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# ON THE HYDROMAGNETIC STABILITY OF A HORIZONTAL NANOFLUID LAYER WITH HALL CURRENTS

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**Abstract.** The paper investigates the hydromagnetic stability of a horizontal nanofluid layer heated from below in the presence of Hall currents. In addition to the Brownian motion and thermophoretic forces, Lorentz force is introduced due to high magnetic field. Boussinesq's approximation is used to linearize the hydromagnetic equations and perturbation equations are analysed using method of normal modes and one term Galerkin approximation. The critical Rayleigh number and critical wave number for alumina-water nanofluid and copper-water nanofluid for different values of Chandrasekhar number and Hall parameter are found. Copper-water nanofluid is found to be far less stable than alumina-water nanofluid. The mode of heat transfer is through stationary convection for the present configuration. Effect of Hall currents is to quicken the onset of convection whereas that of magnetic field is to postpone it.

## 1. Introduction

Chandrasekhar [1] has done a comprehensive work on the problem of thermal convection of a Newtonian fluid layer heated from below by considering different aspects of hydrodynamics and hydromagnetics. The convection problem has applications over a wide range of disciplines, including heat transfer, material science, physics and chemical engineering. Thus, there was a major requirement of highly suitable and conductive fluids, which lead to the invention of new class of fluids known as "nanofluids". Choi [2] was the first scientist who conceived the concept of nanofluid and claimed that by adding nanometer sized particles in base fluids heat transfer rate can be increased. Eastman et al. [3], Vadasz [4] and Choi et al. [5] contributed further in the field and reported the high thermal efficiency of nanofluids. These heat transfer enhancement characteristics influenced Buongiorno [6] to investigate the impact of these characteristics on convective situations and then the conservation equations for a nanofluid layer came into existence. By using conservation equations formulated by Buongiorno [6], Tzou [7, 8], and Nield and Kuznetsov [9, 10] examined the stability of a horizontal nanofluid layer uniformly heated from below. Studies pertaining to the effect of rotation have been conducted by Bhadauria et al. [11], Yadav et al. [12], and Chand and Rana [13]. They remarkably observed that for the case of stationary convection the addition of Coriolis force term and the difference in temperature have stabilizing effects while the reversed effect has been observed due to Brownian motion and thermophoresis. The subject of hydromagnetic stability has attracted the attention of many researchers in view of its applications to astrophysics, geophysics and engineering. Heris et al. [14] examined that magnetic field, nanofluids influenced the thermal performance of a two-phase closed thermosyphon and its thermal efficiency is increased due to magnetic field. Ghasemi et al. [15] and Hamada et al. [16] investigated the behaviour of electrically conducting nanofluids in the presence of magnetic field experimentally. The result of their experimental work is that the fluid within an enclosure experience a Lorentz force due to applied magnetic field, which affects the buoyancy force and the heat transfer rate. Recently, Gupta et al. [17] and Yadav et al.[18] studied the influence of magnetic field on the horizontal layer of nanofluid for bottom heavy configuration of nanoparticles and top heavy configuration, respectively. On the application of magnetic field of high strength perpendicular to the electric field, an electric potential is generated across the conductor, which is perpendicular to both electric field and magnetic field. This phenomenon in the literature is known as Hall Effect. Hall Effect is the direct result of the induced Lorentz force due to applied external magnetic field. Hall Effect is remarkably important in many geophysical and astrophysical systems in addition to the flow of laboratory plasmas.

None of the researchers till today have attempted to study the effect of Hall currents in the presence of Brownian motion and thermophoretic diffusion. Motivated by the applications and importance of Hall currents as mentioned earlier, we are set out to study its effect on thermal instability for alumina-water nanofluid and copper-water nanofluid. In addition to the Lorentz force term in the momentum equation, Maxwell's equations are modified as an additional term is introduced due to the presence of Hall currents. Normal mode technique and Galerkin type weighted residual method is used to obtain the dispersion relation. The critical wave numbers and the critical Rayleigh numbers are obtained for both the types of nanofluids. Stability of copper-water nanofluid is found to be far less than that of alumina water nanofluid. The value of $Ra_c$ for threshold values of the metallic/non-metallic water nanofluids is found to be 63.4 in the absence of Hall parameter and the value decreases appreciably with the increase in Hall parameter since the effect of Hall currents is to hasten the onset of instability.

## 2. Mathematical formulation of the problem

Let us consider the Cartesian coordinate system $x, y, z$ where $x$ - and $y$ - axes are horizontal and $z$ -axis is directing vertically upwards. The region occupied by the nanofluid layer is defined as $S = \{0 \le z \le d, -\infty < x, y < \infty\}$. The fluid layer is heated from below so that $T_0$ and $\phi_1$ are temperature and concentration at the lower boundary and $T_1$ and $\phi_0$ that at the upper boundary, respectively, where $T_0 > T_1$ and $\phi_0 > \phi_1$. Let the system be under the influence of a large magnetic field $\boldsymbol{H}(0,0,H)$. The electric currents generated due to high magnetic field, which are called Hall currents.

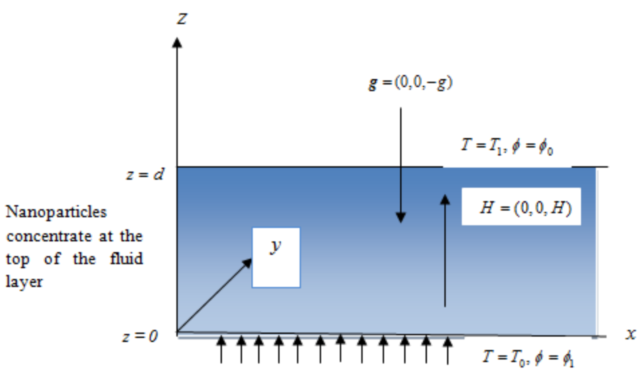


**Fig. 1.** Geometrical Configuration.

The conservation equations under the effect of Hall currents using Buongiorno's assumptions are

$$\nabla.\boldsymbol{v} = 0, \quad (1)$$

$$\frac{\partial\phi}{\partial t}+\boldsymbol{v}.\nabla\phi=\nabla.\left[D_B\nabla\phi+D_T\frac{\nabla T}{T}\right], \tag{2}$$

$$\rho\left(\frac{\partial\boldsymbol{v}}{\partial t}+\boldsymbol{v}.\nabla\boldsymbol{v}\right)=-\nabla p+\mu\nabla^2\boldsymbol{v}+\rho\boldsymbol{g}+\frac{\mu_e}{4\pi}\left(\nabla\times\boldsymbol{h}\right)\times\boldsymbol{H}, \tag{3}$$

$$\left(\rho c\right)_f\left[\frac{\partial T}{\partial t}+\boldsymbol{v}.\nabla T\right]=(k\nabla^2T)+\left(\rho c\right)_p\left[D_B\nabla\phi.\nabla T+D_T\frac{\nabla T.\nabla T}{T}\right], \tag{4}$$

where the density of the nanofluid is given by $\rho=\phi\rho_p+\left(1-\phi\right)\rho_f\cong\phi\rho_p+\left(1-\phi\right)\left\{\rho_{f0}\left(1-\beta\left(T-T_0\right)\right)\right\}$.
In the presence of Hall currents Maxwell's equations are

$$\frac{d\boldsymbol{h}}{dt}=\left(\boldsymbol{H}.\nabla\right)\boldsymbol{v}+\eta\nabla^2\boldsymbol{h}-\frac{1}{4\pi Ne}\nabla\times\left[(\nabla\times\boldsymbol{h})\times\boldsymbol{H}\right], \tag{5}$$

$$\nabla.\boldsymbol{h}=0, \tag{6}$$

where the operator *d/dt* stands for the convective derivative given by $d/dt=\partial/\partial t+\boldsymbol{v}.\nabla$. Here $\boldsymbol{v}=(u,v,w)$, $\phi,\ D_B,\ D_T,\ \mu,\ h=(h_x,h_y,h_z),\ (\rho c)_f,(\rho c)_p,\rho_p,\ \rho_f,\mu_e,\ k,p,\eta,\ N,\ e,\ \beta$ denote, respectively, the nanofluid velocity, the nanoparticles volume fraction, the Brownian diffusion coefficient, the thermophoretic diffusion coefficient, the viscosity of the fluid, the components of magnetic field, the heat capacity of the fluid, the heat capacity of the nanoparticles, the density of nanoparticles, the density of fluid, the magnetic permeability, the thermal conductivity of the medium, the pressure, the magnetic resistivity, number density, charge on an electron and the volumetric coefficient of thermal expansion.
Now introducing the dimensionless variables as

$$(x^*,y^*,z^*)=(x,y,z)/d,\ \ p^*=pd^2/\mu\alpha_m,\ \ h^*=\left(\eta/H\alpha_m\right)h,\ \ t^*=t\alpha_m/d^2,$$

$$(u^*,v^*,w^*)=(u,v,w)d/\alpha_m,\ \ \phi'=(\phi-\phi_1)/(\phi_0-\phi_1),\ \ T'=(T-T_c)/(T_h-T_c),\ \text{where}\ \alpha_m=k/(\rho c)_f, \tag{7}$$

so, that Eqs. (1)-(6) take the forms (after dropping the asterisk sign)

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{8}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{\partial p}{\partial x}+\Pr_1\nabla^2u+Q\left(\frac{\partial h_x}{\partial z}-\frac{\partial h_z}{\partial x}\right), \tag{9}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}+\Pr_1\nabla^2v+Q\left(\frac{\partial h_y}{\partial z}-\frac{\partial h_z}{\partial y}\right), \tag{10}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\Pr_1\nabla^2w-G\left[\left(\phi_1-1\right)\left\{-1+\beta\left(T_1-T_0\right)\right\}+R_\rho\phi_1\right]-$$
$$-\left(\phi_1-1\right)RaT\Pr_1-G\left[\left(\phi_0-\phi_1\right)\phi\left\{-1+\beta\left(T_1-T_0\right)+R_\rho\right\}\right]-\phi\left(\phi_0-\phi_1\right)TRa\Pr_1, \tag{11}$$

$$\frac{\partial\phi}{\partial t}+u\frac{\partial\phi}{\partial x}+v\frac{\partial\phi}{\partial y}+w\frac{\partial\phi}{\partial z}=N_{BT}\nabla^2\phi+N_{TT}\nabla^2T, \tag{12}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\nabla^2T+\frac{1}{Le}\left[\frac{\partial T}{\partial x}\frac{\partial\phi}{\partial x}+\frac{\partial T}{\partial y}\frac{\partial\phi}{\partial y}+\frac{\partial T}{\partial z}\frac{\partial\phi}{\partial z}\right]+\frac{N_A}{Le}\left[\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2+\left(\frac{\partial T}{\partial z}\right)^2\right], \tag{13}$$

$$\frac{\Pr_2}{\Pr_1}\frac{dh_x}{dt}=\frac{\partial u}{\partial z}+\nabla^2h_x-\frac{H}{4\pi Ne\eta}\left[\frac{\partial^2h_z}{\partial z\partial y}-\frac{\partial^2h_y}{\partial z^2}\right], \tag{14}$$

$$\frac{\Pr_2}{\Pr_1}\frac{dh_y}{dt}=\frac{\partial v}{\partial z}+\nabla^2h_y-\frac{H}{4\pi Ne\eta}\left[\frac{\partial^2h_x}{\partial z^2}-\frac{\partial^2h_z}{\partial z\partial x}\right], \tag{15}$$

$$\frac{\Pr_2}{\Pr_1}\frac{dh_z}{dt}=\frac{\partial w}{\partial z}+\nabla^2 h_z-\frac{H}{4\pi Ne\eta}\left[\frac{\partial^2 h_y}{\partial z\partial x}-\frac{\partial^2 h_x}{\partial z\partial y}\right], \tag{16}$$

$$\frac{\partial h_x}{\partial x}+\frac{\partial h_y}{\partial y}+\frac{\partial h_z}{\partial z}=0, \tag{17}$$

where $G=Ra\Pr_1/\beta(T_0-T_1)$ and various non dimensional parameters are

$$M=\left(H/4\pi Ne\eta\right)^2;\ \Pr_1=\frac{\mu}{\rho\alpha_m};\quad \Pr_2=\frac{\mu}{\rho\eta};\quad Le=\frac{k}{\rho_p c_p D_B\left(\phi_0-\phi_1\right)};\ R_\rho=\frac{\rho_p}{\rho_0};\ Ra=\frac{g\beta d^3(T_0-T_1)}{\nu\alpha_m};$$

$$N_A=\frac{N_{TT}}{N_{BT}};\ N_{TT}=\frac{D_T(T_0-T_1)}{\alpha_m T_0\left(\phi_0-\phi_1\right)};\ N_{BT}=\frac{D_B}{\alpha_m};\ Q=\left(\frac{\mu_e H^2 d^2}{4\pi\eta\mu}\right). \tag{18}$$

Here, $Ra$ is the thermal Rayleigh number, $Le$ is the Lewis number, $R_\rho$ is the density ratio, $\Pr_1$ is the Prandtl number, $N_{BT}$ is the Brownian to thermal diffusivity, $N_A$ is the diffusivity ratio, $N_{TT}$ is the coefficient of themophoretic diffusion, $\alpha_m$ is the thermal diffusivity, $Q$ is the Chandrasekhar number and $\Pr_2$ is the magnetic Prandtl number.

## 3. Basic state

Initially, we assume that $(u,v,w)=0$; therefore at the basic state physical quantities undergo variation in $z$-direction only, and is given by

$$\boldsymbol{v}=0,\ T=T_b(z),\ \phi=\phi_b(z),\ p=p_b(z). \tag{19}$$

Here, the 'b' is used to denote the basic state. Using Eq. (19) in Eqs. (12)-(13) and integrating w.r.t. $z$, we obtain

$$\phi_b=-N_A T_b+c_1 z+c_2, \tag{20}$$

$$T_b=\left[1-\exp\left(\frac{(1-z)(1-N_A)}{Le}\right)\right]/\left[1-\exp\left(\frac{(1-N_A)z}{Le}\right)\right]. \tag{21}$$

For most nanofluids $N_A\approx 1-10$, $Le$ is of the order of $10^4$-$10^5$ (Buongiorno [6]). Using this fact, second and higher order terms are neglected in the power series expansion of exponential function. Then a good approximation for the basic state is

$$T_b=1-z,\phi_b=z. \tag{22}$$

## 4. Perturbation equations

Let $\boldsymbol{v}', p', T', \phi', \boldsymbol{h}'\left(h_x,h_y,h_z\right)$ symbolize the disturbances in physical quantities velocity, pressure, temperature, volume fraction of nanoparticles and magnetic field respectively. Let us superimpose these perturbations on the basic state i.e.

$$\boldsymbol{v}(u,v,w)=0+\boldsymbol{v}'(u',v',w'),\ p=p_b+p',\ T=T_b+T',\ \phi=\phi_b+\phi',\quad \boldsymbol{h}(h_x,h_y,h_z)=\boldsymbol{H}+\boldsymbol{h}'(h_x',h_y',h_z'). \tag{23}$$

Thus, the disturbance equations after using the concept of linear theory are (dropping the dashes for the sake of easiness)

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{24}$$

$$\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\Pr_1\nabla^2 u+Q\left(\frac{\partial h_x}{\partial z}-\frac{\partial h_z}{\partial x}\right), \tag{25}$$

$$\frac{\partial v}{\partial t}=-\frac{\partial p}{\partial y}+\Pr_1\nabla^2 v+Q\left(\partial h_y/\partial z-\partial h_z/\partial y\right), \tag{26}$$

$$\frac{\partial w}{\partial t}=-\frac{\partial p}{\partial z}+\Pr_1\nabla^2 w-G\left[(\phi_0-\phi_1)\phi\left\{-1+\beta(T_1-T_0)+R_\rho\right\}\right]-(\phi_1-1)RaT\Pr_1-(\phi_0-\phi_1)(\phi T+T\phi)Ra\Pr_1, \tag{27}$$

$$\frac{\partial \phi}{\partial t}+w=N_{BT}\nabla^2\phi+N_{TT}\nabla^2 T, \tag{28}$$

$$\frac{\partial T}{\partial t}-w=\nabla^2 T+\frac{1}{Le}\left[\frac{\partial T}{\partial z}-\frac{\partial \phi}{\partial z}\right]-2\frac{N_A}{Le}\frac{\partial T}{\partial z}, \tag{29}$$

$$\frac{\Pr_2}{\Pr_1}\frac{dh_x}{dt}=\frac{\partial u}{\partial z}+\nabla^2 h_x-\frac{H}{4\pi N e\eta}\left[\frac{\partial^2 h_z}{\partial z\partial y}-\frac{\partial^2 h_y}{\partial z^2}\right], \tag{30}$$

$$\frac{\Pr_2}{\Pr_1}\frac{dh_y}{dt}=\frac{\partial v}{\partial z}+\nabla^2 h_y-\frac{H}{4\pi N e\eta}\left[\frac{\partial^2 h_x}{\partial z^2}-\frac{\partial^2 h_z}{\partial z\partial x}\right], \tag{31}$$

$$\frac{\Pr_2}{\Pr_1}\frac{dh_z}{dt}=\frac{\partial w}{\partial z}+\nabla^2 h_z-\frac{H}{4\pi N e\eta}\left[\frac{\partial^2 h_y}{\partial z\partial x}-\frac{\partial^2 h_x}{\partial z\partial y}\right], \tag{32}$$

$$\frac{\partial h_x}{\partial x}+\frac{\partial h_y}{\partial y}+\frac{\partial h_z}{\partial z}=0. \tag{33}$$

Eliminating $p$ from Eqs. (25)-(27) and using Eq. (24), we get

$$\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)=\Pr_1\nabla^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-(\phi_1-1)Ra\Pr_1\nabla_H^2 T-$$

$$-G\left[(\phi_0-\phi_1)\nabla_H^2\phi\left\{-1+\beta(T_1-T_0)+R_\rho\right\}\right]-(\phi_0-\phi_1)\left(\phi_b\nabla_H^2 T+T_b\nabla_H^2\phi\right)Ra\Pr_1+Q\Pr_1\frac{\partial}{\partial z}\nabla^2 h_z, \tag{34}$$

where $\nabla_H^2=\partial^2/\partial x^2+\partial^2/\partial y^2$. Eliminating pressure term from Eqs. (25)-(26) and introducing vorticity $\varsigma=(\partial v/\partial x-\partial u/\partial y)$ and current density $\xi=(\partial h_y/\partial x-\partial h_x/\partial y)$ the system of Eqs. (25)-(26) and (30)-(33) reduce to

$$\frac{\partial \zeta}{\partial t}=\Pr_1\nabla^2\zeta+Q\Pr_1\frac{\partial \xi}{\partial z}, \tag{35}$$

$$\frac{\Pr_2}{\Pr_1}\frac{dh_z}{dt}=\frac{\partial w}{\partial z}+\nabla^2 h_z-\frac{H}{4\pi N e\eta}\frac{\partial \xi}{\partial z}, \tag{36}$$

$$\frac{\Pr_2}{\Pr_1}\frac{d\xi}{dt}=\frac{\partial \zeta}{\partial z}+\nabla^2\xi+\frac{H}{4\pi N e\eta}\frac{\partial}{\partial z}\nabla^2 h_z. \tag{37}$$

The above system of Eqs. (28)-(29) and (34)-(37) will be analysed using Normal mode technique and Galerkin type weighted residual method.

## 5. Normal mode technique and Galerkin type weighted residual method

It is assumed that the perturbed quantities are of the form

$$\left(w,T,\phi,h_z,\xi,\zeta\right)=\left(W(z),\Theta(z),\Phi(z),K(z),X(z),Z(z)\right)\exp(ik_x x+ik_y y+nt), \tag{38}$$

where $k_x,k_y$ are the wave numbers along horizontal and vertical directions respectively, $\alpha=\left(k_x^2+k_y^2\right)^{1/2}$ is the resultant wave number and $n$ is the growth rate. Following the normal mode analysis, Eqs. (34)-(37) and (28)-(29) take the forms

$$n\left(D^2-\alpha^2\right)W=\Pr_1\left(D^2-\alpha^2\right)^2 W+(\phi_1-1)Ra\Pr_1\alpha^2\Theta+G\left[(\phi_0-\phi_1)\alpha^2\Phi\left\{-1+\beta(T_1-T_0)+R_\rho\right\}\right]$$

$$+(\phi_0-\phi_1)\left(\bar{\Phi}\alpha^2\Theta+\bar{\Theta}\alpha^2\Phi\right)Ra\Pr_1+Q\Pr_1\left(D^2-\alpha^2\right)DK, \tag{39}$$

$$\Pr_1(D^2-\alpha^2)Z+Q\Pr_1 DX=nZ, \tag{40}$$

$$DZ+\left(D^2-\alpha^2\right)X+\frac{H}{4\pi Ne\eta}D(D^2-\alpha^2)K=\frac{\Pr_2}{\Pr_1}nX, \tag{41}$$

$$DW+\left(D^2-\alpha^2\right)K-\frac{H}{4\pi Ne\eta}DX=\frac{\Pr_2}{\Pr_1}nK, \tag{42}$$

$$n\Theta-W=(D^2-\alpha^2)\Theta+\frac{1}{Le}(D\Theta-D\Phi)-2\frac{N_A}{Le}D\Theta, \tag{43}$$

$$n\Phi+W=N_{BT}\left(D^2-\alpha^2\right)\Phi+N_{TT}\left(D^2-\alpha^2\right)\Theta. \tag{44}$$

Let us consider the system with both boundaries as free. The relevant boundary conditions following Chandrasekhar [1] are

$$W=0, D^2W=0, \Theta=\Phi=0 \text{ at } z=0,1. \tag{45}$$

$K$ = 0; $DK$ = 0, on the boundaries which are perfectly conducting. Eliminating $K, X$ and $Z$ from Eqs. (39)-(42), we get

$$\left[\Pr_1\left(D^2-\alpha^2\right)^2-n\left(D^2-\alpha^2\right)\right]\left[\left(D^2-\alpha^2-n\frac{\Pr_2}{\Pr_1}\right)\left\{\Pr_1\left(D^2-\alpha^2\right)^2-n\left(D^2-\alpha^2\right)-Q\Pr_1 D^2-\right.\right.$$
$$\left.\left.-n\Pr_2\left(D^2-\alpha^2\right)+n^2\frac{\Pr_2}{\Pr_1}\right\}+MD^2\left(D^2-\alpha^2\right)\left(\Pr_1\left(D^2-\alpha^2\right)-n\right)\right]W+\left[\left(D^2-\alpha^2-n\frac{\Pr_2}{\Pr_1}\right)\times\right.$$
$$\times\left(\Pr_1\left(D^2-\alpha^2\right)^2-n\left(D^2-\alpha^2\right)-Q\Pr_1 D^2-n\Pr_2\left(D^2-\alpha^2\right)+n^2\frac{\Pr_2}{\Pr_1}\right)+MD^2\left(D^2-\alpha^2\right)\times$$
$$\left.\times\left(\Pr_1\left(D^2-\alpha^2\right)-n\right)\right]\left[(\phi_1-1)Ra\Pr_1\alpha^2\Theta+G\left(\phi_0-\phi_1\right)\left(-1+\beta\left(T_1-T_0\right)+R_\rho\right)\alpha^2\Phi+\left(\phi_1-\phi_0\right)\alpha^2\times\right.$$
$$\left.\times\left(\bar{\Phi}\Theta+\bar{\Theta}\Phi\right)Ra\Pr_1\right]+Q\Pr_1 D\left(D^2-\alpha^2\right)\left[-D\left(\Pr_1\left(D^2-\alpha^2\right)^2-n\left(D^2-\alpha^2\right)-Q\Pr_1 D^2-\right.\right.$$
$$\left.\left.-n\Pr_2\left(D^2-\alpha^2\right)+n^2\frac{\Pr_2}{\Pr_1}\right)\right]W=0. \tag{46}$$

Applying, Galerkin-type weighted residuals method to solve the system of Eqs. (43)-(44) and (46) along with boundary conditions (45). In this method $W, \Theta$ and $\Phi$ are taken as the linear combination of the base functions

$$W=\sum_{p=1}^{N}A_pW_p,\quad \Theta=\sum_{p=1}^{N}B_p\Theta_p,\quad \Phi=\sum_{p=1}^{N}C_p\Phi_p. \tag{47}$$

$p$ = 1; 2; 3…..N, where N is the natural number. Here, $A_p; B_p$ and $C_p$ are the constants and the base functions $W_p, \Theta_p, \Phi_p$ satisfy the respective boundary conditions. In one term Galerkin method we put $W=A_1\sin\pi z, \Theta=B_1 sin\pi z,\ \Phi=C_1 sin\pi z$, which satisfy the free-free boundary conditions. Substituting this solution in Eqs. (43)-(44) and in Eq. (46) and integrating from $z=0$ to $z=1$, we obtained a system of three equations in three unknowns. Elimination of these unknowns produces the eigenvalue equation as

$$\left[\Pr_1 J^2+nJ\right]\left[\left(J+n\frac{\Pr_2}{\Pr_1}\right)\left\{\Pr_1 J^2+nJ+Q\Pr_1\pi^2+n\Pr_2 J+n^2\frac{\Pr_2}{\Pr_1}\right\}+\right.$$
$$\left.+M\pi^2J\left(\Pr_1 J+n\right)\right](n+J)+\left[\left(J+n\frac{\Pr_2}{\Pr_1}\right)\left(\Pr_1 J^2+nJ+Q\Pr_1\pi^2+n\Pr_2 J+n^2\frac{\Pr_2}{\Pr_1}\right)+M\pi^2J\times\right.$$

$$\times\left(\Pr_1 J+n\right)\right]\left[\left(\phi_1-1\right)Ra\Pr_1\alpha^2\Theta+G\left(\phi_0-\phi_1\right)\left(-1+\beta\left(T_1-T_0\right)+R_\rho\right)\alpha^2\Phi\left(-\frac{n+J(1+N_{TT})}{1+N_{BT}J}\right)+$$

$$+\left(\phi_0-\phi_1\right)\alpha^2\left(1-\frac{n+J(1+N_{TT})}{1+N_{BT}J}\right)Ra\Pr_1\right]+Q\Pr_1\pi^2 J\left[\left(\Pr_1 J^2+nJ+Q\Pr_1\pi^2+n\Pr_2 J+n^2\frac{\Pr_2}{\Pr_1}\right)\right]=0, \quad (48)$$

where $J=\pi^2+\alpha^2$ and $M=\left(H/4\pi Ne\eta\right)^2$ is the Hall parameter.

## 6. Results and discussion

**6.1. Non-oscillatory convection.** For non-oscillatory convection $n=0$, thus the eigen value equation (48) reduces to

$$Ra_{stat}=\frac{2J^3\left(Q\pi^2+J^2+M\pi^2 J\right)N_{BT}+2Q\pi^2\left(Q\pi^2+J^2\right)JN_{BT}}{\alpha^2\left(Q\pi^2+J^2+M\pi^2 J\right)\left[\left(2-\phi_1-\phi_0\right)N_{BT}+\left(\phi_0-\phi_1\right)\left\{1+\frac{2\left\{-1+\beta(T_1-T_0)+R_\rho\right\}}{\beta(T_0-T_1)}\right\}\left(1+N_{TT}\right)\right]}. \quad (49)$$

It is noteworthy that the expression for thermal Rayleigh number is independent of both the Prandtl numbers and Lewis number. For $Q=0$ and $M=0$; Eq. (49) takes the form

$$Ra_{stat}=\frac{2J^3 N_{BT}}{\alpha^2\left[\left(2-\phi_1-\phi_0\right)N_{BT}+\left(\phi_0-\phi_1\right)\left\{1+\frac{2\left\{-1+\beta(T_1-T_0)+R_\rho\right\}}{\beta(T_0-T_1)}\right\}\left(1+N_{TT}\right)\right]}, \quad (50)$$

which is in confirmation with the results obtained by Yadav et al. [12] in the absence of rotation. In the absence of nanoparticles $\left(\phi_0=\phi_1=0\right)$ and Hall current $\left(M=0\right)$, Eq. (49) reduces to

$$Ra_{stat}=\pi^4\frac{(1+x)}{x}\left[\left(1+x\right)^2+\frac{Q}{\pi^2}\right], \text{ where we have put } \alpha^2=\pi^2 x \quad (51)$$

which coincides with the result of Chandrasekhar [1] for Newtonian fluids. In the absence of the Hall current parameter the expression for $Ra_{stat}$ becomes

$$Ra_{stat}=\frac{2J^3 N_{BT}+2Q\pi^2 JN_{BT}}{\alpha^2\left[\left(2-\phi_1-\phi_0\right)N_{BT}+\left(\phi_0-\phi_1\right)\left\{1+\frac{2\left\{-1+\beta(T_1-T_0)+R_\rho\right\}}{\beta(T_0-T_1)}\right\}\left(1+N_{TT}\right)\right]}. \quad (52)$$

To investigate the effect of Chandrasekhar number and Hall parameter on the stability of the system, let us find the derivatives of $Ra_{stat}$ w.r.t $Q$ and $M$ as follows

$$\frac{dRa_{stat}}{dQ}=\left[2\pi^2 J^3\left(Q\pi^2+J^2+M\pi^2 J\right)N_{BT}+2Q\pi^4 J\left(Q\pi^2+J^2\right)N_{BT}+2Q\pi^6 J^2 MN_{BT}\right]\times$$

$$\times\left[\alpha^2\left(Q\pi^2+J^2+M\pi^2 J\right)^2\left[\left(2-\phi_1-\phi_0\right)N_{BT}+\left(\phi_0-\phi_1\right)\left\{1+\frac{2\left\{-1+\beta(T_1-T_0)+R_\rho\right\}}{\beta(T_0-T_1)}\right\}\left(1+N_{TT}\right)\right]\right]^{-1}. \quad (53)$$

Clearly, $Ra_{stat}$ increases with the increase in the Chandrasekhar number as the denominator in Eq. (53) is positive due to small value of nanoparticle volume fraction. Thus, magnetic field plays its well-established role of stabilizing and delays the onset of convection.

$$\frac{dRa_{stat}}{dM}=\left[-2Q\pi^4 J\left(Q\pi^2+J^2\right)N_{BT}\right]\left[\alpha^2\left(Q\pi^2+J^2+M\pi^2 J\right)^2\times\right.$$

$$\left.\times\left[\left(2-\phi_1-\phi_0\right)N_{BT}+\left(\phi_0-\phi_1\right)\left\{1+\frac{2\left\{-1+\beta(T_1-T_0)+R_\rho\right\}}{\beta(T_0-T_1)}\right\}\left(1+N_{TT}\right)\right]\right]^{-1}. \quad (54)$$

It is clear that $Ra_{stat}$ decreases as the Hall parameter increases as expression (54) carries a negative sign in the numerator. Thus, the effect of Hall currents is contrary to that of magnetic field and it hastens the onset of convection. The critical values of Rayleigh number as well as its associated wave number are characterized by the condition

$$(dRa/d\alpha)_{\alpha=\alpha_c} = 0. \tag{55}$$

Here, rather than evaluating $Ra_c$, it is more suitable to evaluate $Ra$ as a function of '$\alpha$' for different values of $Q$ and $M$, and locate the minimum numerically. The critical Rayleigh number $Ra_c$ and the corresponding critical wave number $\alpha_c$ for alumina-water nanofluid and copper-water nanofluid are obtained in this fashion in Section 7.

**6.2. Oscillatory convection.** To consider the oscillatory mode of convection, let us take $n = i\omega \neq 0$ in the eigenvalue Eq. (48). The real and imaginary parts of Eq. (48) take the forms

$$-2J(1+N_{BT}J)\Big[\big(\Pr_1 J^2-\omega^2\big)\Big\{\Pr_1 J^3+Q\Pr_1\pi^2 J-2\omega^2 J\frac{\Pr_2}{\Pr_1}-\omega^2\frac{\Pr_2^2}{\Pr_1}J\Big\}-\omega^2 J(1+\Pr_1)\times$$

$$\times\Big\{J^2+2\Pr_2 J^2+Q\Pr_2\pi^2-\omega^2\Big(\frac{\Pr_2}{\Pr_1}\Big)^2\Big\}\Big]-2J(1+N_{BT}J)M\pi^2 J\Big[\big(\Pr_1 J^2-\omega^2\big)\Pr_1 J-\omega^2 J(1+\Pr_1\Big]-$$

$$-2(\phi_0-1)(1+N_{BT}J)\alpha^2\Big[J\Big\{\Pr_1 J^2+Q\Pr_1\pi^2-\omega^2\frac{\Pr_2}{\Pr_1}\Big\}-\omega^2\frac{\Pr_2}{\Pr_1}J(1+\Pr_2)+M\pi^2 J^2\Pr_1\Big]Ra\Pr_1+$$

$$+2G(\phi_1-\phi_0)\{-1+\beta(T_1-T_0)+R_\rho\}\alpha^2\Big[\Big\{J^2(1+N_{TT})-\omega^2\frac{\Pr_2}{\Pr_1}\Big\}\Big\{\Pr_1 J^2+Q\Pr_1\pi^2-\omega^2\frac{\Pr_2}{\Pr_1}\Big\}-$$

$$-\omega^2 J^2\Big\{1+\frac{\Pr_2}{\Pr_1}(1+N_{TT})\Big\}(1+\Pr_2)+M\pi^2 J\{\Pr_1 J^2(1+N_{TT})-\omega^2\}\Big]-(\phi_1-\phi_0)\alpha^2\big[J\{1+N_{BT}J-$$

$$-J(1+N_{TT})+\omega^2\frac{\Pr_2}{\Pr_1}\Big\}\Big\{\Pr_1 J^2+Q\Pr_1\pi^2-\omega^2\frac{\Pr_2}{\Pr_1}\Big\}-\omega^2\Big(J+\frac{\Pr_2}{\Pr_1}(1+N_{BT}J-J(1+N_{TT}))\Big)\times$$

$$\times J(1+\Pr_2)+M\pi^2 J\{\Pr_1 J(1+N_{BT}J-J(1+N_{TT}))+\omega^2\}\big]Ra\Pr_1-2Q\Pr_1\pi^2 J(1+N_{BT}J)\times$$

$$\times\Big[J\Big\{\Pr_1 J^2+Q\Pr_1\pi^2-\omega^2\frac{\Pr_2}{\Pr_1}-\omega^2 J(1+\Pr_2)\Big\}\Big]=0, \tag{56}$$

$$-2J(1+N_{BT}J)\Big[J(1+\Pr_1)\Big\{\Pr_1 J^2+Q\Pr_1\pi J^2-2\omega^2 J\frac{\Pr_2}{\Pr_1}-\omega^2\frac{\Pr_2^2}{\Pr_1}J\Big\}-\big(\Pr_1 J^2-\omega^2\big)\times$$

$$\times\Big\{J^2+2\Pr_2 J^2+Q\Pr_2\pi^2-\omega^2\Big(\frac{\Pr_2}{\Pr_1}\Big)^2\Big\}\Big]-2J(1+N_{BT}J)M\pi^2 J\Big[J(1+\Pr_1)\Pr_1 J+\big(\Pr_1 J^2-\omega^2\big)\Big]-$$

$$-2(\phi_0-1)(1+N_{BT}J)\alpha^2\Big[J^2(1+\Pr_2)+\frac{\Pr_2}{\Pr_1}\Big\{\Pr_1 J^2+Q\Pr_1\pi^2-\omega^2\frac{\Pr_2}{\Pr_1}\Big\}+M\pi^2 J\Big]Ra\Pr_1+$$

$$+2G(\phi_1-\phi_0)\{-1+\beta(T_1-T_0)+R_\rho\}\alpha^2\Big[J\Big\{1+\frac{\Pr_2}{\Pr_1}(1+N_{TT})\Big\}\Big\{\Pr_1 J^2+Q\Pr_1\pi^2-\omega^2\frac{\Pr_2}{\Pr_1}\Big\}+$$

$$+J\Big\{-\omega^2\frac{\Pr_2}{\Pr_1}+J^2(1+N_{TT})\Big\}(1+\Pr_2)+M\pi^2 J\{\Pr_1 J+J(1+N_{TT})\}\Big]-(\phi_1-\phi_0)\alpha^2\big[J^2\{1+N_{BT}J-$$

$$-J(1+N_{TT})+\omega^2\frac{\Pr_2}{\Pr_1}\Big\}(1+\Pr_2)+\Big(-J+\frac{\Pr_2}{\Pr_1}(1+N_{BT}J)-J(1+N_{TT}))\Big)\Big\{\Pr_1 J^2+Q\Pr_1\pi^2-\omega^2\frac{\Pr_2}{\Pr_1}\Big\}+$$

$$+M\pi^2 J\left\{-\Pr_1 J+(1+N_{BT}J-J(1+N_{TT}))\right\}\Big] Ra\Pr_1-2Q\Pr_1\pi^2 J(1+N_{BT}J)\times$$

$$\times\left[\left\{\Pr_1 J^2+Q\Pr_1\pi^2-\omega^2\frac{\Pr_2}{\Pr_1}-J^2(1+\Pr_2)\right\}\right]=0. \tag{57}$$

Eq. (57) can be rewritten to obtain

$$Ra_{osc}=\Big[2J^2M(1+JN_{BT})\pi^2(J^2\Pr_1+J^2\Pr_1(1+\Pr_1)-\omega^2)-\left\{-J+\frac{(1+JN_{BT}-J(1+N_{TT}))\Pr_2}{\Pr_1}\right\}\times$$
$$\times\left\{\Pr_1 J^2+Q\Pr_1\pi^2-\omega^2\frac{\Pr_2}{\Pr_1}\right\}+2J(1+JN_{BT})Q\Pr_1\pi^2\left\{\Pr_1 J^2+J^2(1+\Pr_2)+Q\Pr_1\pi^2-\omega^2\frac{\Pr_2}{\Pr_1}\right\}\times$$
$$\times\alpha^2(1+\Pr_2)(\phi_1-\phi_0)\left\{J^2(1+N_{BT}J-J(1+N_{TT}))+\omega^2\frac{\Pr_2}{\Pr_1}\right\}+2J(1+JN_{BT})(J^2\Pr_1-\omega^2)\times$$
$$\times\left\{J^2+2\Pr_2 J^2+Q\Pr_2\pi^2-\omega^2\frac{\Pr_2^2}{\Pr_1^2}\right\}+J(1+\Pr_1)\left\{\Pr_1 J^3+Q\Pr_1 J\pi^2-2J\omega^2\frac{\Pr_2}{\Pr_1}-J\omega^2\frac{\Pr^2_2}{\Pr_1}\right\}\Big]\times$$
$$\times\Big[M\pi^2 J\Pr_1(1+JN_{BT}-J(1+N_{TT})-J\Pr_1)-2\alpha^2(1+JN_{BT})\Pr_1(-1+\phi_0)\big\{M\pi^2 J+J^2(1+\Pr_2)\times$$
$$+\frac{\Pr_2}{\Pr_1}\left(J^2\Pr_1+\pi^2 Q\Pr_1-\frac{\omega^2\Pr_2}{\Pr_1}\right)+\big\{2\alpha^2\Pr_1(-1+R_\rho+(T_0-T_1)\beta(-\phi_0+\phi_1)\times$$
$$\times(MJ\pi^2(J(1+N_{TT})+J\Pr_1)+J(1+\Pr_2)(J^2(1+N_{TT})-\Pr_2\omega^2/\Pr_1)+J(1+((1+N_{TT})\times$$
$$\times\Pr_2)/\Pr_1)(J^2\Pr_2+Q\Pr_1\pi^2-\Pr_2\omega^2/\Pr_1)\}/((T_0-T_1)\beta))\}\Big]^{-1}. \tag{58}$$

Eliminating $Ra$ from Eqs. (56) and (57), we get

$$a\omega^8+b\omega^6+c\omega^4+d\omega^2+e=0, \tag{59}$$

where

$$\omega^2=-\frac{b}{4a}-\frac{p_4}{2}\pm\frac{\sqrt{p_5\pm p_6}}{2}, \tag{60}$$

$$p_1=2c^3-9bcd+27ad^2+27b^2e-72ace,\ p_2=p_1+\sqrt{-4\left(c^2-3bd+12ae\right)^3+p_1^2},$$

$$p_3=\frac{c^2-3bd+12ae}{3a\sqrt[3]{p_2/2}}+\frac{\sqrt[3]{p_2/2}}{3a},\ p_4=\sqrt{\frac{b^2}{4a^2}-\frac{2c}{3a}+p_3},\ p_5=\frac{b^2}{4a^2}-\frac{4c}{3a}-p_3,\ p_6=\frac{1}{4p_4}\left(-\frac{b^3}{a^3}+\frac{4bc}{a^2}-\frac{8d}{a}\right). \tag{61}$$

$Ra_{osc}$ is computed for alumina-water nanofluid as well as for copper-water nanofluid for the values of various parameters as given in section 7 using Mathematica software and it has been found that all the values of $Ra_{osc}$ are negative. Therefore, oscillatory motions are not possible for the present configuration of nanoparticles and the instability sets in through stationary convection. This is expected as for oscillatory motions to exist two of the buoyancy forces should be there in opposite directions.

## 7. Numerical results and discussion

The thermal convection problem has been extensively used in oil and gas extraction from beneath the earth. The enhanced heat transfer characteristics of nanofluids can be utilized as a major breakthrough for this extraction process since most of the oil companies are in search of the efficient and economic techniques to maximize the amount of recovered oil. Hall currents are of paramount importance in geophysical problems due to the presence of earth's high magnetic field. We can find sufficiently reliable and accurate value of the critical Rayleigh number, which will determine the most effective temperature for the recovery of maximum oil.

As such, we will determine critical Rayleigh number as the fluid shifts from stable to unstable state. The thermofluid characteristic properties for alumina-water nanofluid and copper-water nanofluid as mentioned by Tzou [7, 8] are

$$\Delta T = 10K,\ \Delta\phi = \phi_0 - \phi_1 = 0.01,\ \ \phi_0 = 0.05,\ N_A = 30.18,\ N_{BT} = 0.2,\ \ \beta = 6\times10^{-3},\ R_\rho = 4, \qquad (62)$$

and $\Delta T = 10K,\ \Delta\phi = \phi_0 - \phi_1 = 0.01,\ \phi_0 = 0.05,\ N_{BT} = 2,\ N_A = 3.018,\ \beta = 6\times10^{-4},\ R_\rho = 9.$ (63)

These values will be used to determine the thermal Rayleigh number as well as critical Rayleigh number in further analysis.

Figures 2a,b represent the effect of Hall current on Rayleigh number $Ra$ for $Al_2O_3$-water nanofluid and Cu-water nanofluid, respectively whereas Figs. 3a,b depict the impact of Chandrasekhar number on the stability of the system for both types of nanofluid. It is observed from the figures that the values of $Ra$ have a significant fall with the rise in Hall current parameter and have an appreciable rise with the rise in Chandrasekhar number. Thus, by applying magnetic field of high strength, effect of Hall current parameter is to quicken the onset of instability whereas the effect of Chandrasekhar number is just opposite to that of Hall current parameter i.e. to postpone the onset of convection. Thus, there are mutually conflicting tendencies of the two parameters. Further these figures show that $Ra$ (Cu-water) < $Ra$ ($Al_2O_3$-water) which means that Cu-water nanofluid is much less stable than $Al_2O_3$-water nanofluid in the presence of Hall current and magnetic field.

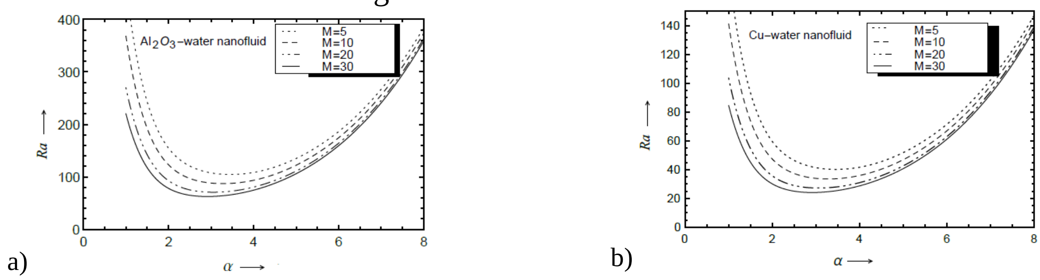


**Fig. 2.** Non oscillatory variation of thermal Rayleigh number with $M$ for (a) $Al_2O_3$-water nanofluid and (b) Cu-water nanofluid for fixed $Q$ = 100.

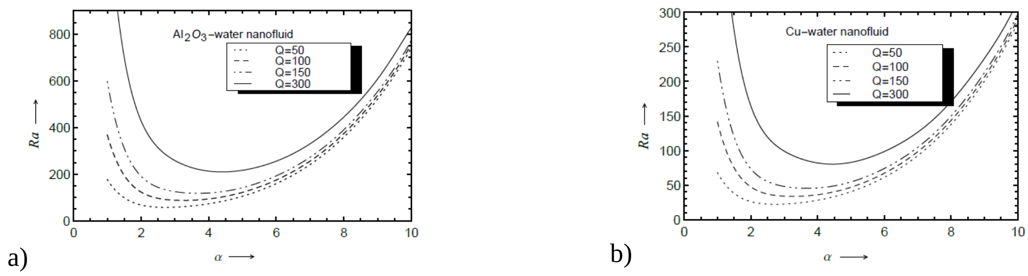


**Fig. 3.** Non oscillatory variation of thermal Rayleigh number with $Q$ for (a) $Al_2O_3$-water nanofluid and (b) Cu-water nanofluid for fixed $M$ = 10.

Figures 4, 5 depict the non-oscillatory variation of $Ra_c$ for alumina-water nanofluid and copper-water nanofluid with the variation in $Q$ and $M$, respectively. It is observed that the values of $Ra_c$ increases/decreases appreciably with the increase in magnetic field/Hall current, respectively. Thus, we can increase the value of the critical Rayleigh number as high as desired by applying magnetic field of high strength and similarly the critical Rayleigh number can be made sufficiently small by increasing the Hall parameter. Further, the curve for alumina-water nanofluid overlies the curve for copper-water nanofluid.

For considering effects of nanoparticle volume fraction, difference in temperature between the boundaries, density ratio and Brownian diffusion coefficient, we take metallic/non-metallic oxide nanofluids with threshold values as given by Tzou [7, 8]

$$\Delta T = 80K,\ \Delta\phi = \phi_0 - \phi_1 = 0.01,\ \phi_0 = 0.05,\ N_{BT} = 0.2,\ N_A = 30.18,\ \beta = 5.30\times10^{-4},\ R_\rho = 6. \qquad (64)$$

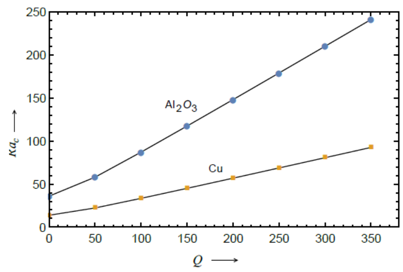


**Fig. 4.** Effect on $Ra_c$ with the variation in $Q$ for fixed value of $M$ =10.

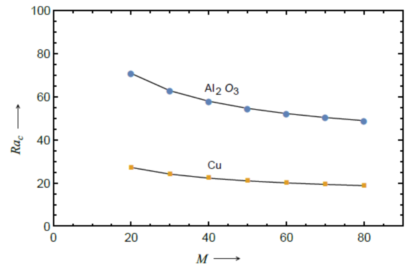


**Fig. 5.** Effect on $Ra_c$ with the variation in $M$ for fixed value of $Q$ =100.

Figure 6 represents the non-oscillatory variation of $Ra$ with '$\alpha$' for $\Delta\phi$ = 0.005, 0.01, 0.02, and 0.03 and for fixed values of the Chandrasekhar number and Hall parameter. Clearly as the difference in concentration of nanoparticles at the boundaries increases there is a significant decrease in the values of $Ra$ which signifies the fact that $\Delta\phi$ destabilizes the nanofluid layer system.

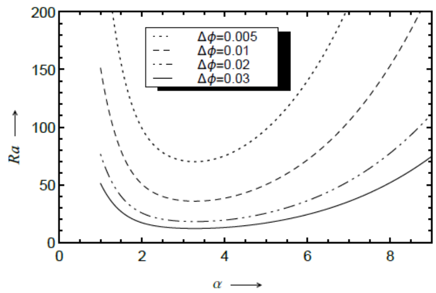


**Fig. 6.** Non oscillatory variation of thermal Rayleigh number with $\Delta\varphi$ for fixed $Q$ = 100; $M$ = 10.

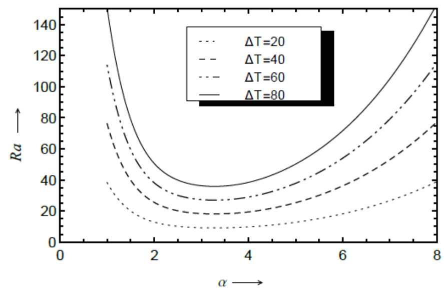


**Fig. 7.** Non oscillatory variation of thermal Rayleigh number with $\Delta T$ for fixed $Q$ = 100; $M$ = 10.

Table 1. Effect of volumetric fraction of nanoparticles on critical Rayleigh number $Ra_c$ for fixed $Q$ = 100.

| | $M=0$ | | $M=10$ | | $M=20$ | | $M=30$ | |
|---|---|---|---|---|---|---|---|---|
| $\Delta\phi$ | $\alpha_c$ | $Ra_c$ | $\alpha_c$ | $Ra_c$ | $\alpha_c$ | $Ra_c$ | $\alpha_c$ | $Ra_c$ |
| 0 | 3.70 | 2677.8 | 3.29 | 1605.5 | 3.07 | 1305.3 | 2.94 | 1157.2 |
| 0.005 | 3.70 | 123.9 | 3.29 | 74.3 | 3.07 | 60.4 | 2.94 | 53.6 |
| 0.01 | 3.70 | 63.4 | 3.29 | 38.03 | 3.07 | 30.9 | 2.94 | 27.4 |
| 0.02 | 3.70 | 32.1 | 3.29 | 19.2 | 3.07 | 15.6 | 2.94 | 13.9 |
| 0.03 | 3.70 | 21.5 | 3.29 | 12.9 | 3.07 | 10.5 | 2.94 | 9.3 |

Further, Table 1 gives the value of $Ra_c$ = 63.4 and $\alpha_c$ = 3.70 for $M=0$ and shows that $Ra_c$ decreases significantly with the rise in Hall parameter as well as nanoparticle volume fraction whereas $\alpha_c$ is independent of $\Delta\phi$ and decreases with rise in M. This confirms that the said parameters hasten the onset of non-oscillatory convection.The behaviour of temperature difference between the boundaries on the onset of thermal instability is analysed from Fig. 7 for $\Delta T$ = 20, 40, 60, and 80. It is observed that buoyancy forces increase with the increase in temperature difference and consequently, Rayleigh number increases. The critical values of the Rayleigh number and wave number are shown in Table 2 and it is observed that $Ra_c$ increases with the rise in $\Delta T$ whereas $\alpha_c$ is independent of the same.

Figure 8 corresponds to the non-oscillatory variation of $Ra$ versus '$\alpha$' for $R_\rho$ = 3, 6, and 9. The figure indicates that rise in nanoparticle density ratio tends to decrease the thermal Rayleigh number. This is because disturbance due to nanoparticles of high density is more as compared to nanoparticles of low density.

Table 2. Effect of temperature difference of nanoparticles on critical Rayleigh number $Ra_c$ for fixed $Q$ = 100.

| | $M=0$ | | $M=10$ | | $M=20$ | | $M=30$ | |
|---|---|---|---|---|---|---|---|---|
| $\Delta T$ | $\alpha_c$ | $Ra_c$ | $\alpha_c$ | $Ra_c$ | $\alpha_c$ | $Ra_c$ | $\alpha_c$ | $Ra_c$ |
| 20 | 3.70 | 16.1 | 3.29 | 9.6 | 3.07 | 7.8 | 2.94 | 6.9 |
| 40 | 3.70 | 32.1 | 3.29 | 19.2 | 3.07 | 15.6 | 2.94 | 13.8 |
| 60 | 3.70 | 47.8 | 3.29 | 28.7 | 3.07 | 23.3 | 2.94 | 20.7 |
| 80 | 3.70 | 63.4 | 3.29 | 38.0 | 3.07 | 30.9 | 2.94 | 27.4 |
| 100 | 3.70 | 78.9 | 3.29 | 47.3 | 3.07 | 38.5 | 2.94 | 34.1 |

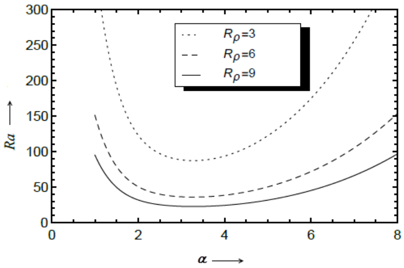


**Fig. 8.** Non oscillatory variation of thermal Rayleigh number with $R_\rho$ for fixed $Q$ = 100; $M$ = 10.

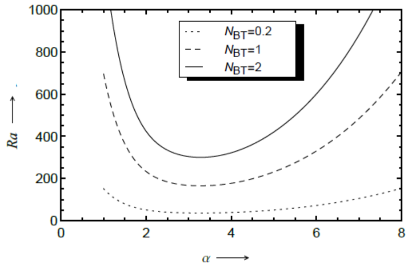


**Fig. 9.** Non oscillatory variation of thermal Rayleigh number with $N_{BT}$ for fixed $Q$ = 100; $M$ = 10.

Table 3. Effect of density ratio on critical Rayleigh number $Ra_c$ for fixed $Q$ = 100.

| | $M=0$ | | $M=10$ | | $M=20$ | | $M=30$ | |
|---|---|---|---|---|---|---|---|---|
| $R_\rho$ | $\alpha_c$ | $Ra_c$ | $\alpha_c$ | $Ra_c$ | $\alpha_c$ | $Ra_c$ | $\alpha_c$ | $Ra_c$ |
| 2 | 3.70 | 294.4 | 3.29 | 176.5 | 3.07 | 143.5 | 2.94 | 127.2 |
| 3 | 3.70 | 154.2 | 3.29 | 92.4 | 3.07 | 75.1 | 2.94 | 66.6 |
| 4 | 3.70 | 104.4 | 3.29 | 62.6 | 3.07 | 50.9 | 2.94 | 45.1 |
| 5 | 3.70 | 78.9 | 3.29 | 47.3 | 3.07 | 38.5 | 2.94 | 34.1 |
| 6 | 3.70 | 63.4 | 3.29 | 38.0 | 3.07 | 31 | 2.94 | 27.4 |

Table 4. Effect of Brownian to thermal diffusivity ratio on critical Rayleigh number $Ra_c$ for fixed $Q$ = 100.

| | $M=0$ | | $M=10$ | | $M=20$ | | $M=30$ | |
|---|---|---|---|---|---|---|---|---|
| $N_{BT}$ | $\alpha_c$ | $Ra_c$ | $\alpha_c$ | $Ra_c$ | $\alpha_c$ | $Ra_c$ | $\alpha_c$ | $Ra_c$ |
| 0.2 | 3.70 | 63.44 | 3.29 | 30.9 | 3.07 | 27.4 | 2.94 | 38 |
| 0.5 | 3.70 | 153.2 | 3.29 | 74.7 | 3.07 | 66.2 | 2.94 | 91.8 |
| 0.7 | 3.70 | 209.7 | 3.29 | 102.2 | 3.07 | 90.6 | 2.94 | 125.7 |
| 1 | 3.70 | 290 | 3.29 | 141.3 | 3.07 | 125.3 | 2.94 | 369.7 |
| 1.5 | 3.70 | 412.6 | 3.29 | 201.1 | 3.07 | 178.3 | 2.94 | 627.8 |

Table 3 confirms the result through critical values. Non-oscillatory variation of $Ra$ with '$\alpha$' for $N_{BT}$ = 0.2, 1, and 2 is plotted in Fig. 9. Clearly, as $N_{BT}$ rises $Ra$ increases predominantly. Table 4 shows a significant increase in the critical Rayleigh number $Ra_c$ with the increase in $N_{BT}$ whereas the critical wave number is independent of the Brownian to thermal diffusivity ratio and decreases with the Hall parameter. Thus, Brownian to thermal diffusivity ratio $N_{BT}$ has stabilizing effect on the nanofluid layer system in the presence of Hall current while Hall current itself has a destabilizing character.

## 8. Conclusions

Present chapter investigates Hall Effect on the hydromagnetic stability of a nanofluid layer for alumina-water nanofluid and copper-water nanofluid using Buongiorno's model. The stability of the system has been examined by using the method of superposition of basic possible modes and single term Galerkin approximation. The effect of different nanofluid parameters have been studied and critical Rayleigh number and critical wave number have been found on the transition from stable to unstable modes. The problem has practical application in the field of oil extraction from beneath the earth where Hall currents are important due to high magnetic field of earth. The present formulation provides the most reliable and accurate value of critical Rayleigh number and associated wave number. This critical value helps to calculate the suitable temperature that should be given in order to get maximum amount of underground content (oil). The value of the critical Rayleigh number $Ra_c$ for threshold values of the metallic/non-metallic water nanofluids is found to be 63.4 in the absence of Hall parameter and this value falls significantly with the rise in Hall parameter. The behaviour of the parameters like magnetic field, volumetric fraction of nanoparticles, nanoparticle density ratio and Brownian to thermal diffusivity ratio has been analysed numerically. It has been found that magnetic field, temperature difference and Brownian to thermal diffusivity ratio delay the onset of convection while Hall current, volumetric fraction of nanoparticles and nanoparticles density ratio hasten the onset of instability. It is established that $Ra_c$ for alumina-water nanofluid is much greater than $Ra_c$ for copper-water nanofluid, which means that alumina-water nanofluid exhibits higher stability as compared to copper-water nanofluid.

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