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DEVELOPMENT OF AN ALGORITHM FOR SOLVING PROBLEMS OF FRACTURE MECHANICS

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Abstract. The implementation of the fundamentals of fracture mechanics in Russian CAx system is discussed. Implementation of two algorithms that use basic fracture mechanics parameters (stress intensity factor (SIF) is calculated using analytical method), is presented. The algorithms use the concept of finite element method (FEM). Simulation of fracture process of the computational model is made by "Birth and Death" method. The developed algorithm eliminates the disadvantages inherent in foreign CAx systems.

1. Introduction

We are discussing fracture mechanics for lineal problems - Lineal Elastic Fracture Mechanics (LEFM). LEFM describes problems that have relatively small plastic areas at the crack tip in comparison to the size of the crack [1]. One parameter approach is used for analysis of the structure with crack and the parameter is one of the criteria of fracture mechanics.

The basic criteria of fracture mechanics [2]:

- Stress Intensity Factor (SIF) K for the three types of deformation (KI, KII, KIII);
- Invariant J–integral;
- Energy Release Rate G for the three types of deformation (GI, GII, GIII).

The relevance of this research is substantiated by the lack of Russian CAx systems, implementing LEFM. Foreign CAx system have a number of disadvantages:

- limited use of finite element (FE) to calculate the parameters of fracture mechanics;
- no automatic mesh refinement in the vicinity of stress concentrators;
- no consideration of crack initiation process;
- criteria of fracture mechanics are not evaluated continuously during the process of crack extension;
- size and location of the crack front have to be defined in advance;
- value of critical SIF K_c have to be defined.

The goal of this work is the development of algorithm for solution of LEFM problems.

The abovementioned algorithm consists of two branches. The first is designed for the problems with the FE model without crack, and the second - with the crack parameters already defined in model.

2. The algorithm using strength theory and LEFM for model without cracks

We analyze FE model with applied loads without macroscopic defects using maximum strain theory, and calculation of principal stresses and their directions [3]. The schematic block diagram of the first branch of the algorithm is shown in Fig. 1.

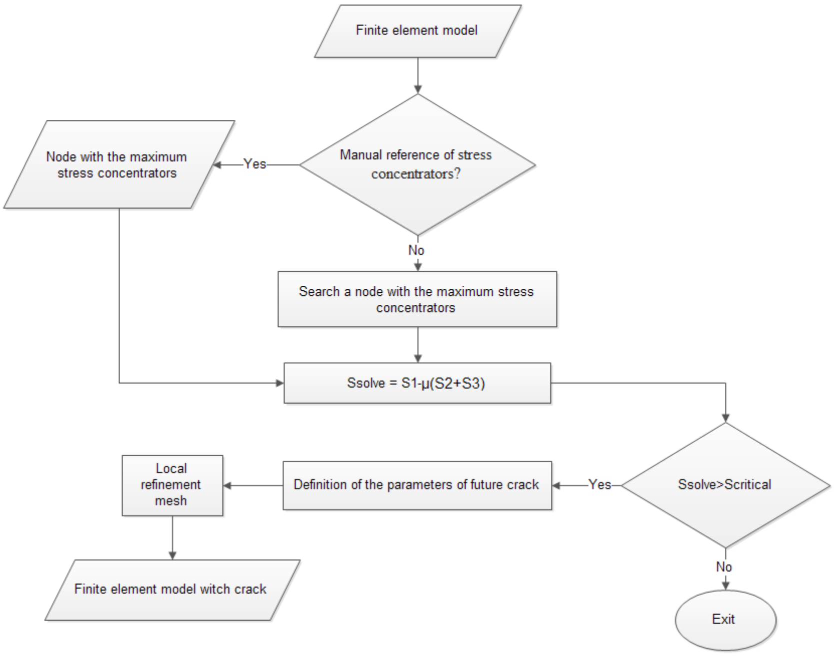

Fig. 1. The schematic block diagram of the first branch of the algorithm.

3. The algorithm using the principles LEFM in model with crack

This part of the algorithm allows us to analyze two problems. The first - is to continue calculations after the first branch of the algorithm is finished, and second - to solve the problem when the position and size of the crack front are defined. The dimensions of the crack are found using the critical SIF (Kc), obtained by finding the cyclic fracture toughness (Kfc) [4] with subsequent calculation of the fracture toughness (critical SIF) [5]. The schematic block diagram of the second branch of the algorithm is shown in Fig. 2.

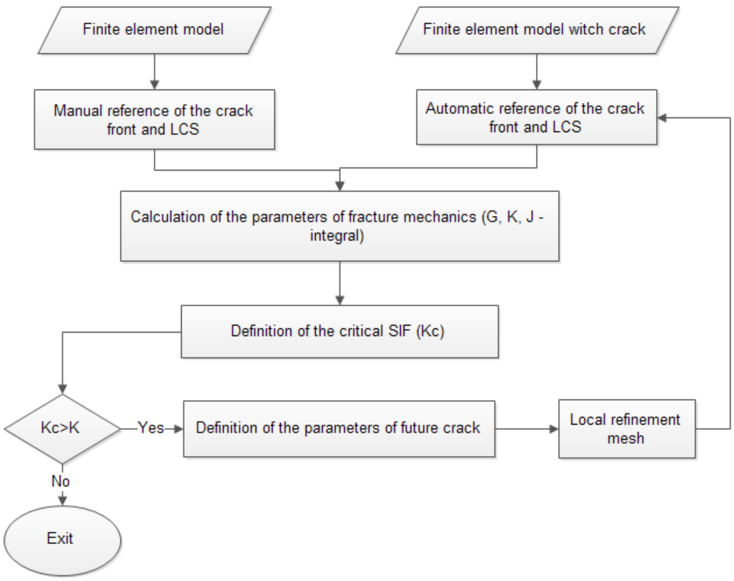

Fig. 2. The schematic block diagram of the second branch of the algorithm.

Consider rectangular plate with crack in the state of plain strain, given tensile load (Fig. 3). Boundary conditions: distributed load P = 100 MPa or equivalent axial force P = 20 MN. Dimensions of the half of the plate: width b = 1000 mm; length L = 3000 mm; thickness t = 100 mm; crack size l = 200 mm; distance from the base of the specimen to the crack a = 1500 mm.

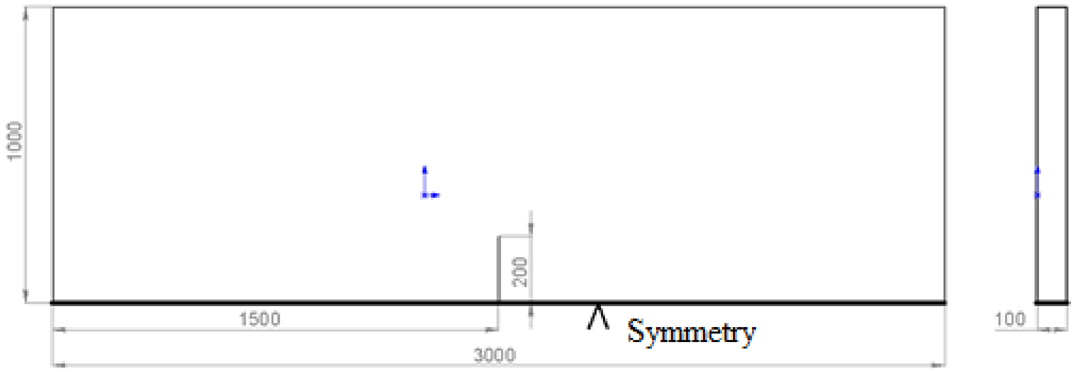

Fig. 3. Test problem dimensions.

SIF calculation:

1. Regional standard GOST 25.506-85

$$K_I = \frac{P}{t\sqrt{b}} Y_1, \tag{1}$$

where Y_1 – dimensionless function:

$$Y_1 = 0.38[1 + 2.308\left(\frac{2l}{b}\right) + 2.439\left(\frac{2l}{b}\right)^2] \text{ with } 0.3b \le 2l \le 0.5b.$$

2. Regional standard GOST 25.506-85

$$K_I = \frac{P\sqrt{l}}{tb} Y_1, \tag{2}$$

$$Y_1 = 1.77 + 0.227\left(\frac{2l}{b}\right) - 0.51\left(\frac{2l}{b}\right)^2 + 2.7\left(\frac{2l}{b}\right)^3.$$

3. Murakami [6], subsection 1.1.1, formula 2.

$$K_I = \sigma\sqrt{\pi l}[1 - 0.025\left(\frac{2l}{b}\right)^2 + 0.06\left(\frac{2l}{b}\right)^4]\sqrt{\sec\left(\frac{\pi\lambda}{2}\right)}. \tag{3}$$

The results of SIF calculations are presented in Table 1.

Table 1.

Calculation methods	$K_I, MPa\sqrt{mm}$
APM WinMachine, stress method	2030.62
APM WinMachine, J-integral method	2571.74
Formula (1)	2779.64
Formula (2)	2760.66
Formula (3)	2498.19

4. Comparison with other CAx systems

The algorithm implemented by the authors in Russian CAx system APM WinMachine (APM Structure 3D software module). Consider FE model of test problem – plate with a crack [7].

FE model: number of nodes – 24850, number of elements – 19200, 8node hexahedral elements. The obtained values of fracture mechanics criteria are presented in Table 2.

As shown in Table 2, the value of KI, resulting in the module APM Structure 3D, closer to the value obtained by the analytical method.

Table 2. Parameters of fracture mechanics.

	CAx system № 1	CAx system № 2	APM Structure 3D
KI (MPa· $\sqrt{mm}$)	471.4	-1230	2030.62
GI (MPa·mm)	-63.549	-	-60.57
J-integral (MPa·mm)	63.62	63	57.32

5. Results

This algorithm removes restrictions on the usage of FE method in application to LEFM problems, allows us to calculate the LEFM parameters at the crack tip considering either dynamic or static problem, allows automatic mesh refinement in the vicinity of stress concentrators. Solution of dynamic problems is possible using «Birth and Death» method. The SIF values obtained using this algorithm, are more reliable in comparison with foreign CAx systems.

The disadvantages of this algorithm yet to be solved:

- no tools for the nonlinear fracture mechanics problems;
- material models for composites are not supported.

These disadvantages are a promising direction for further development of the algorithm.

The implementation of this algorithm in a software module APM Structure 3D will allow for the design of secure technical systems, and significantly reduce material costs and shorten the "design - production" cycle.

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