corresponding results for generalized thermoelastic with mass diffusion model.

Subcases. (i) If $\alpha = 0$ in particular case (i), we obtain the results for thermoelastic with mass diffusion for Lord Shulman (L-S) model. (ii) If $\alpha = 0$ in particular case (ii), we obtain the corresponding results for thermoelastic with mass diffusion for Green Lindsay (G-L) model.

8. Numerical results and discussion

For numerical computations, we take the copper material Sherief and Saleh (2005) [24] (thermoelastic diffusion solid) as:
$\lambda = 7.76 \times 10^{10} Kg\, m^{-1} s^{-2},\ \mu = 3.86 \times 10^{10} Kg\, m^{-1} s^{-2},\ T_0 = 0.293 \times 10^3 K\ ,\ c_e = 0.3831\ \times 10^3 JKg^{-1}K^{-1},\ \alpha_t = 1.78 \times 10^{-5} K^{-1},\ \alpha_c = 1.98 \times 10^{-4} m^3 Kg^{-1},\ a = 1.02 \times 10^4 m^2 s^{-2} K^{-1}, b = 9 \times 10^5 Kg^{-1} m^5 s^{-2}, D = 0.85 \times 10^{-8} Kgsm^{-3}, \rho = 8.954 \times 10^3 Kgm^{-3},\ K^* = 0.383 \times 10^3 Wm^{-1}K^{-1},\ \alpha = 0.05\ Kg\, m\, s^{-2},\ t = 0.5\, s,\ \tau_0 = 0.01\, s,\ \tau^0 = 0.02\ s, \tau_1 = 0.07\ s,\ \tau^1 = 0.08\ s.$
The software Matlab 7.10.4 has been used to determine the values of phase velocity, attenuation coefficient, specific loss and penetration depth of plane waves. The variations of phase velocity, attenuation coefficients, specific loss and penetration depth with respect to frequency are shown in Figs. 1-14, respectively.

In Figures 1-3 and 6-14, the solid line corresponds to couple stress LS model (CTLS), small dash line corresponds to couple stress GL model (CTGL), solid line with centre symbols (×) corresponds to without couple stress LS model (WCTLS), small dash line with centre symbols (×) corresponds to without couple stress GL model (WCTGL). In Figure 4, solid line corresponds to couple stress (CT), small dash line corresponds to without couple stress (WCT). In Figure 5, small dash dotted line corresponds to couple stress (CT) for $\alpha = 0.01$, small dash line corresponds to couple stress (CT) for $\alpha = 0.05$, solid line corresponds to couple stress (CT) for $\alpha = 0.1$.

Phase velocity. From Figures 1 - 3, it is noticed that the values of V_1, V_2 and V_3 increase rapidly with the increase in the values of frequency for all the theories of thermoelasticity. We find that the values of V_1 are more for WCT(LS) and CT(LS) in comparison to WCT(GL) and CT(GL) theories. The values of V_2 are higher for CT(LS) and WCT(LS) in comparison to CT(GL) and WCT(GL). Due to relaxation times the value of V_3 are more for WCT(GL) and CT(GL) in comparison to CT(LS) and WCT(LS). Figure 4 shows that the values of phase velocity V_4 is less for lower frequency and more for higher frequency for CT with comparison to WCT. Figure 5 shows that the values of phase velocity V_5 for $\alpha = 0.1$ are higher than for other two values of $\alpha = 0.01, 0.05$.

Attenuation coefficient. Figure 6 shows that the values of attenuation coefficient Q_1 increase as frequency increase for CT(LS) and WCT(GL), whereas it increase monotonically in the range $0 < \omega \leq 6$ for CT(GL) and WCT(GL) and for $\omega > 6$, the opposite behaviour is noticed. Figure 7 shows that the values of attenuation coefficient Q_2 increase with increase of frequency but the values of Q_2 are more for WCT(LS) in comparison to CT(LS). The value of Q_2 increases with small magnitude in the range $0 < \omega < 5$ for CT(GL) and WCT(GL) and for $\omega \geq 5$, the opposite behaviour is noticed. Figure 8 indicates that the value of attenuation coefficient Q_3 increases monotonically for both CT (LS) and CT(GL) theories. The value of Q_3 for CT (GL) increases with high magnitude as compared to CT(LS) theory.

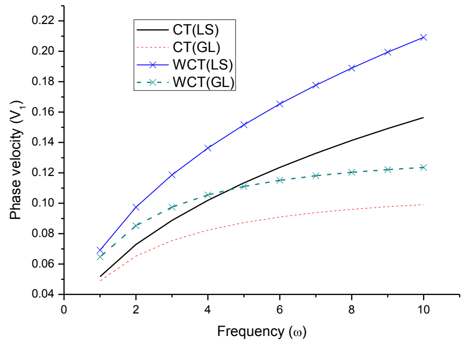

Fig. 1. Variations of phase velocity V_1 w.r.t frequency

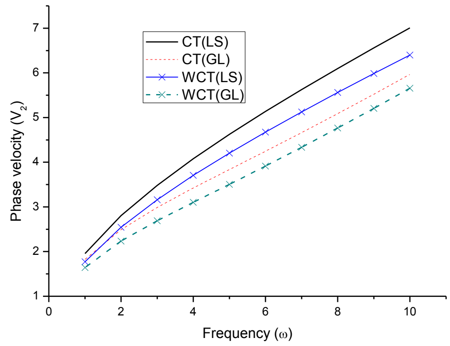

Fig. 2. Variations of phase velocity V_2 w.r.t frequency.

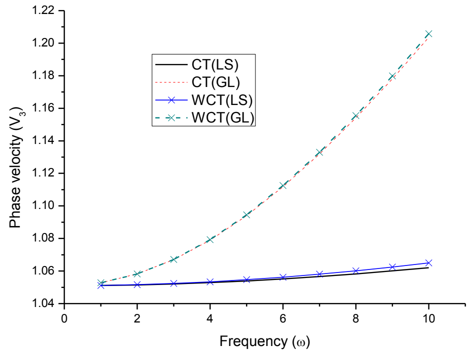

Fig. 3. Variations of phase velocity V_3 w.r.t frequency.

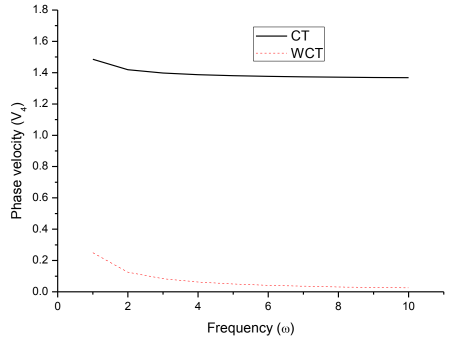

Fig. 4. Variations of phase velocity V_4 w.r.t frequency.

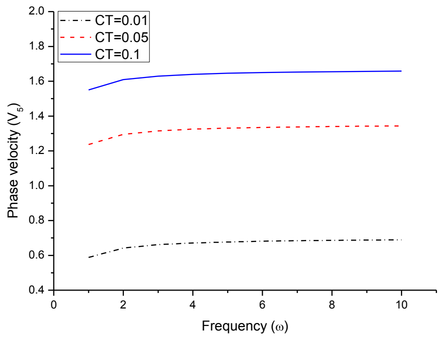

Fig. 5. Variations of phase velocity V_5 w.r.t frequency.

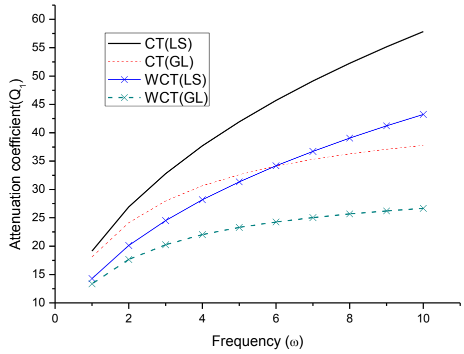

Fig. 6. Variations of attenuation coefficient Q_1 w.r.t. frequency.

Specific loss. We notice from Fig. 9 that the values of specific loss R_1 decrease as frequency increase for CT(LS), CT(GL), WCT(LS) and WCT(GL). The values of specific loss for CT(LS) and WCT(LS) are more in comparison to CT(GL) and WCT(GL). Figure 10 depicts that the values of specific loss R_2 increase monotonically for CT(LS) and WCT(LS) and decrease sharply for CT(GL) and WCT(GL) with higher magnitude. Figure 11 indicates that the values of specific loss R_3 increase with increase in the value of frequency. It is evident that the values of R_3 for CT(GL) is higher as compared to CT(LS) theory.

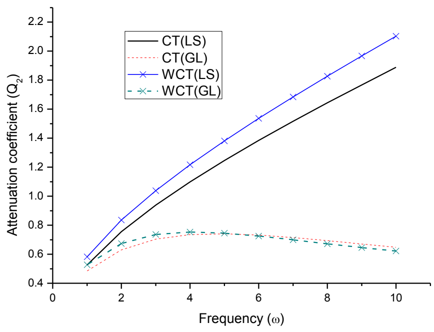

Fig. 7. Variations of attenuation coefficient Q_2 w.r.t. frequency.

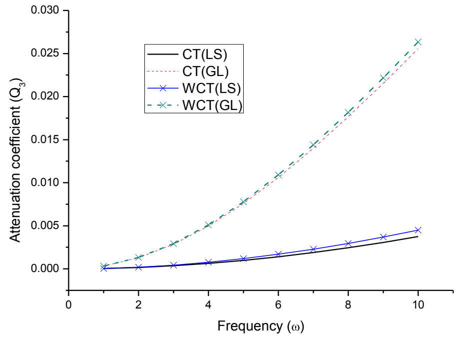

Fig. 8. Variations of attenuation coefficient Q_3 w.r.t. frequency.

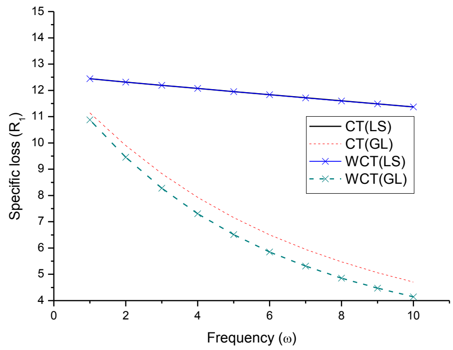

Fig. 9. Variations of specific loss R_1 w.r.t. frequency.

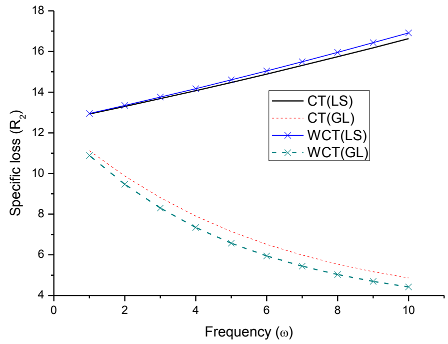

Fig. 10. Variations of specific loss R_2 w.r.t. frequency.

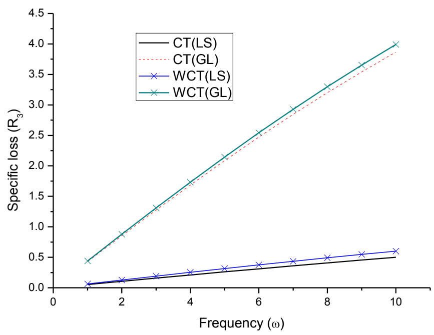

Fig. 11. Variations of specific loss R_3 w.r.t. frequency.

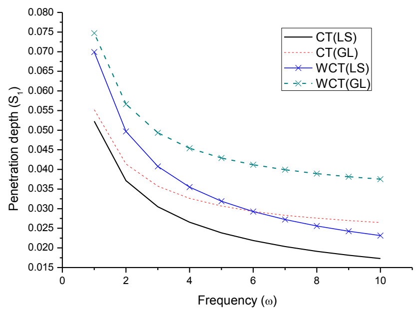

Fig. 12. Variations of penetration S_1 depth w.r.t. frequency.

Penetration depth. Figure 12 shows that the values of penetration depth S_1 decrease sharply with increase in the value of frequency for CT(LS), CT(GL), WCT(LS) and WCT(GL). The values of S_1 are less for lower frequency and more for high frequency for CT(GL) with comparison to CT(LS). Figure 13 depicts that the value of penetration depth S_2 decreases monotonically for CT(LS) and WCT(LS). The value of S_2 for CT(GL) and WCT(GL) initially increase in the range $0 < \omega < 5$ and for $\omega \geq 5$, the behavior is reversed. Figure 14 indicates that the values of S_3 decrease with the increase of frequency for CT(LS), CT(GL), WCT(LS) and WCT(GL). The values of S_3 for CT(LS) and WCT(LS) are more as compared to CT(GL) and WCT(GL) theories.

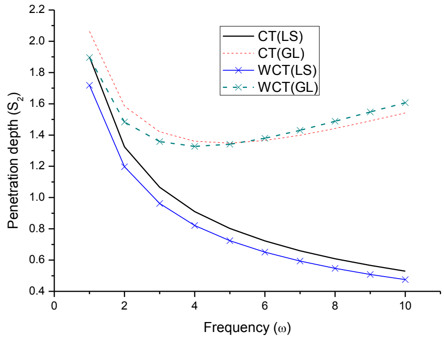

Fig. 13. Variations of penetration depth S_2 w.r.t. frequency.

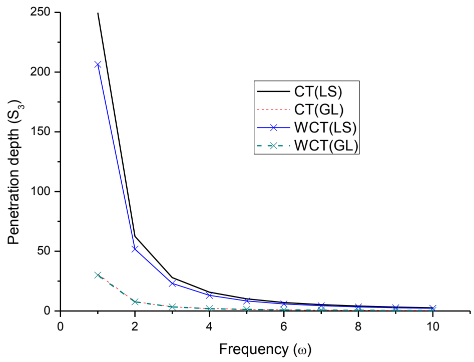

Fig. 14. Variations of penetration depth S_3 w.r.t. frequency.

9. Conclusions

In the present paper, we have studied the propagation of plane waves in a homogeneous isotropic modified couple stress generalized thermoelastic elastic with mass diffusion medium. For two dimensional model of isotropic modified couple stress generalized thermoelastic elastic with mass diffusion medium, there exists three longitudinal waves namely Longitudinal wave (P-wave), thermal wave (T- wave), mass diffusion wave (MD-wave) in addition to a set of coupled transverse waves (SV1 and SV2 waves) which are not affected by thermal and mass diffusion fields. It is observed that couple stress decrease the values of phase velocitiy V_1, V_3 and it increase the values of attenuation coefficient Q_1 and Q_3 for P-wave and MD-wave respectively. It is also observed that couple stress increase the values of phase velocity V_2 and decrease the value of attenuation coefficient Q_2 for T-wave. Significant effect of couple stress is observed on the phase velocity of coupled SV-waves. Appreciable effect is observed on specific loss and penetration depth for different waves. The fundamental solution of system of equations in the generalized theories of thermoelastic elastic with mass diffusion in case of steady oscillations in terms of elementary functions has also been constructed.

The fundamental solution $\boldsymbol{G}(\boldsymbol{x})$ of the system of equations (25)-(27) makes it possible to investigate three-dimensional boundary value problems of generalized theories of modified couple stress thermoelastic diffusion by potential method Kupradze et al. (1979) [39].

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