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EXPERIMENTAL AND ANALYTICAL INVESTIGATION OF THE STRESSED STRAINED STATE OF A CYLINDRICAL SHELL UNDER THE ACTION OF CONCENTRATED RADIAL FORCES

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Abstract. Experimental study of deformation of a cylindrical shell by radial forces and comparison with the analytical solution are being performed.

1. Introduction

The analytical solution of the problem of deformation of the cylindrical shell by the radial forces and comparison with the results obtained by the asymptotic formulas showed the difference in the values of the unknown quantities to order. In the analytical solution of this problem it is necessary to prove the reliability of the results, which are in the proximity of the point of the action of the forces to the shell, as it is known that the Fourier series of separation of variables of differential equations in the point of impact does not converge. In order to prove the reliability of the analytical solution of the problem the comparison with the results of the experiment on the example of shell deformation by radial forces is being held.

The problem of estimation of the error of the solution of boundary value problems is being eliminated by the definition of solutions of differential equations in the form of values of the functions of the Cauchy-Krylov formula [1]. The control of errors is being held a priori by comparison of the partial sums of matrix series.

A priori estimation of errors at the solution of boundary value problems requests to take into account that the computer operates with approximate values of variables. Otherwise, the estimation of the error is required as a result of rounding errors when run on a computer.

The vast number of machine operations at solution of boundary value problems by coupling intervals [2] accounts for matrix solution of algebraic equations. Typically, such systems of algebraic equations are solved by the Gauss method on a computer. In that case it is also known [3] that it is possible to assess a priori the number of wrong significant digits N, as a result of rounding errors. If the decimal number system $N = \lg m$, in the case of the binary system $N = \log_2 m$, where *m*- the order of the system of algebraic equations.

The estimation of the number of the wrong significant digits of the formulas is the top. It is necessary to keep in mind also that in the method of the coefficient the matrix of the system of algebraic equations is quasidiagonal (low filled), which additionally reduces the error in the results of solutions of boundary value problems due to rounding errors when run on a computer.

Unresolved is the question of error, which is caused by the use of the Fourier method of separation of variables. Otherwise, the evaluation of the error associated with the deterioration of the convergence of trigonometric series in determining the solutions in a small neighborhood at the point of impact forces on the shell is required.

2. Convergence of Fourier series

At the action of the radial force P on the cylindrical shell the asymptotic formulas have a logarithmic singularity at the point of application of force. [4] These formulas are of the form:

$$y = a\ln\rho\,,\ \ \rho = \sqrt{\xi^2+\varphi^2}\,,$$

where y - the unknown value of the problem, a - load parameter, ξ and φ - longitudinal and circumferential position.

Assume that $\xi = 0$. Then $y = a\ln\varphi$. It is known that $\ln(2\sin\frac{\varphi}{2}) = \sum_{n=1}^{\infty}\frac{\cos n\varphi}{n}$.

Shown, that $\ln\varphi = \sum_{n=1}^{\infty}\frac{\cos n\varphi}{n}$ with an error not exceeding 0.05 %.

It should be kept in mind that the asymptotic formula – is the part of the overall solution for the unknown quantities in the form of trigonometric series. Discarded part of the overall solution - trigonometric series that converge like $\frac{\cos n\varphi}{n^5}$, i.e. very quickly [5]. Thus, it is obvious that trigonometric series of solutions converge quickly in vicinity of point of shell loading

Numerical experiments and solutions of many problems in loading of shells intently showed that the error in determination of the required quantities of less than 3 %, with the exception of a small neighborhood at the points of loading, when the members of the trigonometric series retained n=100. The radius of the neighborhood, where the error can not be controlled, does not exceed the thickness of four shells.

3. Experimental

The purpose of the experiment was to determine the deformation of the shell in the vicinity of the application of concentrated forces. To calculate the values of the bending moments in the shell, and to compare them with the results of the solution of the same problem analytically by coupling intervals in order to confirm the reliability of the solution of the boundary value problem on a computer.

The experiment used a shell from the AMG-6 material with modulus $E = 0{,}67\cdot 10^6$ kg/cm^2 (65,7 GPa). Shell size: l=200 mm, radius R=55,75 mm, h=1,5 mm. Its dimensionless parameters l/R=3,6; R/h=37,2. The shell is loaded by radial forces concentrated at diametrically opposite points located on the middle of the length of the image, wherein $\varphi = 0$. Elements that transmit forces to the shell, have a frustoconical shape with a diameter of a circle for contact with the shell - 1,5 mm. One of the co-axial elements of loading of the shell is stationary and the other loaded the shell by coaxial moving. The load is transmitted through the dynamometer, which determines the force P of radial exposure.

The deflection was measured with an accuracy of 0.01 mm along the zero image $\varphi = 0$ on the inner surface of the shell by a tracer head moving along the axis. For each point along the forming deflection w defined as $w = w_2 - \frac{w_2}{2}$, where w_2 – readings move the sensor head indicator.

To determine the strain in area of load application were used the strain gauges of 1 mm with a small base which resistance 50 ohms. Others load cells have a base of 5 mm and a resistance of 100 ohms. To register signals from strain gauges were used the strain gauges with a remote switch. The indications of strain gauges were recorded at loading the shell by forces 0; 284,4 H; 549,2 H; 814 H; 1075,7 H.

Approximate values of strain ε of shell were obtained by subtraction the readings of strain gauges for the unloaded membrane from those that were registered under the action of forces P .

The deformation at each point was calculated, taking into account factors of tensosensitivity of the bridge, tensosensitivity of sensors respectively with the base 1 mm and 5 mm.

The values of the longitudinal M_1 and circumferential M_2 bending moments were calculated by the formula

$$M_1 = D(\chi_1 + \nu\chi_2)\,,\; M_2 = D(\chi_2 + \nu\chi_1)\,,$$

where $\chi_1 = \dfrac{\xi_B + \xi_H}{h}$, $\chi_2 = \dfrac{\xi'_B + \xi'_H}{h}$, $D = \dfrac{Eh^3}{12(1-\nu^2)}$, $E = 65{,}7$ GPa, $h = 1{,}5$ mm, $\nu = 0{,}3$.

Here ξ_B and ξ_H – relative deformations produced for point using strain gauges glued respectively on the inner and outer surface of the shell along a generatrix $\varphi = 0$.

The values of the dimensionless bending moments $\overline{M_i} = \dfrac{\pi M_i}{2P_z}$ ($i = 1{,}2$), obtained by deformation of the shell forces P_z = 284,4 N; 549,2 N; 814,0 N; 1078,7 N and using for their calculations the indications of various strain gauges, it turned out to be different from the random errors.

The reliability P of the estimate depends on the number of measurements [6].

The confidence interval of values M_1, M_2, obtained by the results of the experiment [7] and the values M_1 и M_2, obtained by solving the problem by coupling intervals, i.e. analytically are shown in Table 1.

Table 1. Confidence interval of values M_1, M_2 and the values of M_1 и M_2 in solving the problem by coupling intervals.

Distance to the tensor - sensor, mm / Moments bending	10	20	30
(experiment) $\overline{M_1}$	0,0346<*a*<0,0368 *P*=0,996	0,0238<*a*<0,0251 *P*=0,997	0,0251<*a*<0,0255 *P*=0,997
(calculation) $\overline{M_1}$	0,0320	0,0270	0,0271
(experiment) $\overline{M_2}$	0,170<*a*<0,177 *P*=0,996	0,114<*a*<0,118 *P*=0,997	0,105<*a*<0,106 *P*=0,997
(calculation) $\overline{M_2}$	0,175	0,126	0,105

For presentation in the form of graphs are shown in Fig. 1: longitudinal changes M_1 and circumferential M_2 bending moments, respectively, along the way, when $\varphi = 0$. The points mark the results obtained experimentally. Thus, the accuracy of the analytical calculations confirmed.

4. Conclusion

Analytical method of solution of boundary value problems regarding concentrated action to the cylindrical shell allows to obtain reliable results with an error not exceeding 3 % at the distance from the point of concentrated action not exceeding three shell thickness.

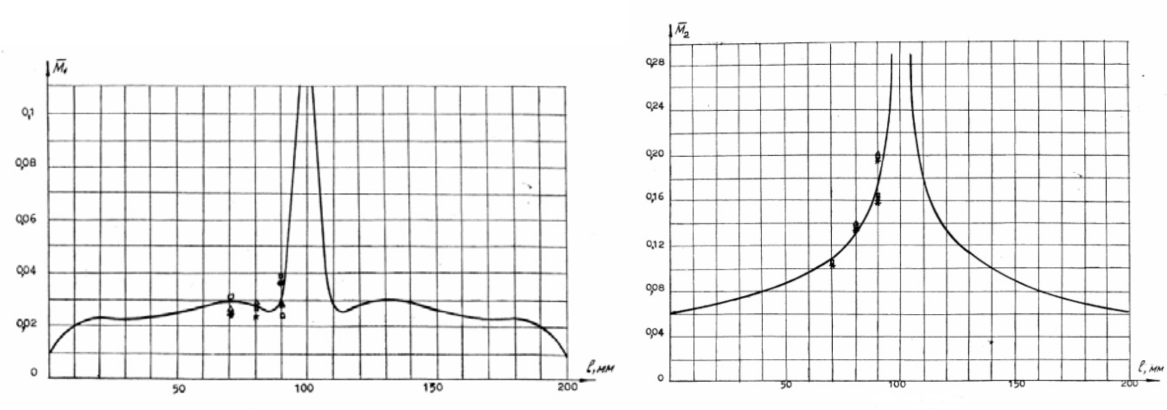

Fig. 1. Changing of linear bending moments M_1 and M_2 along the zero formating shell.

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