Materials Physics and Mechanics 26 (2016) 77-80 Received: September 24, 2015

USE OF ADDITIONAL FINITE ELEMENT METHOD FOR NONLINEAR ANALYSIS OF BAR SYSTEMS AT LIMIT STATES

A. Ermakova

South Ural State University (National Research University),

Lenin prospect 76, Chelyabinsk, 454080, Russia

Abstract. This paper considers some problems connected with description of limit states of the plane bar FE and spatial bar one for nonlinear analysis of reinforced concrete structures by means of developed AFEM. These problems are connecting with necessity of taking account of reinforcement and changing physical nonlinear properties of these FEs when this method is used.

1. Introduction

At present one of the most important problem in the field of structural analysis is the computer realization of nonlinear analysis of bar reinforced concrete structures at limit states. Solving of some problems met in this process by means of Additional Finite Element Method (AFEM) [1] is shown here. AFEM is a variant of the Finite Element Method (FEM) for nonlinear analysis of reinforced concrete structures at limit states as n-nonlinear systems. AFEM adds to the traditional sequence of problem solving by FEM the units of two well- known methods of the structural design: Ultimate Equilibrium (Limit States) Method and Method of Elastic Decisions (Method of Additional Loads).

2. Limit state of the finite element

There is its own stress-strain state at each point of the structure during operating under load. The failure begins in the point with ultimate limit state. The degree of inclusion of each point in general behavior of the structure is determined by the degree of reaching of ultimate limit state in it. The separate finite element is considered instead of the point in analysis of structure by finite element method. The structure is considered as an assembly of such elements where each finite element is the separate small structure of simple form fixed in nodes. This finite element has its own limit state in operating under load. The contribution of each finite element in behavior of the structure is determined by the degree of limit state reaching in it too. It means that rigid properties of the main finite element introducing in analysis ought to be formed in connection with the degree of limit state reaching stage in all its points.

3. Finite elements "Bar of general attitude" and "Plane bar"

The finite element "Bar of general attitude" is destined for analysis of spatial bar systems (Fig. 1). Each node has six degrees of freedom, i.e. three possible linear displacements along coordinate axes and tree angular displacements around the same axes. The next system of stresses characterizes this finite element: *N* - longitudinal force; *My* - bending moment relative to one axis of inertia; *Mz* - bending moment relative to another axis of inertia; *Mk* - twisting moment relative to longitudinal axis of bar; *Qy* - shear force acting along one axis of inertia; *Qz* - shear force acting along another axis of inertia.

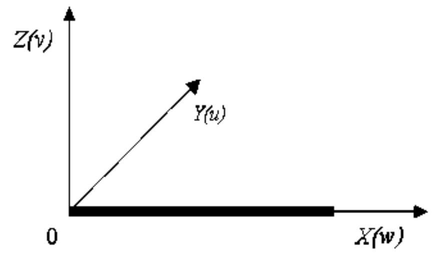

Fig. 1. The finite element “Bar of general attitude”

The finite element “Plane bar” is destined for analysis of plane bar systems. Each node has tree degrees of freedom, i.e. two possible linear displacements along coordinate axes and one angular displacement around the third axis. This element has next system of stresses: *N* - longitudinal force; *My* - bending moment relative to one axis of inertia; *Qz* - shear force acting along another axis of inertia.

4. Limit states of the concrete bar of general attitude and concrete plane bar

Each stress of “The bar of general attitude” has its own limit states, which are determined by the properties of a finite element material; concrete in this case (Table 1).

Table 1. Limit states of concrete finite elements “The bar of general attitude and “The plane bar” with reinforcement.

№	Force	Limit state	Character of failure	Limit state of m and its number			
				The bar of general attitude		The plane bar	
				Concrete	Rein-t	Concrete	Rein-t
1	N - longitudinal force	1. Tensile stress reaches the value of tensile strength	Tension	1	13	1	13
		2. Compressive stress reaches the value of compressive strength	Compression	2	14	2	14
		3. Longitudinal compressive force reaches the ultimate value in statically stability design	Lost of stability	3	15	3	15
2	M_y - bending moment about axis *y*	1. Tensile bent stress in *y* direction reaches the value of tensile strength	Failure of tensile zone of the border of section	4	16	4	16
		2. Compressive bent stress in*y* direction reaches the value of compressive strength	Failure of compressive zone of the border of section	5	17	5	17
3	M_z - bending moment about axis *z*	1. Tensile bent stress in ***y*** direction reaches the value of tensile strength	Failure of tensile zone of perpendicular border of section	8	20	–	–
		2. Compressive bent stress in z direction reaches the value of tensile strength	Failure of compressive zone of perpendicular border of section	9	21	–	–
4	M_k -torsional moment about axis *x*	1. Torsional moment reaches the ultimate value in torsion with bent	Failure because of torsion with bent	10	10	–	–

5	Q_y - shear force in ***y*** direction	1. Shear force in ***y*** direction reaches the ultimate value in strength because of diagonal crack in *xoy* plane	Failure because of diagonal crack in *xoy* plane	11	22	–	–
		2. Tangential stress in *xoy* plane reaches the shear strength	Failure because of shear in *xoy* plane	12	23	–	–
6	Q_z - shear force in z direction	1. Shear force in z direction reaches the ultimate value in strength because of diagonal crack in *xoz* plane	Failure because of diagonal crack in *xoz* plane	6	18	6	18
		2. Tangential stress in *xoz* plane reaches the ultimate value of shear strength	Failure because of shear in *xoz* plane	7	19	7	19

Concrete is characterized by two types of strength (compressive and tensile). It is a determining fact for limit states of this finite element [1, 2]. Three limit states appear due to action of longitudinal force N: tension (№ 1), compression (№ 2) and lost of stability (№ 3). Every bending moment *My* or *Mz* may be caused two limit states for each moment: failure of tensile (№ 4 and № 8) or compressive (№ 5 and № 9) zone of section. Torsion moment *Mk* gives one limit state (№ 10). Shear forces *Qz* or *Qy* give two limit states for each force: failure because of diagonal crack (№ 6 and № 11) and shear (№ 7 and № 12). Thus, this FE has twelve limit states. The plane concrete bar has seven limit states (Table 1): three for longitudinal force *N* (№ 1, 2, 3), two for bending moment *My* (№ 4, 5) and two for shear forces *Qz* (№ 6, 7). Behavior of the concrete bar finite elements at each limit state demands a special theoretical research. In particular, since we may come across the proper shear in reinforced concrete rarely, the appearance of shear limit state is low-probability.

a) Scheme of action of bar AFE for the allowance of the first nonlinear property

Bar FE with the first nonlinear property	=	Bar FE with linear property	+	Bar AFE for the allowance of the first nonlinear property

b) Scheme of action of bar AFE for the allowance of и-th nonlinear property in limit state

Bar FE with n-th nonlinear property	=	Bar FE with (n-1)-th nonlinear property	+	Bar AFE for the allowance of n-th nonlinear property in limit state

c) Scheme of action of bar AFE for the allowance of reinforcement in concrete FE

Concrete bar FE with the reinforcement	=	Concrete bar FE with linear property	+	Bar AFE for the allowance of the reinforcement

d) Example of presentation of bottom and top reinforcement of bar FE under the action of bending moment *My* with the help of bar AFE

Bar AFE for the allowance of the reinforcement	=	Bar AFE for the allowance of the bottom reinforcement	+	Bar AFE for the allowance of the top reinforcement

Fig. 2. Schemes of formation and action of bar AFE for the allowance of the reinforcement and nonlinear properties in limit state.

5. Limit states of the concrete bar of general attitude and concrete plane bar with reinforcement

Reinforcement in bar finite elements adds number of limit states (Table 1). Eleven limit states appear for concrete “The bar of general attitude” (from № 13 to № 23) and seven limit states appear for concrete “The plane bar” (from № 13 to № 19). These limit states connected with behavior of reinforcement in failure moment [2].

6. Properties of additional finite elements “Bar of general attitude” and “Plane bar”

In nonlinear analysis of bar reinforced concrete structures by limit states, it is necessary to define of the way for formation of corresponded additional finite elements (AFEs). Each AFE transforms a finite element (FE) with linear properties into FE with nonlinear properties depending on the stage of behavior of the main FE on the way to its ultimate limit state. Schemes of formation and action of bar AFE present in Fig. 2.

7. Conclusions

Bar AFEs for taking account of reinforcement open possibility for nonlinear analysis of bar reinforced concrete structures by limit states. Different models of behavior of reinforced concrete structures may be include in analysis [3, 4]. Efficiency of AFEM is proved by analysis of space shell [5].

References

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