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INFLUENCE OF THERMO-PIEZOELECTRIC FIELD IN A CIRCULAR BAR SUBJECTED TO THERMAL LOADING DUE TO LASER PULSE

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Abstract. The influence of thermo-piezoelectric field in a circular bar subjected to thermal loading due to non-Gaussian laser pulse is discussed using linear theory of thermoelasticity. The equations of motion of the rod are formulated using the constitutive equations of a thermo-piezoelectric material. Displacement potential functions are introduced to uncouple the equations of motion, heat conduction and charge equation. The frequency equation of the system is developed under the assumption of stress free, thermal and electrically shorted boundary conditions. The numerical calculations are carried out for the material PZT-5A and the computed stress, strain and temperature distribution are plotted as the dispersion curves and their physical characteristics are discussed for longitudinal and flexural (symmetric and antisymmetric) modes.

1. Introduction

Laser-induced vibrations of micro-beam resonators have attracted considerable attention recently due to their many important technological applications in Micro-Electro-Mechanical Systems (MEMS) and Nano- Electro-Mechanical Systems (NMES). Very rapid thermal processes, under the action of a thermal shock (e.g., ultra-short laser pulse) are interesting from the standpoint of thermoelasticity, since they require an analysis of the coupled temperature and deformation fields. This means that the temperature shock induces very rapid movements in the structure elements, thus causing the rise of very significant inertial forces, and thereby, the rise of vibrations.

In recent years, polymers piezoelectric materials have been used in numerous fields taking advantage of the flexible characteristics of these polymers. Some of the applications of these polymers include Audio device-microphones, high frequency speakers, tone generators and acoustic modems; Pressure switches – position switches, accelerometers, impact detectors, flow meters and load cells; Actuators- electronic fans and high shutters. Since piezoelectric polymers allow their use in a multitude of compositions and geometrical shapes for a large variety of applications from transducers in acoustics, ultrasonic's and hydrophone applications to resonators in band pass filters, power supplies, delay lines, medical scans and some industrial non-destructive testing instruments.

The coupling between the thermal/electric/elastic fields in piezo electric materials provides a mechanism for sensing thermomechanical disturbances from measurements of induced electric potentials, and for altering structural responses via applied electric fields. One of the applications of the piezo thermoelastic material is to detect the responses of a structure by measuring the electric charge, sensing or to reduce excessive responses by applying additional electric forces or thermal forces actuating. If sensing and actuating can be integrated smartly, a so-called intelligent structure can be designed. The piezoelectric materials are also often used as resonators whose frequencies need to be precisely controlled. The coupling between the thermoelastic and pyroelectric effects, it is important to qualify the effect of heat dissipation on the propagation of wave at low and high frequencies.

The governing equations of a thermo-piezoelectric plate was first proposed by Mindlin [1]. The physical laws for the thermo-piezoelectric materials have been discussed by Nowacki [2]. Chandrasekhariah [3] presented the generalized theory of thermo-piezoelectricity by taking into account the finite speed of propagation of thermal disturbance. Yang and Batra [4] studied the effect of heat conduction on shift in the frequencies of a freely vibrating linear piezoelectric body with the help of perturbation methods. Sharma and Pal [5] discussed the propagation of Lamb waves in a transversely isotropic piezothermoelastic plate.

Tang and Xu [6] derived the general dynamic equations, which include mechanical, thermal and electric effects, based on the anisotropic composite laminated plate theory. They also obtained analytical dynamical solutions for the case of general force acting on a simply supported piezo thermoelastic laminated plate and harmonic responses to temperature variation and electric field have been examined as a special case. Tauchert [7] applied thermo-piezoelectricity theory to composite plate. The generalized theory of thermoelasticity was developed by Lord and Shulman [8] involving one relaxation time for isotropic homogeneous media, which is called the first generalization to the coupled theory of elasticity. These equations determine the finite speeds of propagation of heat and displacement distributions, the corresponding equations for an isotropic case were obtained by Dhaliwal and Sheried [9]. The second generalization to the coupled theory of elasticity is what is known as the theory of thermoelasticity, with two relaxation times or the theory of temperature – dependent thermoelasticity. Longitudinal vibrations of a circular cylindrical shell coupled with a thermal field were also obtained independently by Suhubi [10]. Dispersion analysis of generalized magneto-thermoelastic waves in a transversely isotropic cylindrical panel is analyzed by Ponnusamy and Selvamani [11]. Later, extensional waves in a transversely isotropic solid bar immersed in an inviscid fluid calculated using Chebyshev polynomials were discussed by Selvamani and Ponnusamy [12].

This theory contains two constants that act as relaxation times and modify not only the heat equations, but also all the equations of the coupled theory. Wang and Xu [13] have studied the stress wave induced by nanoseconds, picoseconds, and femtoseconds laser pulses in a semi-infinite solid. The Precursor in Laser-Generated Ultrasound Waveforms in Metals were discussed by McDonald [14].The so-called ultra-short lasers are those with pulse duration ranging from nanoseconds to femtoseconds in general. In the case of ultra-short-pulsed laser heating, the high-intensity energy flux and ultra-short duration laser beam, have introduced situations where very large thermal gradients or an ultra-high heating speed may exist on the boundaries Sun et al. [15].

In this paper, the influence of piezo thermoelastic field in a circular bar subjected to thermal loading due to laser pulse is studied. The frequency equations are derived for stress free, temperature and electrically shorted boundary conditions. The numerical calculations are carried out for the material PZT-5A and the computed stress, strain and temperature distribution are plotted as the dispersion curves and their physical characteristics are discussed for longitudinal and flexural (symmetric and antisymmetric) modes.

2. Formulation of the problem

We consider homogeneous transversely isotropic, thermally and electrically conducting piezoelectric bar of finite length with uniform temperature T_0 in the undistributed state initially. This piezoelectric rod is subjected to laser pulse with Non-Gaussian temporal profile discussed in Sun et al. [15].

2.1. The Non-Gaussian Laser pulse model. The piezo thermoelastic rod is heated uniformly by a laser beam of Non-Gaussian form defined by Sun et al. [15]

$$I(t)=\frac{L_0}{t_p^2}e^{\left(\frac{-t}{t_p}\right)}, \tag{1}$$

where $t_p = 2\,ps$ is the characterstic time of the laser pulse, the total energy carried by laser pulse per unit area of the laser beam is denoted by L_0 (laser intensity). The conduction of heat transfer in the rod can be modeled with an energy source $Q(z,t)$ as

$$Q(z,t)=\frac{1-R}{\delta}e^{\left(\frac{z-h/2}{\delta}\right)I(t)}=\frac{R_a L_0}{\delta t_p^2}te^{\left(\frac{z-h/2}{\delta}-\frac{t}{t_p}\right)}, \tag{2}$$

where δ is the absorption depth of heating energy and R_a is the surface reflexivity, when we take the laser pulse lie on the surface of the rod when $z=0$, we can get the energy source in the following form

$$Q(z,t)=\frac{R_a L_0}{\delta t_p^2}te^{\left(\frac{-h}{2\delta}-\frac{t}{t_p}\right)}. \tag{3}$$

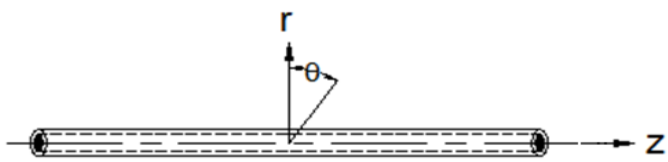

Fig. 1. Geometry of the bar.

The equations of motion, heat conduction and electric potentials in the absence of body force are

$$\sigma_{rr,r}+r^{-1}\sigma_{r\theta,\theta}+\sigma_{rz,z}+r^{-1}\left(\sigma_{rr}-\sigma_{\theta\theta}\right)=\rho u_{r,tt}, \qquad \sigma_{r\theta,r}+r^{-1}\sigma_{\theta\theta,\theta}+\sigma_{\theta z,z}+2r^{-1}\sigma_{r\theta}=\rho u_{\theta,tt},$$
$$\sigma_{rz,r}+r^{-1}\sigma_{\theta z,\theta}+\sigma_{zz,z}+r^{-1}\sigma_{rz}=\rho u_{z,tt}. \tag{4}$$

The heat conduction equation is

$$K_1\left(T_{,rr}+r^{-1}T_{,r}+r^{-2}T_{,\theta\theta}\right)+K_3T_{,zz}-\rho c_v T_{,t}=T_o\left(\beta_1\left(e_{rr}+e_{\theta\theta}\right)+\beta_3 e_{zz}-p_3\phi_{,z}-\rho Q\right)_{,t}. \tag{5}$$

The electric charge equation is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rD_r\right)+\frac{1}{r}\frac{\partial D_\theta}{\partial \theta}+\frac{\partial D_z}{\partial r}=0. \tag{6}$$

The elastic, the piezoelectric, and dielectric matrices of the 6mm crystal class, the thermo-piezoelectric relations are

$$\sigma_{rr}=c_{11}e_{rr}+c_{12}e_{\theta\theta}+c_{13}e_{zz}-\beta_1T-e_{31}E_z \;,\; \sigma_{\theta\theta}=c_{12}e_{rr}+c_{11}e_{\theta\theta}+c_{13}e_{zz}-\beta_1T-e_{31}E_z,$$
$$\sigma_{zz}=c_{13}e_{rr}+c_{13}e_{\theta\theta}+c_{33}e_{zz}-\beta_3T-e_{33}E_z \;,\quad \sigma_{r\theta}=c_{66}e_{r\theta}, \quad \sigma_{\theta z}=c_{44}e_{\theta z}-r^{-1}e_{15}E_\theta,$$
$$\sigma_{rz}=2c_{44}e_{rz}-e_{15}E_r,$$
$$D_r=e_{15}e_{rz}+\varepsilon_{11}E_r,\; D_\theta=e_{15}e_{\theta z}+r^{-1}\varepsilon_{11}E_\theta,\; D_z=e_{31}\left(e_{rr}+e_{\theta\theta}\right)+e_{33}e_{zz}+\varepsilon_{33}E_z+p_3T, \tag{7}$$

where $\sigma_{rr},\sigma_{\theta\theta},\sigma_{zz},\sigma_{r\theta},\sigma_{\theta z},\sigma_{rz}$ are the stress components, $e_{rr},e_{\theta\theta},e_{zz},e_{r\theta},e_{\theta z},e_{rz}$ are the strain components, T is the temperature change about the equilibrium temperature $T_o, c_{11}, c_{12}, c_{13}, c_{33}, c_{44}$ and $c_{66}=\left(c_{11}-c_{12}\right)/2$ are the five elastic constants, β_1,β_3 and K_1,K_3

respectively thermal expansion coefficients and thermal conductivities along and perpendicular to the symmetry, ρ is the mass density, c_v is the specific heat capacity , p_3 is the pyroelectric effect.

The strain e_{ij} are related to the displacements are given by

$$e_{rr} = u_{r,r},\ e_{\theta\theta} = r^{-1}\left(u_r + u_{\theta,\theta}\right),\ e_{zz} = u_{z,z} \tag{8a}$$

$$e_{r\theta} = u_{\theta,r} + r^{-1}\left(u_{r,\theta} - u_\theta\right),\ e_{z\theta} = \left(u_{\theta,z} + r^{-1}u_{z,\theta}\right),\ e_{rz} = u_{z,r} + u_{r,z} \tag{8b}$$

The comma in the subscripts denotes the partial differentiation with respect to the variables.

Substituting the Eqs.(8), (7) in the Eqs. (4)-(6), results in the following three-dimensional equations of motion, heat electric conductions are obtained as follows:

$$\begin{aligned} &c_{11}\left(u_{rr,r} + r^{-1}u_{r,r} - r^{-2}u_r\right) - r^{-2}\left(c_{11} + c_{66}\right)u_{\theta,\theta} + r^{-2}c_{66}u_{r,\theta\theta} + c_{44}u_{r,zz} + \left(c_{44} + c_{13}\right)u_{z,rz} \\ &+ r^{-1}\left(c_{66} + c_{12}\right)u_{\theta,r\theta} + \left(e_{31} + e_{15}\right)\phi_{,rz} - \beta_1 T_{,r} = \rho u_{r,tt}, \end{aligned} \tag{9a}$$

$$\begin{aligned} &r^{-1}\left(c_{12} + c_{66}\right)u_{r,r\theta} + r^{-2}\left(c_{66} + c_{11}\right)u_{r,\theta} + c_{66}\left(u_{\theta,rr} + r^{-1}u_{\theta,r} - r^{-2}u_\theta\right) + r^{-2}c_{11}u_{\theta,\theta\theta} + c_{44}u_{\theta,zz} \\ &+ r^{-1}\left(c_{44} + c_{13}\right)u_{z,\theta z} + \left(e_{31} + e_{15}\right)\phi_{,\theta z} - \beta_1 r^{-1}T_{,\theta} = \rho u_{\theta,tt}, \end{aligned} \tag{9b}$$

$$c_{44}\left(u_{z,rr} + r^{-1}u_{z,r} + r^{-2}u_{z,\theta\theta}\right) + r^{-1}\left(c_{44} + c_{13}\right)\left(u_{r,z} + u_{\theta,\theta z}\right) + \left(c_{44} + c_{13}\right)u_{r,rz} + c_{33}u_{z,zz} + e_{33}\phi_{,zz} - \beta_3 T_{,z} = \rho u_{z,tt}, \tag{9c}$$

$$K_1\left(T_{,rr} + r^{-1}T_{,r} + r^{-2}T_{,\theta\theta}\right) + K_3 T_{,zz} - \rho c_v T_{,t} = T_o \frac{\partial}{\partial t}\left(\beta_1\left(u_{r,r} + r^{-1}u_{\theta,\theta} + r^{-1}u_r\right) + \beta_3 u_{z,z} - p_3\phi_{,z} - \rho Q\right), \tag{9d}$$

$$\begin{aligned} &e_{15}\left(u_{z,rr} + r^{-1}u_{z,r} + r^{-2}u_{z,\theta\theta}\right) + \left(e_{31} + e_{15}\right)\left(u_{r,zr} + r^{-1}u_{r,z} + r^{-1}u_{\theta,z\theta}\right) + e_{33}u_{z,zz} - \varepsilon_{33}\phi_{,zz} \\ &- \varepsilon_{11}\left(\phi_{,rr} + r^{-1}\phi_{,r} + r^{-2}\phi_{,\theta\theta}\right) + p_3 T_{,z} = 0. \end{aligned} \tag{9e}$$

3. Solutions of the Field Equation

To obtain the propagation of harmonic waves in thermo-piezoelectric circular bar, we assume the solutions of the displacement components to be expressed in terms of derivatives and we seek the solution of the Eqs. (9) in the form

$$\begin{aligned} &u_r(r,\theta,z,t) = \left(\phi_{,r} + r^{-1}\psi_{,\theta}\right)e^{i(kz+\omega t)},\ u_\theta(r,\theta,z,t) = \left(r^{-1}\phi_{,\theta} - \psi_{,r}\right)e^{i(kz+\omega t)},\ u_z(r,\theta,z,t) = \left(\frac{i}{a}\right)We^{i(kz+\omega t)}, \\ &V(r,\theta,z,t) = iVe^{i(kz+\omega t)},\ E_r(r,\theta,z,t) = -E_{,r}e^{i(kz+\omega t)},\ E_\theta(r,\theta,z,t) = -r^{-1}E_{,\theta}e^{i(kz+\omega t)},\ E_z(r,\theta,z,t) = E_{,z}e^{i(kz+\omega t)} \end{aligned} \tag{10}$$

where $i = \sqrt{-1}$, k is the wave number, ω is the angular frequency, $\phi(r,\theta)$, $W(r,\theta)$, $\psi(r,\theta)$ and $E(r,\theta)$ are the displacement potentials and $V(r,\theta)$, is the electric potentials, $T(r,\theta)$ is the temperature change and a is the geometrical parameter of the bar.

By introducing the dimensionless quantities such as $x = r/a$, $\zeta = ka$, $\Omega^2 = \rho\omega^2 a^2/c_{44}$, $\bar{c}_{11} = c_{11}/c_{44}$, $\bar{c}_{13} = c_{13}/c_{44}$, $\bar{c}_{33} = c_{33}/c_{44}$, $\varepsilon = \frac{\rho R_a L_0}{\beta_3 T_0 \delta t_p^2} e^{-h/2\delta}$, $\bar{c}_{66} = c_{66}/c_{44}$, $\bar{\beta} = \beta_1/\beta_3$, $\varpi = t/t_p$ $\bar{k} = \left(\rho c_{44}\right)^{1/2}/\beta_3^2 T_o a\Omega$, $\bar{d} = \rho c_v c_{44}/\beta_3^2 T_o$, $\bar{p}_3 = p_3 c_{44}/\beta_3 e_{33}$ and substituting Eq. (10) in Eqs. (9), we obtain

$$\left(\bar{c}_{11}\nabla^2 + \left(\Omega^2 - \zeta^2\right)\right)\phi - \zeta\left(1 + \bar{c}_{13}\right)W - \zeta\left(\bar{e}_{31} + \bar{e}_{15}\right)V - \bar{\beta}T = 0,$$

$$\zeta\left(1 + \bar{c}_{13}\right)\nabla^2\phi + \left(\nabla^2 + \left(\Omega^2 - \zeta^2\bar{c}_{33}\right)\right)W + \left(\bar{e}_{15}\nabla^2 - \zeta^2\right)V - \zeta T = 0,$$

$$\zeta\left(\bar{e}_{31} + \bar{e}_{15}\right)\nabla^2\phi + \left(\bar{e}_{15}\nabla^2 - \zeta^2\right)W + \zeta\bar{p}_3 T + \left(\zeta^2\bar{\varepsilon}_{33} - \bar{\varepsilon}_{11}\nabla^2\right)V = 0,$$

$$\bar{\beta}\nabla^2\phi-\zeta W+\left(\bar{d}+\varepsilon e^{-\varpi}(1-\varpi)+i\bar{k}_1\nabla^2-i\bar{k}_3\zeta^2\right)T+\zeta\bar{p}_3V=0, \tag{11}$$

and $$\left(\bar{c}_{66}\nabla^2+\left(\Omega^2-\zeta^2\right)\right)\psi=0, \tag{12}$$

where $\nabla^2=\dfrac{\partial^2}{\partial x^2}+x^{-1}\dfrac{\partial}{\partial x}+x^{-2}\dfrac{\partial^2}{\partial\theta^2}$

The Eq. (11) can be written as

$$\begin{vmatrix} \left(\bar{c}_{11}\nabla^2+\left(\Omega^2-\zeta^2\right)\right) & -\zeta\left(1+\bar{c}_{13}\right) & -\bar{\beta} & -\zeta\left(\bar{e}_{31}+\bar{e}_{15}\right) \\ \zeta\left(1+\bar{c}_{13}\right)\nabla^2 & \left(\nabla^2+\left(\Omega^2-\zeta^2\bar{c}_{33}\right)\right) & -\zeta & \left(\bar{e}_{15}\nabla^2-\zeta^2\right) \\ \bar{\beta}\nabla^2 & -\zeta & \left(\bar{d}+\varepsilon e^{-\varpi}(1-\varpi)+i\bar{k}_1\nabla^2-i\bar{k}_3\zeta^2\right) & \zeta\bar{p}_3 \\ \zeta\left(\bar{e}_{31}+\bar{e}_{15}\right)\nabla^2 & \left(\bar{e}_{15}\nabla^2-\zeta^2\right) & \zeta\bar{p}_3 & \left(\zeta^2\bar{\varepsilon}_{33}-\bar{\varepsilon}_{11}\nabla^2\right) \end{vmatrix}(\phi,W,T,V)=0 \tag{13}$$

Evaluating the determinant given in Eq. (13), we obtain a partial differential equation of the form

$$\left(A\nabla^8+B\nabla^6+C\nabla^4+D\nabla^2+E\right)(\phi,W,T,V)=0, \tag{14}$$

where

$$A=-i\bar{k}_1\bar{c}_{11}g_{18},\ B=\bar{c}_{11}\left(g_9-i\bar{k}_1\bar{\varepsilon}_{11}g_2+\bar{e}_{15}g_{17}\right)+i\bar{k}_1\left(g_1g_{18}-\zeta^2g_3^2\right)-\bar{\beta}^2g_{18},$$

$$C=\bar{c}_{11}\left(g_2g_9+\zeta^2\left(g_5+\bar{\varepsilon}_{11}+2\bar{e}_{15}g_8-i\bar{k}_1\zeta^2\right)\right)+g_1\left(g_9-i\bar{k}_1\bar{\varepsilon}_{11}g_2+\bar{e}_{15}g_{17}\right)$$

$$-\bar{\beta}\left(\bar{\beta}\bar{\varepsilon}_{11}g_2-\zeta^2\left(\bar{\beta}\bar{\varepsilon}_{33}-\bar{p}_3g_3-g_3\bar{e}_{15}+2\bar{\beta}\bar{e}_{15}\right)\right)$$

$$+\zeta^2g_4\left(g_4g_{14}+\bar{\beta}\bar{e}_{15}+\zeta g_3g_{15}\right)-\zeta^2g_3\left(i\bar{k}_1\bar{e}_{15}g_4+\bar{\beta}\left(\bar{p}_3+\bar{e}_{15}\right)-g_3\left(g_{16}+i\bar{k}_1g_2\right)\right),$$

$$D=\zeta^2\bar{c}_{11}\left(g_2g_5-\zeta^2\left(g_8-g_7\right)\right)+g_1\left(g_2g_9+\zeta^2\left(g_5+\bar{\varepsilon}_{11}+2\bar{e}_{15}g_8-i\bar{k}_1\zeta^2\right)\right)$$

$$+\zeta^2g_4\left(g_4\left(g_{11}+\bar{p}_3\zeta\bar{\varepsilon}_{11}\right)-\zeta g_3g_{12}-\bar{\beta}\zeta\left(\zeta+\bar{p}_3^2\right)\right)$$

$$-\bar{\beta}\zeta^2\left(g_4g_{13}-g_2\left(\bar{\beta}\bar{\varepsilon}_{33}-\bar{p}_3g_3\right)-\zeta\left(g_3-\bar{\beta}\right)\right)-\zeta^2g_3\left(g_4g_{11}+g_2\left(\bar{\beta}\bar{p}_3-g_3g_{16}\right)+\zeta^2\left(g_3-\bar{\beta}\right)\right),$$

$$E=\zeta^2g_1g_2g_5-\zeta^2\left(g_1g_8-g_1g_7+g_4^2g_6+\bar{\beta}g_4g_7-g_4g_3g_8\right),$$

in which $g_1=\Omega^2-\zeta^2$, $g_2=\Omega^2-\zeta^2\bar{c}_{33}$, $g_3=\bar{e}_{31}+\bar{e}_{15}$, $g_4=1+\bar{c}_{13}$, $g_5=\bar{d}\bar{\varepsilon}_3-i\bar{k}_3\bar{\varepsilon}_{33}\zeta^2-\bar{p}_3^2$,

$g_6=\bar{d}+\bar{p}_3\zeta\bar{\varepsilon}_{33}+\varepsilon e^{-\varpi}(1-\varpi)$, $g_7=\bar{p}_3-\bar{\varepsilon}_{33}$, $g_8=\bar{d}-\bar{p}_3-i\bar{k}_3\zeta^2$, $g_{11}=\bar{d}\bar{e}_{15}-i\bar{k}_1\zeta^2-i\bar{k}_3\zeta^2\bar{e}_{15}$

$g_9=i\bar{k}_1\zeta^2\bar{\varepsilon}_{33}+\varepsilon e^{-\varpi}(1-\varpi)+i\bar{k}_3\bar{\varepsilon}_{11}\zeta^2-\bar{d}\bar{\varepsilon}_{11}$, $g_{10}=\bar{\varepsilon}_{11}+2\bar{e}_{15}g_8-i\bar{k}_1\zeta^2$,

$g_{12}=\zeta^2\bar{\varepsilon}_{33}+\varepsilon e^{-\varpi}(1-\varpi)+\bar{d}\bar{p}_3-i\bar{k}_3\bar{p}_3\zeta^2$, $g_{13}=\bar{\varepsilon}_{11}-\bar{p}_3\bar{e}_{15}$, $g_{14}=i\left(\bar{k}_1\bar{e}_{15}+\bar{k}_3\right)$,

$g_{15}=\bar{\varepsilon}_{11}-i\bar{p}_3\bar{k}_1$, $g_{16}=\bar{d}^2-\zeta^2i\bar{k}_3$, $g_{17}=i\bar{k}_1\zeta^2-g_{11}$, $g_{18}=\bar{\varepsilon}_{11}+\bar{e}_{15}^2$.

Factorizing the relation given in Eq. (14) into biquadratic equation for $(\alpha_i a)^2$, $i=1,2,3,4$, the solutions for the symmetric modes are obtained as

$$\phi=\sum_{i=1}^{4}A_iJ_n(\alpha_i ax)\cos n\theta,\qquad W=\sum_{i=1}^{4}a_iA_iJ_n(\alpha_i ax)\cos n\theta,$$

$$T=\sum_{i=1}^{4}b_iA_iJ_n(\alpha_i ax)\cos n\theta,\qquad V=\sum_{i=1}^{4}c_iA_iJ_n(\alpha_i ax)\cos n\theta. \tag{15}$$

Here $(\alpha_i a)^2 > 0$, $i=1,2,3,4$ are the roots of the algebraic equation

$$A(\alpha a)^8 - B(\alpha a)^6 + C(\alpha a)^4 - D(\alpha a)^2 + E = 0. \quad (16)$$

The solutions corresponding to the root $(\alpha_i a)^2 = 0$ is not considered here, since $J_n(0)$ is zero, except for $n=0$. The Bessel function J_n is used when the roots $(\alpha_i a)^2$, $i=1,2,3,4$ are real or complex and the modified Bessel function I_n is used when the roots $(\alpha_i a)^2$, $i=1,2,3,4$ are imaginary.

The constants a_i, b_i and c_i defined in the Eq. (15) can be calculated from the equations

$$a_i = \left(\overline{\beta}\,\overline{p}_3 - g_3 L\right) \Big/ \left(g_4 L + \overline{\beta}\right),\ b_i = -\left(g_3 \overline{\beta}(\alpha_i a)^2 + \overline{p}_3\left(g_1 + \overline{c}_{11}(\alpha_i a)^2\right)\right) \Big/ \left(g_4 L + \overline{\beta}\right),$$

$$c_i = \left((\alpha_i a)^2\left(g_4 \overline{\beta} - \overline{c}_{11}\right) - g_1\right) \Big/ \left(g_4 L + \overline{\beta}\right), \quad (17)$$

where $L = \left(\overline{d} - i\overline{k}_1(\alpha_i a)^2 - i\overline{k}_3 \zeta^2\right)$.

Solving the Eq.(12), the solution to the symmetric mode is obtained as

$$\psi = A_5 J_n(\alpha_5 a x)\sin n\theta, \quad (18)$$

where $(\alpha_5 a)^2 = \Omega^2 - \zeta^2$. If $(\alpha_5 a)^2 < 0$, the Bessel function J_n is replaced by the modified Bessel function I_n.

4. Boundary conditions and Frequency equations

In this problem, the free vibration of transversely isotropic thermo-piezoelectric solid bar of circular cross-section is considered. The stress free, temperature and electrically shorted can be written as

$$(\sigma_{rr}, \sigma_{r\theta}, \sigma_{rz}, V, T) = (0,0,0,0), \qquad r = a. \quad (19)$$

Substituting the solutions given in the Eqs. (16) and (18) in the boundary condition in the Eq. (19), we obtain a system of five linear algebraic equation as follows:

$$[A]\{X\} = \{0\}, \quad (20)$$

where $[A]$ is a 5×5 matrix of unknown wave amplitudes, and $\{X\}$ is an 5×1 column vector of the unknown amplitude coefficients A_1, A_2, A_3, A_4, A_5. The solution of Eq. (20) is nontrivial when the determinant of the coefficient of the wave amplitudes $\{X\}$ vanishes, that is

$$|A| = 0. \quad (21)$$

Eq. (21) is the frequency equation of the coupled system consisting of a transversely isotropic thermo-piezoelectric solid circular bar.

$$a_{1i} = 2\overline{c}_{66}\{n(n-1)J_n(\alpha_i ax) + (\alpha_i ax)J_{n+1}(\alpha_i ax)\} - x^2\left[(\alpha_i a)^2 \overline{c}_{11} + \zeta \overline{c}_{13} a_i + \zeta b_i + \overline{\beta} c_i\right] J_n(\alpha_i ax),\ i=1,2,3,4,$$

$$a_{15} = 2\overline{c}_{66}\{n(n-1)J_n(\alpha_5 ax) - (\alpha_5 ax)J_{n+1}(\alpha_5 ax)\},$$

$$a_{2i} = 2n\{(\alpha_i ax)J_{n+1}(\alpha_i ax) - (n-1)J_n(\alpha_i ax)\},\ i=1,2,3,4,\ a_{25} = \left\{\left[(\alpha_5 ax)^2 - 2n(n-1)\right]J_n(\alpha_5 ax) - 2(\alpha_5 ax)J_{n+1}(\alpha_5 ax)\right\},$$

$$a_{3i} = \left(\zeta + a_i + \overline{e}_{15} b_i\right)\left[nJ_n(\alpha_i ax) - (\alpha_i ax)J_{n+1}(\alpha_i ax)\right],\ i=1,2,3,4,\ a_{35} = n\zeta J_n(\alpha_5 ax),$$

$a_{4i}=\left(\bar{e}_{15}\zeta a_i-\bar{\varepsilon}_{11}b_i\right)\left\{nJ_n\left(\alpha_i ax\right)-\left(\alpha_i ax\right)J_{n+1}\left(\alpha_i ax\right)\right\}$, $i=1,2,3,4$, $a_{45}=\bar{e}_{15}\zeta nJ_n\left(\alpha_5 ax\right)$,

$a_{5i}=c_i\left\{nJ_n(\alpha_i ax)-(\alpha_i ax)J_{n+1}\left(\alpha_i ax\right)\right\}$, $i=1,2,3,4$, $a_{55}=0$.

5. Numerical results and discussion

The free vibration of transversely isotropic thermo-piezoelectric solid bar of circular cross-section is considered. The frequency equation is numerically solved for the material PZT-5A and the material properties of PZT-5A is given below:

$c_{11}=13.9\times10^{10}\,Nm^{-2}$, $c_{12}=7.78\times10^{10}\,Nm^{-2}$, $c_{13}=7.54\times10^{10}\,Nm^{-2}$, $c_{33}=11.3\times10^{10}\,Nm^{-2}$, $c_{44}=2.56\times10^{10}\,Nm^{-2}$, $e_{31}=-6.98Cm^{-2}$, $e_{33}=13.8Cm^{-2}$, $e_{15}=13.4Cm^{-2}$, $\varepsilon_{11}=60.0\times10^{-10}C^2N^{-1}m^{-2}$, $\varepsilon_{33}=54.7\times10^{-10}C^2N^{-1}m^{-2}$, $\rho=7750Kgm^{-2}$, $\beta_1=1.52\times10^6\,NK^{-1}m^{-2}$, $\beta_3=1.53\times10^6\,NK^{-1}m^{-2}$, $C_e=420J\,kg^{-1}K^{-1}$, $K_1=K_3=1.5Wm^{-1}K^{-1}$. $\beta_3=1.53\times10^6\,NK^{-1}m^{-2}$, $T_0=298K$, $p_3=-452\times10^{-6}CK^{-1}m^{-2}$.

In this problem, there are two kinds of basic independent modes of wave propagation have been considered, namely, the longitudinal and flexural modes. By choosing respectively n=0 and n=1, we can obtain the non-dimensional frequencies of longitudinal and flexural modes of vibrations.

5.1. Dispersion curves. The results of longitudinal and flexural modes (symmetric and antisymmetric) are plotted in the form of dispersion curves. The notation used in the figures, namely LM, FLS and FLAS respectively denote the longitudinal mode, flexural symmetric mode and, flexural anti symmetric mode. The dispersion curves are drawn for stress distribution versus the different position of the thermo-piezoelectric bar for longitudinal and flexural (symmetric and antisymmetric) modes are respectively shown in Figs. 2 and 3. From the Figs. 2 and 3, it is clear that the stress distributions have the same trend in both piezoelectric and thermo-piezoelectric solid bar with respect to its different positions. But there is a small deviation in the lower limit of the position in Fig. 3 due to the influence of laser pulse.

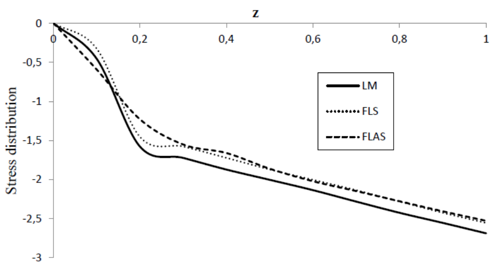

Fig. 2. Stress distribution verses different position of the piezoelectric bar.

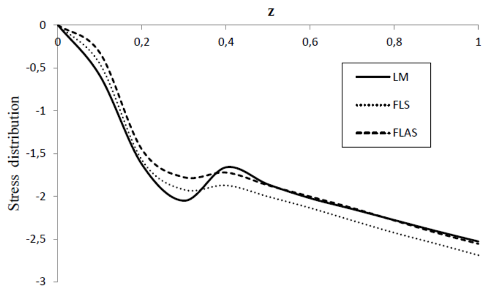

Fig. 3. Stress distribution verses different position of the thermo-piezoelectric bar.

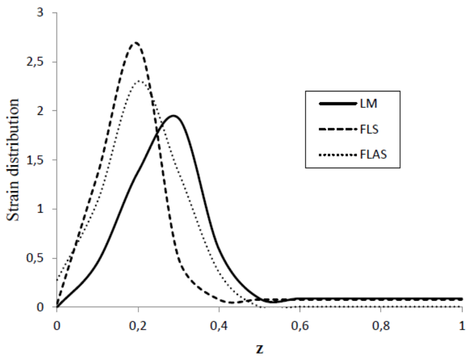

Fig. 4. Strain distribution verses different position of the piezoelectric bar.

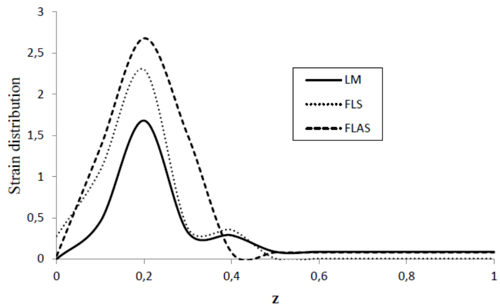

Fig. 5. Strain distribution verses different position of the thermo-piezoelectric bar.

A comparison is made among the longitudinal and flexural symmetric and antisymmetric mode of strain distribution of piezoelectric and thermo-piezoelectric vibrations are respectively shown in the Figs. 3 and 4 respectively. From the Figs. 3 and 4, it is observed that, both the piezoelectric and thermo-piezoelectric bar the strain energy experiences some cross over and peak values for a particular position after that, it starts decreases. The inclusion of thermal energy due to laser pulse is oscillating the strain energy in Fig. 4.

A dispersion curve is drawn to compare the temperature distribution of longitudinal and flexural symmetric and antisymmetric modes of vibration for a piezoelectric and thermo-piezoelectric bar is shown respectively in the Figs. 5 and 6. From the Figs. 5 and 6, it is observed that the temperature distribution are decreases with respect to the position of bar.

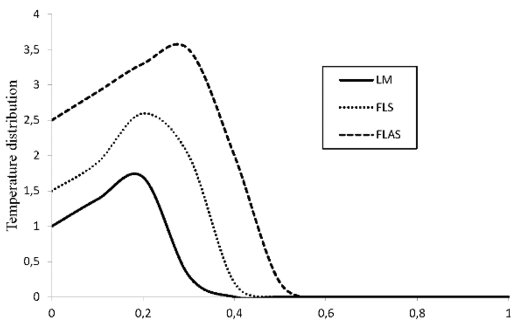

Fig. 6. Temperature distribution verses different position of the piezoelectric bar.

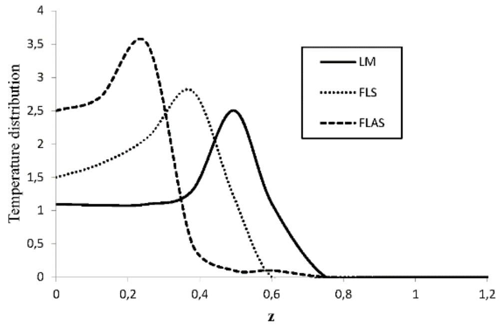

Fig. 7. Temperature distribution verses different position of the thermo-piezoelectric bar.

6. Conclusion

The influence of thermo-piezoelectric field in a circular bar subjected to thermal loading due to non-Gaussian laser pulse is discussed using linear theory of thermoelasticity. The frequency equation of the system is developed under the assumption of stress free, thermal and electrically shorted boundary conditions. The numerical calculations are carried out for the material PZT-5A and the computed stress, strain and temperature distribution are plotted as the dispersion curves and their physical characteristics are discussed for longitudinal and flexural (symmetric and antisymmetric) modes of piezoelectric and thermo-piezoelectric bar. From the result, it is clear that the thermal load due to non-Gaussian laser pulse has the significant effect on the thermal and mechanical interactions.

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