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INFLUENCE OF THERMO-PIEZOELECTRIC FIELD IN A CIRCULAR BAR SUBJECTED TO THERMAL LOADING DUE TO LASER PULSE

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Abstract. The influence of thermo-piezoelectric field in a circular bar subjected to thermal loading due to non-Gaussian laser pulse is discussed using linear theory of thermoelasticity. The equations of motion of the rod are formulated using the constitutive equations of a thermo-piezoelectric material. Displacement potential functions are introduced to uncouple the equations of motion, heat conduction and charge equation. The frequency equation of the system is developed under the assumption of stress free, thermal and electrically shorted boundary conditions. The numerical calculations are carried out for the material PZT-5A and the computed stress, strain and temperature distribution are plotted as the dispersion curves and their physical characteristics are discussed for longitudinal and flexural (symmetric and antisymmetric) modes.

1. Introduction

Laser-induced vibrations of micro-beam resonators have attracted considerable attention recently due to their many important technological applications in Micro-Electro-Mechanical Systems (MEMS) and Nano- Electro-Mechanical Systems (NMES). Very rapid thermal processes, under the action of a thermal shock (e.g., ultra-short laser pulse) are interesting from the standpoint of thermoelasticity, since they require an analysis of the coupled temperature and deformation fields. This means that the temperature shock induces very rapid movements in the structure elements, thus causing the rise of very significant inertial forces, and thereby, the rise of vibrations.

In recent years, polymers piezoelectric materials have been used in numerous fields taking advantage of the flexible characteristics of these polymers. Some of the applications of these polymers include Audio device-microphones, high frequency speakers, tone generators and acoustic modems; Pressure switches – position switches, accelerometers, impact detectors, flow meters and load cells; Actuators- electronic fans and high shutters. Since piezoelectric polymers allow their use in a multitude of compositions and geometrical shapes for a large variety of applications from transducers in acoustics, ultrasonic’s and hydrophone applications to resonators in band pass filters, power supplies, delay lines, medical scans and some industrial non-destructive testing instruments.

The coupling between the thermal/electric/elastic fields in piezo electric materials provides a mechanism for sensing thermomechanical disturbances from measurements of induced electric potentials, and for altering structural responses via applied electric fields. One of the applications of the piezo thermoelastic material is to detect the responses of a structure by measuring the electric charge, sensing or to reduce excessive responses by applying additional electric forces or thermal forces actuating. If sensing and actuating can be integrated smartly, a so-called intelligent structure can be designed. The piezoelectric materials are also often used as resonators whose frequencies need to be precisely controlled. The coupling

$$I(t)=\frac{L_0}{t_p^2}e^{\left(\frac{-t}{t_p}\right)}, \tag{1}$$

where $t_p=2\,ps$ is the characterstic time of the laser pulse, the total energy carried by laser pulse per unit area of the laser beam is denoted by L_0 (laser intensity). The conduction of heat transfer in the rod can be modeled with an energy source $Q(z,t)$ as

$$Q(z,t)=\frac{1-R}{\delta}e^{\left(\frac{z-h/2}{\delta}\right)I(t)}=\frac{R_aL_0}{\delta t_p^2}te^{\left(\frac{z-h/2}{\delta}-\frac{t}{t_p}\right)}, \tag{2}$$

where δ is the absorption depth of heating energy and R_a is the surface reflexivity, when we take the laser pulse lie on the surface of the rod when $z=0$, we can get the energy source in the following form

$$Q(z,t)=\frac{R_aL_0}{\delta t_p^2}te^{\left(\frac{-h}{2\delta}-\frac{t}{t_p}\right)}. \tag{3}$$

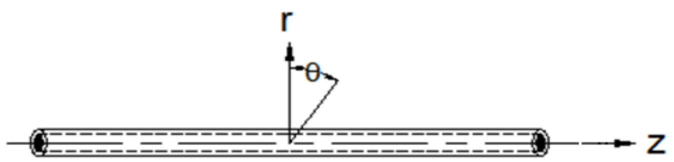

Fig. 1. Geometry of the bar.

The equations of motion, heat conduction and electric potentials in the absence of body force are

$$\sigma_{rr,r}+r^{-1}\sigma_{r\theta,\theta}+\sigma_{rz,z}+r^{-1}\left(\sigma_{rr}-\sigma_{\theta\theta}\right)=\rho u_{r,tt}, \qquad \sigma_{r\theta,r}+r^{-1}\sigma_{\theta\theta,\theta}+\sigma_{\theta z,z}+2r^{-1}\sigma_{r\theta}=\rho u_{\theta,tt},$$
$$\sigma_{rz,r}+r^{-1}\sigma_{\theta z,\theta}+\sigma_{zz,z}+r^{-1}\sigma_{rz}=\rho u_{z,tt}. \tag{4}$$

The heat conduction equation is

$$K_1\left(T_{,rr}+r^{-1}T_{,r}+r^{-2}T_{,\theta\theta}\right)+K_3T_{,zz}-\rho c_vT_{,t}=T_o\left(\beta_1\left(e_{rr}+e_{\theta\theta}\right)+\beta_3e_{zz}-p_3\phi_{,z}-\rho Q\right)_{,t}. \tag{5}$$

The electric charge equation is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rD_r\right)+\frac{1}{r}\frac{\partial D_\theta}{\partial\theta}+\frac{\partial D_z}{\partial r}=0. \tag{6}$$

The elastic, the piezoelectric, and dielectric matrices of the 6mm crystal class, the thermo-piezoelectric relations are

$$\sigma_{rr}=c_{11}e_{rr}+c_{12}e_{\theta\theta}+c_{13}e_{zz}-\beta_1T-e_{31}E_z\;,\; \sigma_{\theta\theta}=c_{12}e_{rr}+c_{11}e_{\theta\theta}+c_{13}e_{zz}-\beta_1T-e_{31}E_z,$$
$$\sigma_{zz}=c_{13}e_{rr}+c_{13}e_{\theta\theta}+c_{33}e_{zz}-\beta_3T-e_{33}E_z\;, \qquad \sigma_{r\theta}=c_{66}e_{r\theta}, \qquad \sigma_{\theta z}=c_{44}e_{\theta z}-r^{-1}e_{15}E_\theta,$$
$$\sigma_{rz}=2c_{44}e_{rz}-e_{15}E_r,$$
$$D_r=e_{15}e_{rz}+\varepsilon_{11}E_r,\; D_\theta=e_{15}e_{\theta z}+r^{-1}\varepsilon_{11}E_\theta,\; D_z=e_{31}\left(e_{rr}+e_{\theta\theta}\right)+e_{33}e_{zz}+\varepsilon_{33}E_z+p_3T, \tag{7}$$

where $\sigma_{rr},\sigma_{\theta\theta},\sigma_{zz},\sigma_{r\theta},\sigma_{\theta z},\sigma_{rz}$ are the stress components, $e_{rr},e_{\theta\theta},e_{zz},e_{r\theta},e_{\theta z},e_{rz}$ are the strain components, T is the temperature change about the equilibrium temperature $T_o, c_{11}, c_{12}, c_{13}, c_{33}, c_{44}$ and $c_{66}=\left(c_{11}-c_{12}\right)/2$ are the five elastic constants, β_1,β_3 and K_1,K_3

$$\bar{\beta}\nabla^2\phi-\zeta W+\left(\bar{d}+\varepsilon e^{-\varpi}(1-\varpi)+i\bar{k}_1\nabla^2-i\bar{k}_3\zeta^2\right)T+\zeta\bar{p}_3V=0, \tag{11}$$

and $\left(\bar{c}_{66}\nabla^2+\left(\Omega^2-\zeta^2\right)\right)\psi=0$, (12)

where $\nabla^2=\frac{\partial^2}{\partial x^2}+x^{-1}\frac{\partial}{\partial x}+x^{-2}\frac{\partial^2}{\partial\theta^2}$

The Eq. (11) can be written as

$$\begin{vmatrix} \left(\bar{c}_{11}\nabla^2+\left(\Omega^2-\zeta^2\right)\right) & -\zeta\left(1+\bar{c}_{13}\right) & -\bar{\beta} & -\zeta\left(\bar{e}_{31}+\bar{e}_{15}\right) \\ \zeta\left(1+\bar{c}_{13}\right)\nabla^2 & \left(\nabla^2+\left(\Omega^2-\zeta^2\bar{c}_{33}\right)\right) & -\zeta & \left(\bar{e}_{15}\nabla^2-\zeta^2\right) \\ \bar{\beta}\nabla^2 & -\zeta & \left(\bar{d}+\varepsilon e^{-\varpi}(1-\varpi)+i\bar{k}_1\nabla^2-i\bar{k}_3\zeta^2\right) & \zeta\bar{p}_3 \\ \zeta\left(\bar{e}_{31}+\bar{e}_{15}\right)\nabla^2 & \left(\bar{e}_{15}\nabla^2-\zeta^2\right) & \zeta\bar{p}_3 & \left(\zeta^2\bar{\varepsilon}_{33}-\bar{\varepsilon}_{11}\nabla^2\right) \end{vmatrix}\left(\phi,W,T,V\right)=0 \tag{13}$$

Evaluating the determinant given in Eq. (13), we obtain a partial differential equation of the form

$$\left(A\nabla^8+B\nabla^6+C\nabla^4+D\nabla^2+E\right)\left(\phi,W,T,V\right)=0, \tag{14}$$

where

$$A=-i\bar{k}_1\bar{c}_{11}g_{18},\ B=\bar{c}_{11}\left(g_9-i\bar{k}_1\bar{\varepsilon}_{11}g_2+\bar{e}_{15}g_{17}\right)+i\bar{k}_1\left(g_1g_{18}-\zeta^2g_3^2\right)-\bar{\beta}^2g_{18},$$

$$C=\bar{c}_{11}\left(g_2g_9+\zeta^2\left(g_5+\bar{\varepsilon}_{11}+2\bar{e}_{15}g_8-i\bar{k}_1\zeta^2\right)\right)+g_1\left(g_9-i\bar{k}_1\bar{\varepsilon}_{11}g_2+\bar{e}_{15}g_{17}\right)$$

$$-\bar{\beta}\left(\bar{\beta}\bar{\varepsilon}_{11}g_2-\zeta^2\left(\bar{\beta}\bar{\varepsilon}_{33}-\bar{p}_3g_3-g_3\bar{e}_{15}+2\bar{\beta}\bar{e}_{15}\right)\right)$$

$$+\zeta^2g_4\left(g_4g_{14}+\bar{\beta}\bar{e}_{15}+\zeta g_3g_{15}\right)-\zeta^2g_3\left(i\bar{k}_1\bar{e}_{15}g_4+\bar{\beta}\left(\bar{p}_3+\bar{e}_{15}\right)-g_3\left(g_{16}+i\bar{k}_1g_2\right)\right),$$

$$D=\zeta^2\bar{c}_{11}\left(g_2g_5-\zeta^2\left(g_8-g_7\right)\right)+g_1\left(g_2g_9+\zeta^2\left(g_5+\bar{\varepsilon}_{11}+2\bar{e}_{15}g_8-i\bar{k}_1\zeta^2\right)\right)$$

$$+\zeta^2g_4\left(g_4\left(g_{11}+\bar{p}_3\zeta\bar{\varepsilon}_{11}\right)-\zeta g_3g_{12}-\bar{\beta}\zeta\left(\zeta+\bar{p}_3^2\right)\right)$$

$$-\bar{\beta}\zeta^2\left(g_4g_{13}-g_2\left(\bar{\beta}\bar{\varepsilon}_{33}-\bar{p}_3g_3\right)-\zeta\left(g_3-\bar{\beta}\right)\right)-\zeta^2g_3\left(g_4g_{11}+g_2\left(\bar{\beta}\bar{p}_3-g_3g_{16}\right)+\zeta^2\left(g_3-\bar{\beta}\right)\right),$$

$$E=\zeta^2g_1g_2g_5-\zeta^2\left(g_1g_8-g_1g_7+g_4^2g_6+\bar{\beta}g_4g_7-g_4g_3g_8\right),$$

in which $g_1=\Omega^2-\zeta^2$, $g_2=\Omega^2-\zeta^2\bar{c}_{33}$, $g_3=\bar{e}_{31}+\bar{e}_{15}$, $g_4=1+\bar{c}_{13}$, $g_5=\bar{d}\bar{\varepsilon}_3-i\bar{k}_3\bar{\varepsilon}_{33}\zeta^2-\bar{p}_3^2$,
$g_6=\bar{d}+\bar{p}_3\zeta\bar{\varepsilon}_{33}+\varepsilon e^{-\varpi}(1-\varpi)$, $g_7=\bar{p}_3-\bar{\varepsilon}_{33}$, $g_8=\bar{d}-\bar{p}_3-i\bar{k}_3\zeta^2$, $g_{11}=\bar{d}\bar{e}_{15}-i\bar{k}_1\zeta^2-i\bar{k}_3\zeta^2\bar{e}_{15}$
$g_9=i\bar{k}_1\zeta^2\bar{\varepsilon}_{33}+\varepsilon e^{-\varpi}(1-\varpi)+i\bar{k}_3\bar{\varepsilon}_{11}\zeta^2-\bar{d}\bar{\varepsilon}_{11}$, $g_{10}=\bar{\varepsilon}_{11}+2\bar{e}_{15}g_8-i\bar{k}_1\zeta^2$,
$g_{12}=\zeta^2\bar{\varepsilon}_{33}+\varepsilon e^{-\varpi}(1-\varpi)+\bar{d}\bar{p}_3-i\bar{k}_3\bar{p}_3\zeta^2$, $g_{13}=\bar{\varepsilon}_{11}-\bar{p}_3\bar{e}_{15}$, $g_{14}=i\left(\bar{k}_1\bar{e}_{15}+\bar{k}_3\right)$,
$g_{15}=\bar{\varepsilon}_{11}-i\bar{p}_3\bar{k}_1$, $g_{16}=\bar{d}^2-\zeta^2i\bar{k}_3$, $g_{17}=i\bar{k}_1\zeta^2-g_{11}$, $g_{18}=\bar{\varepsilon}_{11}+\bar{e}_{15}^2$.

Factorizing the relation given in Eq. (14) into biquadratic equation for $\left(\alpha_ia\right)^2$, $i=1,2,3,4$, the solutions for the symmetric modes are obtained as

$$\phi=\sum_{i=1}^{4}A_iJ_n\left(\alpha_iax\right)\cos n\theta,\qquad W=\sum_{i=1}^{4}a_iA_iJ_n\left(\alpha_iax\right)\cos n\theta,$$

$$T=\sum_{i=1}^{4}b_iA_iJ_n\left(\alpha_iax\right)\cos n\theta,\qquad V=\sum_{i=1}^{4}c_iA_iJ_n\left(\alpha_iax\right)\cos n\theta. \tag{15}$$

$a_{4i} = \left(\bar{e}_{15}\zeta a_i - \bar{\varepsilon}_{11} b_i\right)\left\{nJ_n(\alpha_i ax) - (\alpha_i ax) J_{n+1}(\alpha_i ax)\right\}$, $i=1,2,3,4$, $a_{45} = \bar{e}_{15}\zeta n J_n(\alpha_5 ax)$,
$a_{5i} = c_i\left\{nJ_n(\alpha_i ax) - (\alpha_i ax) J_{n+1}(\alpha_i ax)\right\}$, $i=1,2,3,4$, $a_{55} = 0$.

5. Numerical results and discussion

The free vibration of transversely isotropic thermo-piezoelectric solid bar of circular cross-section is considered. The frequency equation is numerically solved for the material PZT-5A and the material properties of PZT-5A is given below:

$c_{11} = 13.9\times10^{10}\, Nm^{-2}$, $c_{12} = 7.78\times10^{10}\, Nm^{-2}$, $c_{13} = 7.54\times10^{10}\, Nm^{-2}$, $c_{33} = 11.3\times10^{10}\, Nm^{-2}$, $c_{44} = 2.56\times10^{10}\, Nm^{-2}$, $e_{31} = -6.98 Cm^{-2}$, $e_{33} = 13.8 Cm^{-2}$, $e_{15} = 13.4 Cm^{-2}$, $\varepsilon_{11} = 60.0\times10^{-10}\, C^2N^{-1}m^{-2}$, $\varepsilon_{33} = 54.7\times10^{-10}\, C^2N^{-1}m^{-2}$, $\rho = 7750 Kgm^{-2}$, $\beta_1 = 1.52\times10^6\, NK^{-1}m^{-2}$, $\beta_3 = 1.53\times10^6\, NK^{-1}m^{-2}$, $C_e = 420 J\,kg^{-1}K^{-1}$, $K_1 = K_3 = 1.5 Wm^{-1}K^{-1}$.
$\beta_3 = 1.53\times10^6\, NK^{-1}m^{-2}$, $T_0 = 298 K$, $p_3 = -452\times10^{-6}\, CK^{-1}m^{-2}$.

In this problem, there are two kinds of basic independent modes of wave propagation have been considered, namely, the longitudinal and flexural modes. By choosing respectively n=0 and n=1, we can obtain the non-dimensional frequencies of longitudinal and flexural modes of vibrations.

5.1. Dispersion curves. The results of longitudinal and flexural modes (symmetric and antisymmetric) are plotted in the form of dispersion curves. The notation used in the figures, namely LM, FLS and FLAS respectively denote the longitudinal mode, flexural symmetric mode and, flexural anti symmetric mode. The dispersion curves are drawn for stress distribution versus the different position of the thermo-piezoelectric bar for longitudinal and flexural (symmetric and antisymmetric) modes are respectively shown in Figs. 2 and 3. From the Figs. 2 and 3, it is clear that the stress distributions have the same trend in both piezoelectric and thermo-piezoelectric solid bar with respect to its different positions. But there is a small deviation in the lower limit of the position in Fig. 3 due to the influence of laser pulse.

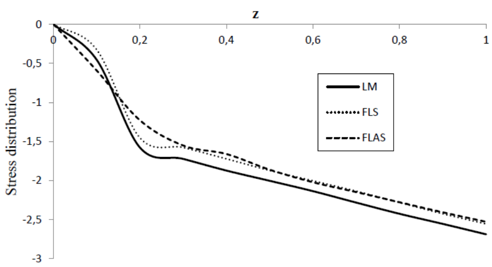

Fig. 2. Stress distribution verses different position of the piezoelectric bar.

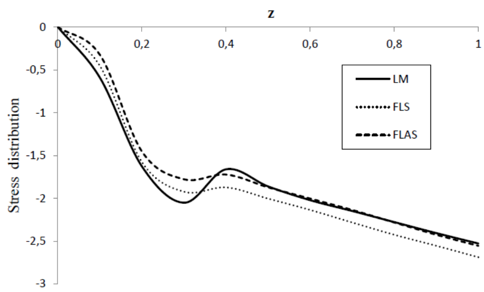

Fig. 3. Stress distribution verses different position of the thermo-piezoelectric bar.

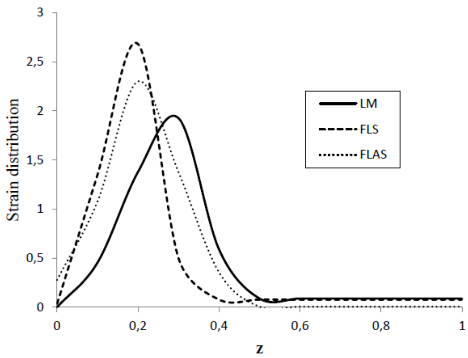

Fig. 4. Strain distribution verses different position of the piezoelectric bar.

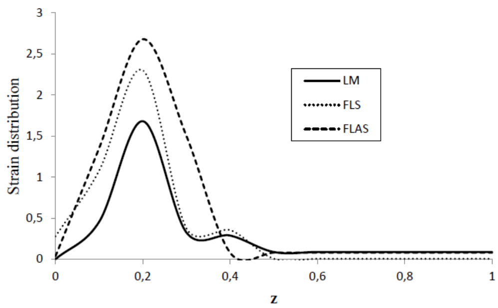

Fig. 5. Strain distribution verses different position of the thermo-piezoelectric bar.

A comparison is made among the longitudinal and flexural symmetric and antisymmetric mode of strain distribution of piezoelectric and thermo-piezoelectric vibrations are respectively shown in the Figs. 3 and 4 respectively. From the Figs. 3 and 4, it is observed that, both the piezoelectric and thermo-piezoelectric bar the strain energy experiences some cross over and peak values for a particular position after that, it starts decreases. The inclusion of thermal energy due to laser pulse is oscillating the strain energy in Fig. 4.

A dispersion curve is drawn to compare the temperature distribution of longitudinal and flexural symmetric and antisymmetric modes of vibration for a piezoelectric and thermo-piezoelectric bar is shown respectively in the Figs. 5 and 6. From the Figs. 5 and 6, it is observed that the temperature distribution are decreases with respect to the position of bar.

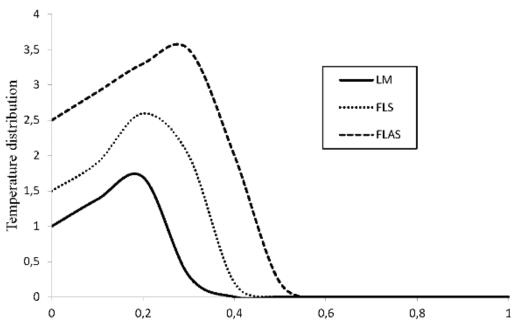

Fig. 6. Temperature distribution verses different position of the piezoelectric bar.

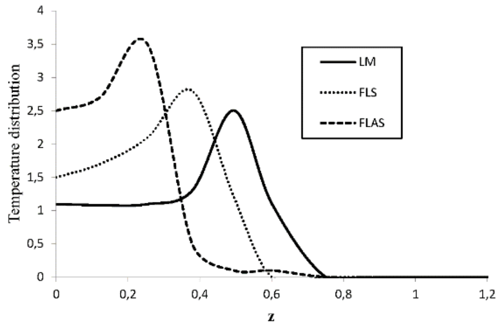

Fig. 7. Temperature distribution verses different position of the thermo-piezoelectric bar.

6. Conclusion

The influence of thermo-piezoelectric field in a circular bar subjected to thermal loading due to non-Gaussian laser pulse is discussed using linear theory of thermoelasticity. The frequency equation of the system is developed under the assumption of stress free, thermal and electrically shorted boundary conditions. The numerical calculations are carried out for the material PZT-5A and the computed stress, strain and temperature distribution are plotted as the dispersion curves and their physical characteristics are discussed for longitudinal and flexural (symmetric and antisymmetric) modes of piezoelectric and thermo-piezoelectric bar. From the result, it is clear that the thermal load due to non-Gaussian laser pulse has the significant effect on the thermal and mechanical interactions.

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