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DEFORMATION DUE TO VARIOUS SOURCES IN TRANSVERSELY ISOTROPIC THERMOELASTIC MATERIAL WITHOUT ENERGY DISSIPATION AND WITH TWO-TEMPERATURE

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Abstract. A general solution to the field equations of a transversely isotropic thermoelastic without energy dissipation and with two temperatures due to various sources has been obtained in the transformed domain using the Laplace and Fourier transforms. As an application, concentrated or distributed sources have been taken to illustrate the utility of the approach. The transformed solutions are inverted numerically using a numerical inversion technique. The result in the form displacement components, conductive temperature and stress components are obtained numerically and illustrated graphically for particular model. Some special cases of interest are also discussed for investigation.

1. Introduction

The classical theory of thermoelasticity as exposed, for example, in Carlson's article [1] has found generalizations and modifications into various thermoelastic models that run under the label hyperbolic thermoelasticity, see the survey of Chandrasekharaiah [2] and Hitnarski and Ignazack [3]. The notation 'hyperbolic' reflects the fact that thermal waves are modeled, avoiding the physical paradox of infinite propagation speed of the classical model. In the 1990's Green and Naghdi [4-6] proposed three new thermoelastic theories based on an entropy equality rather than usual entropy inequality. The constitutive assumptions for heat flux vector are different in each theory. Thus, they obtained three theories that they called thermoelasticity of type I, II & III. When the theory of type I is linearized, we obtain the classical system of thermoelasticity. The theory of type II (a limited case of type III) does not admit energy dissipation. In the context of the linearized version of this theory, theorem on uniqueness of solutions has been established by Hitnarski & Ignazack [4] and Green and Naghdi [6].

Thermoelasticity with two temperatures is one of the non-classical theories of thermoelasticity of elastic solids. The thermal dependence is the main difference of this theory with respect to the classical one. A theory of heat conduction in deformable bodies depend on the two distinct temperatures, the conductive temperature ϕ and thermodynamic temperature T which has been formulated by Chen et al. [7, 8]. The difference between these two temperatures is proportional to the heat supply for time independent situations. For time dependent problems and for wave propagation problem in particular, the two temperatures are in general different, regardless of the presence of heat supply. Warren and Chen [9] investigated the wave propagation in the two-temperature theory of thermoelasticity.

A new theory of generalized thermoelasticity by considering the theory of heat conduction in deformable bodies, which depends on two distinct temperatures, the conductive temperature and thermodynamic temperature was presented by Youssef [10]. A uniqueness theorem for generalized linear thermoelasticity involving a homogeneous and isotropic body was also recorded in this study. Also a new theory of generalized thermoelasticity by considering the theory of two-temperature generalized thermoelasticity for homogeneous and isotropic body without energy dissipation was given by Youssef [11]. Various authors e.g. Puri and Jorden [12], Youssef and Al-Lehaibi [13], Youssef and Al-Harby [14], Magana and Quintanilla [15], Mukhopadhyay and Kumar [16], Abbas et al. [17], Roushan and Santwana [18], Kaushal et al. [19, 20] studied the problems of thermoelastic media with two temperatures. Miglani and Kaushal [21] studied axi-symmteric problem in the context of generalized theories with two temperatures, Sharma et al. [22] studied propagation of waves in thermoelastic micropolar solid with two temperatures bordered with layers or half-spaces of inviscid liquid. Kumar et al. [23] studied plane waves in transversely isotropic viscothermoelastic medium with two temperatures and rotation, Sharma and Marin [24] investigated wave propagation in micropolar thermoelastic solid half space with distinct conductive and thermodynamic temperature.

In this paper, a general solution has been obtained in the transformed form using the Laplace and Fourier transforms to the field equations of a transversely isotropic thermoelastic without energy dissipation and with two temperature due to various sources. Concentrated sources have been taken to illustrate the utility of the approach as an application. Numerical inversion technique is applied to invert numerically the transformed solutions. Numerically, the results of displacement components, conductive temperature and stress components have been obtained and have been illustrated graphically for particular model. Some special cases of interest are also discussed for investigation.

2. Basic Equations

Following Youssef [10, 11] the constitutive relations and field equations are

$$t_{ij} = C_{ijkl}e_{kl} - \beta_{ij}T \tag{1}$$

$$C_{ijkl}e_{kl,j} - \beta_{ij}T_{,j} + \rho F_i = \rho \ddot{u}_i \tag{2}$$

$$K_{ij}\phi_{,ij} = \beta_{ij}T_0\ddot{e}_{ij} + \rho C_E\ddot{T} \tag{3}$$

where $T = \phi - a_{ij}\phi_{,ij}$, $\beta_{ij} = C_{ijkl}\alpha_{ij}$, $e_{ij} = (u_{i,j} + u_{j,i})/2$, $(i, j=1,2,3)$, and C_{ijkl} , β_{ij} are the constitutive coefficients, T is temperature, T_0 being the reference temperature, t_{ij} is component of stress tensor, e_{kl} is the component of the strain tensor, u_i are the displacement components, ρ is density, C_E is the specific heat, K_{ij} is the thermal conductivity, a_{ij} is the two temperature parameters, ϕ is the conductive temperature and α_{ij} is coefficient of linear thermal expansion.

3. Formulation of the Problem

We consider homogeneous transversely isotropic thermoelastic solid half space without energy dissipation and with two temperatures. We take rectangular Cartesian coordinate system (x_1, x_2, x_3) having origin on the surface x_3=0 with x_3-axis pointing vertically downward into the half space. A mechanical (normal or tangential) force and thermal source is assumed to be acting at the plane surface x_3=0. We restrict our analysis in two dimensions subject to the plane parallel to x_1x_3-plane.

The displacement vector for two dimensional problem is taken as

$$\vec{u} = (u_1,\ 0,\ u_3), \qquad \vec{\phi} = (0,\ \phi_2,\ 0) \tag{4}$$

The basic governing equations (2) & (3) with the aid of (4) are given by

$$c_{11}u_{1,11} + c_{44}u_{1,33} + (c_{13}+c_{44})u_{3,13} - \beta_1 \frac{\partial}{\partial x_1}\{\phi - (a_1\phi_{,11} + a_3\phi_{,33})\} = \rho\ddot{u}_1, \tag{5}$$

$$c_{33}u_{3,33} + c_{44}u_{3,11} + (c_{13}+c_{44})u_{1,13} - \beta_3 \frac{\partial}{\partial x_3}\{\phi - (a_1\phi_{,11} + a_3\phi_{,33})\} = \rho\ddot{u}_3, \tag{6}$$

$$k_1\phi_{,11} + k_3\phi_{,33} = T_0(\beta_1\ddot{e}_{11} + \beta_3\ddot{e}_{33}) + \rho C_E\{\ddot{\phi} - (a_1\ddot{\phi}_{,11} + a_3\ddot{\phi}_{,33})\}. \tag{7}$$

The relation between T and ϕ is,

$$T = \left[1 - \left(a_1\frac{\partial^2}{\partial x_1^{\ 2}} + a_3\frac{\partial^2}{\partial x_3^{\ 2}}\right)\right]\phi \tag{8}$$

and $\beta_1 = C_{11}\alpha_1 + C_{13}\alpha_3$, $\beta_3 = C_{31}\alpha_1 + C_{33}\alpha_3$.

To facilitate the solution, following dimensionless quantities are introduced:

$$x_1' = \frac{x_1}{L},\ x_3' = \frac{x_3}{L},\ u_1' = \frac{\rho c_1^{\ 2}}{L\beta_1 T_0}u_1,\ u_3' = \frac{\rho c_1^{\ 2}}{L\beta_1 T_0}u_3,\ T' = \frac{T}{T_0},\ t' = \frac{c_1}{L}t,\ t_{11}' = \frac{t_{11}}{\beta_1 T_0},$$

$$t_{33}' = \frac{t_{33}}{\beta_1 T_0},\ t_{31}' = \frac{t_{31}}{\beta_1 T_0},\ \phi' = \frac{\phi}{T_0},\ a_1' = \frac{a_1}{L},\ a_3' = \frac{a_3}{L}, \tag{9}$$

where $c_1^2 = \dfrac{c_{11}}{\rho}$ and L is a constant of dimension length.

Using the dimensionless quantities defined by (9) in equations (5)-(7), we obtain (after suppressing the primes),

$$\frac{\partial^2 u_1}{\partial x_1^2} + b_1\frac{\partial^2 u_1}{\partial x_3^2} + b_2\frac{\partial^2 u_3}{\partial x_1\partial x_3} - \left[1 - \left(a_1\frac{\partial^2}{\partial x_1^2} + a_3\frac{\partial^2}{\partial x_3^2}\right)\right]\frac{\partial\varphi}{\partial x_1} = \frac{\partial^2 u_1}{\partial t^2}, \tag{10}$$

$$b_3\frac{\partial^2 u_3}{\partial x_3^2} + b_1\frac{\partial^2 u_3}{\partial x_1^2} + b_2\frac{\partial^2 u_1}{\partial x_1\partial x_3} - b_4\left[1 - \left(a_1\frac{\partial^2}{\partial x_1^2} + a_3\frac{\partial^2}{\partial x_3^2}\right)\right]\frac{\partial\phi}{\partial x_3} = \frac{\partial^2 u_3}{\partial t^2}, \tag{11}$$

$$\frac{\partial^2\phi}{\partial x_1^2} + b_5\frac{\partial^2\phi}{\partial x_3^2} - \frac{\partial}{\partial t^2}\left[b_6\left(\frac{\partial u_1}{\partial x_1}\right) + b_7\left(\frac{\partial u_3}{\partial x_3}\right)\right] = b_8\left[1 - \left(a_1\frac{\partial^2}{\partial x_1^2} + a_3\frac{\partial^2}{\partial x_3^2}\right)\right]\frac{\partial^2\phi}{\partial t^2}, \tag{12}$$

where $b_1 = \dfrac{c_{44}}{c_{11}}$, $b_2 = \dfrac{c_{13}+c_{44}}{c_{11}}$, $b_3 = \dfrac{c_{33}}{c_{11}}$, $b_4 = \dfrac{\beta_3}{\beta_1}$, $b_5 = \dfrac{k_3}{k_1}$, $b_6 = \dfrac{T_0\beta_1^2}{k_1\rho}$, $b_7 = \left(\dfrac{T_0\beta_3\beta_1}{k_1\rho}\right)$,

$b_8 = \dfrac{c_E c_{11}}{k_1}$.

To solve the system of equations, we define Laplace and Fourier transforms as

$$\tilde{f}(x_1,x_3,s) = \int_0^\infty e^{-st} f(x_1,x_3,t)\,dt,\ \ \hat{\tilde{f}}(\xi,x_3,s) = \int_{-\infty}^{\infty} e^{i\xi}\tilde{f}(x_1,x_3,s)\,dx. \tag{13}$$

Apply Laplace and Fourier transforms defined by (13) on equations (10)-(12) and simplifying, we obtain

$$\left(\frac{d^6}{dx_3{}^6}+Q\frac{d^4}{dx_3{}^4}+R\frac{d^2}{dx_3{}^2}+S=0\right)\left(\hat{\tilde{u}}_1,\hat{\tilde{u}}_3,\hat{\tilde{\phi}}\right), \tag{14}$$

where $Q=\frac{1}{P}\left[A_{12}\left(-b_7-A_{11}\right)+s^2\xi^2\alpha_1A_{13}+s^2\xi^2\alpha_3A_{14}+s^2b_1A_{15}+\xi^2b_1A_{16}\right]$,

$R=\frac{1}{P}\left[A_{12}\left(s^2\xi^2A_{17}+s^2A_{18}+A_{19}\right)+s^2\xi^2A_{20}+A_{21}+A_{22}\right]$, $S=\frac{1}{P}\left[A_{12}\left(-s^2\xi^2A_{23}-\xi^7b_3\right)+s^2\xi^2A_{24}\right]$,

$P=-b_1b_7-b_1A_{11}$, and $A_{11}=\alpha_3s^2b_{10}+\alpha_3s^2b_5b_6b_9$, $A_{12}=s^2-\xi^2$, $A_{13}=b_1b_{10}-b_1b_5b_6b_9$,

$A_{14}=b_1b_3b_{10}-b_2b_4b_{10}-b_2b_5b_6b_8-b_4b_9+b_8$, $A_{15}=b_6b_7b_{10}+\alpha_3s^2b_6b_{10}+b_9$, $A_{16}=\xi^3+b_3b_7-b_2b_4b_7$,

$A_{17}=\alpha_1b_{10}+\alpha_3b_3b_{10}+\alpha_1b_5b_6b_7$, $A_{18}=b_{10}+b_6b_7+\alpha_3s^2b_6b_{10}+b_9$, $A_{19}=\xi^5+\xi^2b_3b_7$,

$A_{21}=\xi^7\left(b_2b_4-b_1b_3\right)b_6b_7b_{10}+s^4b_1b_6b_{10}$,

$A_{20}=-b_1b_3b_{10}-\alpha_1s^2b_1b_6b_{10}+b_2b_4b_{10}+b_2b_5b_6b_8+b_4b_9-b_8-s^2\alpha_3b_6b_8$,

$A_{22}=s^2\xi^4\left(-\alpha_1b_1b_3b_{10}+\alpha_1b_2b_4b_{10}-\xi b_1b_6+\alpha_1b_2b_5b_6b_8+\alpha_1b_4b_9+\alpha_1b_8-\alpha_3b_3b_8\right)$,

$A_{23}=-\alpha_1\xi^2b_3b_{10}-b_3b_{10}-\xi^3b_6-s^2b_6b_{10}-\alpha_1s^2b_6b_{10}$,

$A_{24}=\xi^2b_3b_8+b_6b_8s^2+\alpha_1\xi^4b_3b_8+\alpha_1\xi^2b_6b_8$, $b_9=c_{13}/c_{11}$.

Making use of the radiation conditions $\hat{\tilde{u}}_1,\hat{\tilde{u}}_3,\hat{\tilde{\phi}}\to 0$ as $x_3\to 0$, the solution of Eq. (14) can be written as

$$\left(\hat{\tilde{u}}_1,\hat{\tilde{u}}_3,\hat{\tilde{\phi}}\right)=\left(\sum_{i=1}^{3}A_ie^{-m_i\,x_3},\sum_{i=1}^{3}p_iA_ie^{-m_i\,x_3},\sum_{i=1}^{3}\eta_iA_ie^{-m_i\,x_3}\right), \tag{15}$$

where $\pm m_i\left(i=1,2,3\right)$ are the roots of the characteristic equation

$$\frac{d^6}{dx_3{}^6}+Q\frac{d^4}{dx_3{}^4}+R\frac{d^2}{dx_3{}^2}+S=0, \tag{16}$$

and $p_i=\left(\frac{\lambda_1m_i{}^3+\lambda_2m_i}{\lambda_3m_i{}^4+\lambda_4m_i{}^2+\lambda_5}\right)$, $n_i=\left(\frac{\lambda_6m_i{}^2-\lambda_7}{\lambda_3m_i{}^4+\lambda_4m_i{}^2+\lambda_5}\right)$ $(i=1,2,3)$, $\lambda_1=a_5a_{13}-a_9a_{10}$,

$\lambda_2=a_5a_{12}+a_7a_{10}+a_8a_{10}$, $\lambda_3=-a_{13}-a_9a_{11}$, $\lambda_4=a_6a_{13}-a_{12}+a_7a_{11}+a_8a_{11}$, $\lambda_5=a_6a_{12}$,

$\lambda_6=a_{10}-a_5a_{11}$, $\lambda_7=a_6a_8$, $a_1=s^2+\xi^2$, $a_2=i\xi\xi_2$, $a_3=i\xi+i\xi^3\alpha_1$, $a_4=i\xi\xi_3$, $a_5=i\xi\xi_4$,

$a_6=s^2b_6+\xi^2b_3$, $a_7=b_5b_6$, $a_8=\alpha_1\xi^2b_5b_6$, $a_9=\alpha_3b_5b_6$, $a_{10}=-i\xi\xi^2b_8$, $a_{11}=s^2b_9$,

$a_{12}=\xi^5+s^2b_{10}+s^2\xi^2\alpha_1b_{10}$, $a_{13}=b_7+\alpha_3s^2b_{10}$.

4. Boundary Conditions

We consider a Mechanical (Normal or Tangential) force and Thermal sources be acting on the surface of the half spaces. The boundary conditions on the surface $x_3=0$ are

$$t_{33}=-P_1P(x,t),\; t_{31}=-P_2P(x,t),\; \frac{\partial\phi}{\partial x_3}=P_3P(x,t), \tag{17}$$

where $t_{11}=\frac{\partial u_1}{\partial x_1}+b_9\frac{\partial u_3}{\partial x_3}-\phi$, $t_{33}=b_9\frac{\partial u_1}{\partial x_1}+b_3\frac{\partial u_3}{\partial x_3}-b_4\phi$, $t_{31}=b_1\frac{\partial u_3}{\partial x_1}+b_1\frac{\partial u_3}{\partial x_3}$, and P_1,P_2 are the magnitudes of the forces, and P_3 is the constant temperature applied on the boundary. Using the dimensionless quantities given by (8) and considering $P_1'=P_1/aT_0$, $P_2'=P_2/aT_0$, $P_3'=LP_3/T_0$, yield the boundary conditions in non-dimensional form (after suppressing the primes) and then applying the Laplace and Fourier transforms defined by (12) on the resulting expressions, we obtain,

$$\hat{\tilde{t}}_{33} = -P_1 \hat{\tilde{P}}(\xi, s),\ \hat{\tilde{t}}_{31} = -P_2 \hat{\tilde{P}}(\xi, s),\ \frac{\partial \hat{\tilde{\phi}}}{\partial x_3} = P_3 \hat{\tilde{P}}(\xi, s). \tag{18}$$

Substituting the values of $\hat{\tilde{u}}_1$, $\hat{\tilde{u}}_3$, $\hat{\tilde{\phi}}$ from equation (15) in the boundary conditions (18). After simplification, we obtain the components of displacement, conductive temperature and stress components in the transformed form as

$$\hat{\tilde{u}}_1 = \frac{\hat{\tilde{P}}(\xi, s)}{\Delta}\{P_1 D_{71} + P_2 D_{72} + P_3 D_{73}\}, \tag{19}$$

$$\hat{\tilde{u}}_3 = \frac{\hat{\tilde{P}}(\xi, s)}{\Delta}\{P_1 D_{81} + P_2 D_{82} + P_3 D_{83}\}, \tag{20}$$

$$\hat{\tilde{\phi}} = \frac{\hat{\tilde{P}}(\xi, s)}{\Delta}\{P_1 D_{91} + P_2 D_{92} + P_3 D_{93}\}, \tag{21}$$

$$\hat{\tilde{t}}_{33} = \frac{\hat{\tilde{P}}(\xi, s)}{\Delta}\{P_1 E_{11} + P_2 E_{12} + P_3 E_{13}\}, \tag{22}$$

$$\hat{\tilde{t}}_{31} = \frac{\hat{\tilde{P}}(\xi, s)}{\Delta}\{P_1 E_{21} + P_2 E_{22} + P_3 E_{23}\}, \tag{23}$$

where $\Delta = -i\xi b_1 b_{11}(M_{11} + M_{12} + M_{13}) + \xi^2 b_1 b_{11}(M_{21} + M_{22} + M_{23}) + b_1 b_{12}(M_{31} + M_{32} + M_{33})$
$-i\xi\xi_1 b_{12}(M_{41} + M_{42} + M_{43}) + b_1 b_5 b_{13}(M_{51} + M_{52} + M_{53}) - i\xi\xi_1 b_5 b_{13}(M_{61} + M_{62} + M_{63})$,
$D_{7i} = M_{7i} e^{-m_1 x_3} + M_{8i} e^{-m_2 x_3} + M_{9i} e^{-m_3 x_3}$, $D_{8i} = p_1 M_{7i} e^{-m_1 x_3} + p_2 M_{8i} e^{-m_2 x_3} + p_3 M_{9i} e^{-m_3 x_3}$,
$D_{9i} = n_1 M_{7i} e^{-m_1 x_3} + n_2 M_{8i} e^{-m_2 x_3} + n_3 M_{9i} e^{-m_3 x_3}$, $E_{1i} = Q_{11} M_{7i} e^{-m_1 x_3} + Q_{12} M_{8i} e^{-m_2 x_3} + Q_{13} M_{9i} e^{-m_3 x_3}$,
$E_{2i} = Q_{21} M_{7i} e^{-m_1 x_3} + Q_{22} M_{8i} e^{-m_2 x_3} + Q_{23} M_{9i} e^{-m_3 x_3}$, $Q_{1i} = -i\xi b_{11} - m_i p_i b_{12} - n_i b_5 b_{13}$,
$Q_{2i} = -n_i b_1 (i\xi + m_i)$, $i = 1, 2, 3$, $M_{11} = m_1 m_2 (n_2 - n_1)$, $M_{13} = m_1 m_3 (n_1 - n_3)$, $M_{21} = m_1 n_1 (p_2 - p_3)$,
$M_{22} = m_2 n_2 (p_3 - p_1)$, $M_{23} = m_3 n_3 (p_1 - p_2)$, $M_{31} = m_1 m_2 m_3 p_1 (n_2 - n_3)$,
$M_{32} = m_1 m_2 m_3 p_2 (n_3 - n_1)$, $M_{33} = m_1 m_2 m_3 p_3 (n_1 - n_2)$, $M_{41} = p_1 p_2 m_3 n_3 (m_1 - m_2)$,
$M_{42} = p_1 p_3 m_2 n_2 (m_3 - m_1)$, $M_{43} = p_2 p_3 m_1 n_1 (m_2 - m_3)$, $M_{51} = m_2 n_1 n_3 (m_1 - m_3)$,
$M_{52} = m_1 n_2 n_3 (m_3 - m_2)$, $M_{53} = m_3 n_1 n_2 (m_1 - m_2)$, $M_{61} = n_1 n_3 p_2 (m_3 - m_1)$,
$M_{62} = n_2 n_3 p_1 (m_2 - m_3)$, $M_{63} = n_1 n_2 p_3 (m_1 - m_2)$, $M_{71} = b_1 m_2 m_3 (n_2 - n_3) - i\xi b_1 (p_2 m_3 n_3 - p_3 m_2 n_2)$,
$M_{72} = -i\xi b_{11}(m_2 n_2 - m_3 n_3) + b_{12} m_2 m_3 (p_3 n_2 - p_2 n_3) + b_5 b_{13} n_2 n_3 (m_3 - m_2)$,
$M_{73} = -i\xi b_1 b_{11}(m_2 - m_3) + b_1 b_{12} m_2 m_3 (p_2 - p_3) + b_1 b_5 b_{13}(m_3 n_2 - m_2 n_3) - \xi^2 b_1 b_{11}(p_3 - p_2)$
$-i\xi b_1 b_{12} p_2 p_3 (m_3 - m_2) - i\xi b_1 b_5 b_{13}(p_2 n_3 - p_3 n_2)$, $M_{81} = b_1 m_1 m_3 (n_3 - n_1) - i\xi b_1 (p_3 m_1 n_1 - p_1 m_3 n_3)$,
$M_{82} = -i\xi b_{11}(m_3 n_3 - m_1 n_1) + b_{12} m_1 m_3 (p_3 n_1 - p_1 n_3) + b_5 b_{13} n_1 n_3 (m_1 - m_3)$,
$M_{83} = -i\xi b_1 b(m_3 - m_1) + b_1 b_{12} m_1 m_3 (p_3 - p_1) + b_1 b_5 b_{13}(m_1 n_3 - m_3 n_1) - \xi^2 b_1 b_{11}(p_1 - p_3)$
$-i\xi b_1 b_{12} p_1 p_3 (m_1 - m_3) - i\xi b_1 b_5 b_{13}(p_1 n_3 - p_3 n_1)$, $M_{91} = b_1 m_1 m_2 (n_1 - n_2) - i\xi\xi_1 (p_1 m_2 n_2 - p_2 m_1 n_1)$,
$M_{92} = -i\xi\xi_{11}(m_1 n_1 - m_2 n_2) + b_{12} m_1 m_2 (p_1 n_2 - p_2 n_1) + b_5 b_{13} n_1 n_2 (m_1 - m_2)$,
$M_{93} = -i\xi b_1 b_{11}(m_1 - m_2) + b_1 b_{12} m_1 m_2 (p_1 - p_2) + b_1 b_5 b_{13}(m_2 n_1 - m_1 n_2) - \xi^2 b_1 b_{11}(p_2 - p_1)$
$-i\xi b_1 b_{12} p_1 p_2 (m_2 - m_1) - i\xi b_1 b_5 b_{13}(p_1 n_2 - p_2 n_1)$.

5. Applications

Concentrated force or source

$$P(x,t)=\delta(x)\delta(t),\ \hat{\tilde{P}}(\xi,s)=1. \tag{24}$$

(i) Normal force P_3=0=P_2, (ii) Tangential force P_3=0=P_1, (iii) Thermal source P_1=0=P_2.
Making use of the values of (24) in the equations (19)-(23) yield the components of displacement, conductive temperature and stress components respectively.

6. Particular Cases

6.1. If a=0 then we obtain corresponding expression for transversely isotropic thermoelastic medium without energy dissipation.

6.2. If $C_{11}=C_{33}=\lambda+2\mu$, $C_{31}=C_{13}=\lambda$ **,** $C_{44}=\mu$, we obtain the corresponding expression for isotropic thermoelastic without energy dissipation and with two temperature.

7. Numerical Discussion

In order to illustrating theoretical results obtained in proceeding section, we now present some numerical results. The physical data for a single crystal of copper material are given below:

C_{11} = 18.78×10^{10} kg·m^{-1}·s^2, C_{13} = 8.0×10^{10} kg·m^{-1}·s^2, C_{31} = 8.2×10^{10} kg·m^{-1}·s^2,
C_{33} = 18.2×10^{10} kg·m^{-1}·s^2, C_{44} = 5.06×10^{10} kg·m^{-1}·s^2, T_0 = 0.293×10^3 K,
C_E = 0.6331×10^3 J·kg^{-1}·K^{-1}, α_1 = 2.98×10^{-5} K^{-1}, α_3 = 2.4×10^{-5} K^{-1}, $a=0.0104$,
K_1 = 0.433×10^3 W·m^{-1}·K^{-1}, K_3 = 0.450×10^3 W·m^{-1}·K^{-1}, ρ = 8.954×10^3 kg m^{-3}.

The variations of different components for transversely isotropic thermoelastic solid with two temperature (ATT) and without two temperature parameter (AWTT), for isotropic thermoelastic solid with two temperature (ITT) and without two temperature (IWTT) are shown graphically in Figs. (1) - (15) for the range $0\le x\le 10$. The solid line corresponds for ATT, small dashed line corresponds for AWTT, solid line with centre symbol 'Triangle' represents ITT and small dashed line with centre symbol 'Diamond' indicates IWTT.

7.1. Concentrated Normal Force. It is evident from the Fig. 1 that trends of u_1 for anisotropic and isotropic theory are similar in nature for both values of a i.e. $a=0.0$ and $a=0.0104$ with significant difference in their magnitude.

Figure 2 depicts the variation of u_3, it is noticed that near the loading surface, values of u_3 increase for transversely and isotropic case, magnitude of values is greater for $a=0.0104$ as compared to those obtained at $a=0.0$. As x increases values of u_3 follows an oscillating behavior and approaches to the boundary surface.

It is evident from Fig. 3, that values of ϕ for transversely and isotropic case decrease sharply in the range $0\le x\le 2$, magnitude being greater at $a=0.0104$ as compared to those obtained at $a=0.0$, with further increase in x, values of ϕ at $a=0.0$ and $a=0.0104$ decrease in oscillating manner.

It is noticed from Fig. 4 that variations of t_{31} at $a=0.0$ increase abruptly in the range $0\le x\le 2$ and then values at $a=0.0$ oscillates about zero value, whereas the values of t_{31} for ATT and ITT at $a=0.0104$ increase steadily in the initial range and as x increases values of t_{31} shows small variations about origin.

It is evident from Fig. 5 that trends of t_{33} at $a=0.0104$ is opposite in comparison to those obtained at $a=0.0$ with significant difference in their magnitude, which reveals the impact of two temperature parameter for both transversely and isotropic case.

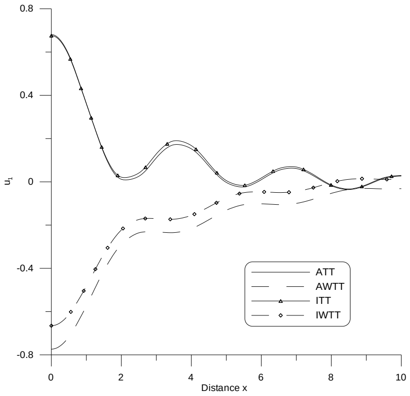

Fig. 1. Variation of u_1 with distance x.
(Concentrated normal force).

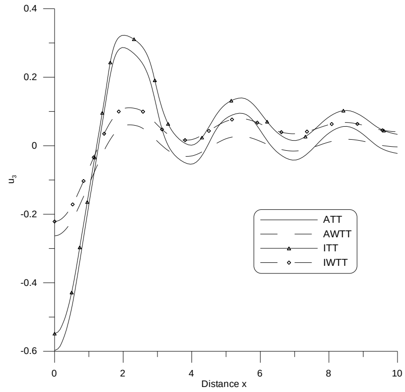

Fig. 2. Variation of u_3 with distance x.
(Concentrated normal force).

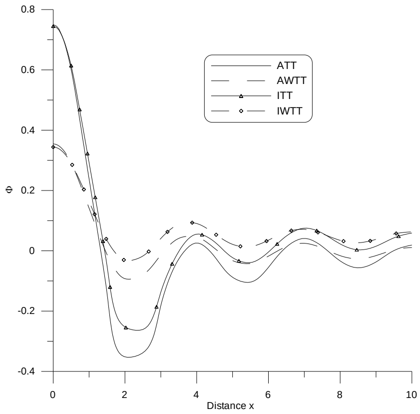

Fig. 3. Variation of ϕ with distance x.
(Concentrated normal force).

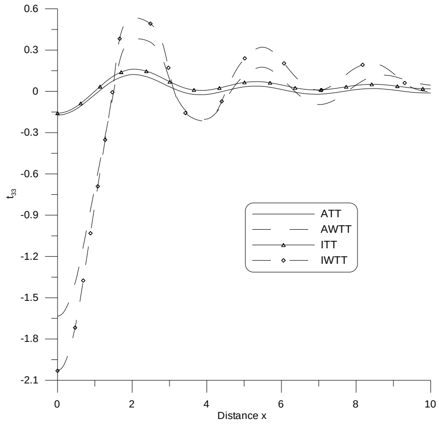

Fig. 4. Variation of t_{31} with distance x.
(Concentrated normal force).

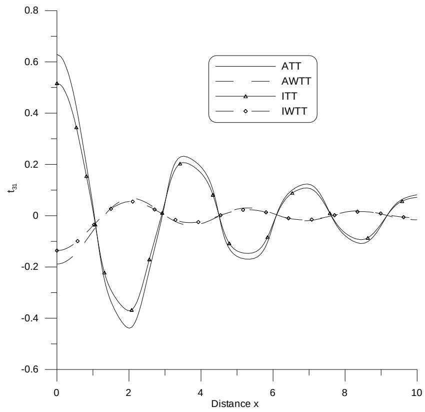

Fig. 5. Variation of t_{33} with distance.
(Concentrated normal force).

7.2. Concentrated Tangential Force. It is noticed from Fig. 6, which is plot of u_1 that values of ATT and ITT decrease abruptly in the interval $0 \le x \le 2$, whereas values of AWTT and IWTT increase, with further increase in x, values of ATT and ITT at $a = 0.0104$ show oscillating behaviour with decreasing magnitude, whereas values of AWTT and IWTT show small variations at origin, which is accounted as effect of two temperature parameter.

It is evident from Fig. 7 that trend of u_3 for ATT and ITT are similar in nature as observed for u_1 (in Fig. 6) with significant difference in their magnitude. Also, values of AWTT and IWTT of u_3 at $a = 0.0$ decrease near loading surface and as x increases, values of u_3 oscillate about origin.

It is noticed from Fig. 8 that values of ϕ for both transversely and isotropic case at $a = 0.0$ and $a = 0.0104$ increase in the range $0 \le x \le 2$, $3.5 \le x \le 5$, $7 \le x \le 8.5$ and decrease in remaining range with significant difference in the magnitude.

Figure 9 shows the variation of t_{31} with *x*. It is noticed that trends of t_{31} for all the considered cases are similar in nature as observed for u_3 with significant difference in their magnitude.

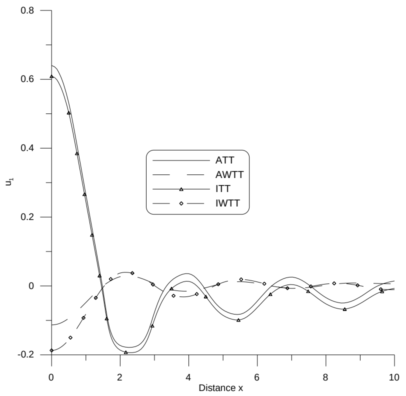

Fig. 6. Variation of u_1 with distance.
(Concentrated tangential force).

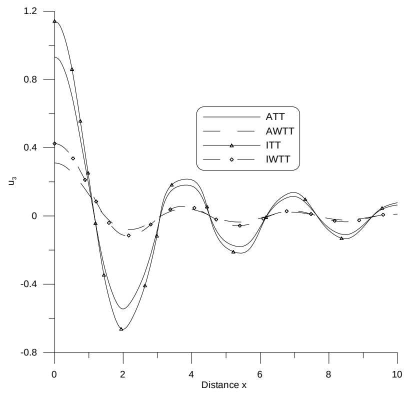

Fig. 7. Variation of u_3 with distance x.
(Concentrated tangential force).

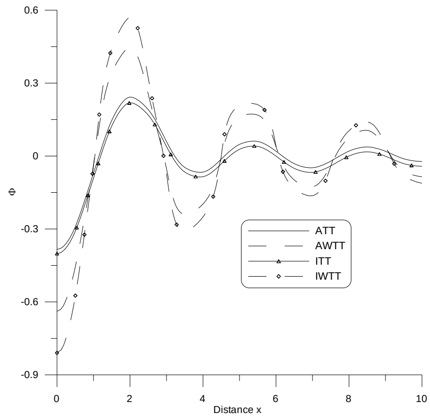

Fig. 8. Variation of ϕ with distance x.
(Concentrated tangential force).

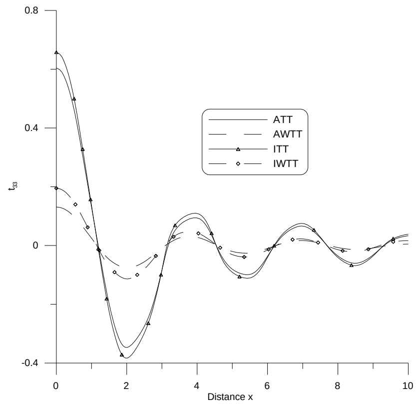

Fig. 9. Variation of t_{31} with distance x.
(Concentrated tangential force).

It is evident from the Fig. 10 that the values of t_{33} for AWTT and IWTT increase prominently in the range $0 \le x \le 2$ and as x increases values of t_{33} at $a = 0.0$ follows an oscillatory pattern whereas at $a = 0.0104$ trends of ATT and ITT are opposite in nature as noticed for AWTT and IWTT in the entire range.

7.3. Thermal Point Source. Figure 11 depicts the variation of u_1, it is noticed that values of u_1 for AWTT and IWTT at $a = 0.0$ decrease in entire range whereas values of u_1 at $a = 0.0104$ increase and decrease alternately about zero value, which show significant impact of two temperature parameters.

It is observed from Fig. 12 that trends of u_3 at $a = 0.0$ and $a = 0.0104$ for both transversely and isotropic are similar in nature in the entire range, magnitude of values for u_3 at $a = 0.0$ are greater as compared to those obtained at $a = 0.0104$, which is accounted as presence of two temperature parameter.

It is evident from Fig. 13, which is plot of ϕ for transversely case increase with greater magnitude as compared to isotropic case for values of two temperature parameter and as x

increases values of ϕ for ATT and AWTT increase and decrease alternately, whereas values of ϕ for ITT and IWTT shows small variations about origin.

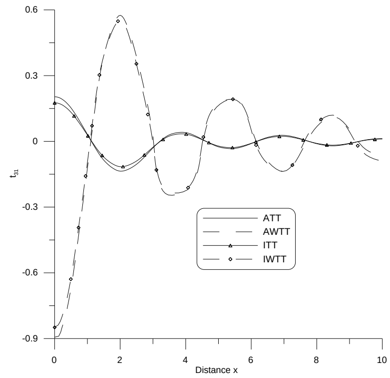

Fig. 10. Variation of t_{33} with distance x.
(Concentrated tangential force).

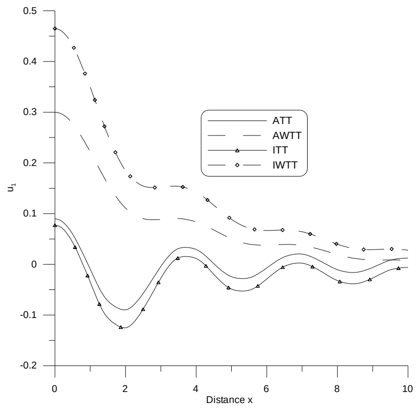

Fig. 11. Variation of u_1 with distance.
(Thermal point source).

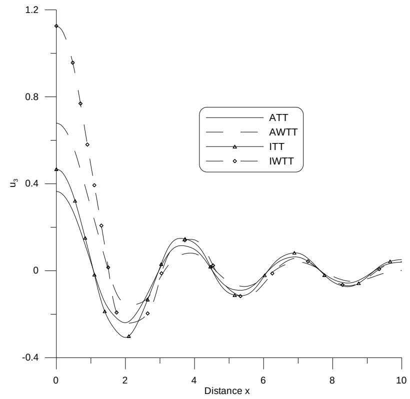

Fig. 12. Variation of u_3 with distance x.
(Thermal point source).

It is noticed from Fig. 14 that trends of t_{31} for all the cases considered are similar in nature, values of t_{31} at $a = 0.0$ are greater as compared to those obtained at $a = 0.0104$ for both transversely and isotropic case.

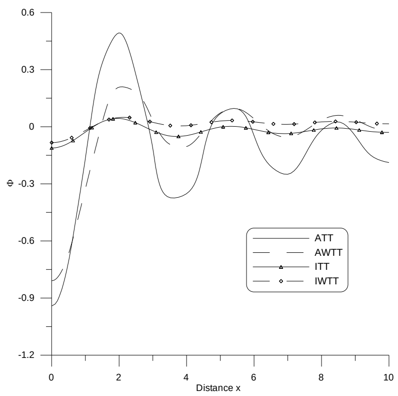

Fig. 13. Variation of ϕ with distance x.
(Thermal point source).

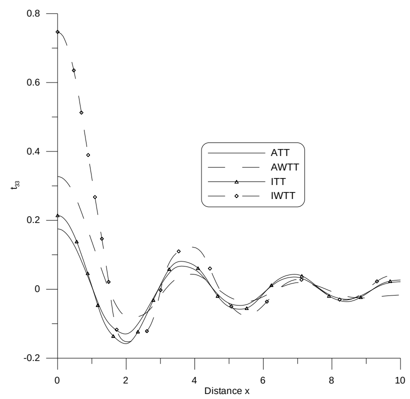

Fig. 14. Variation of t_{31} with distance x.
(Thermal point source).

It is evident from Fig. 15, which is plot of t_{33} that values of t_{33} for both transversely and isotropic at $a = 0.0$ and $a = 0.0104$ decrease in the interval $0 \le x \le 2$, $3.5 \le x \le 5$, and $7 \le x \le 8.5$ and increase in remaining interval.

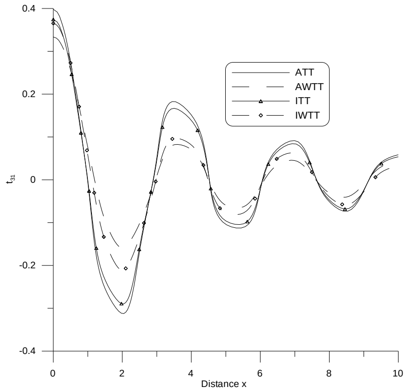

Fig. 15. Variation of t_{33} with distance x.
(Thermal point source).

8. Conclusion

It is observed from the above discussion that effect of two temperature is significant on the components of displacement, stresses and conductive temperature. It is observed from the Figs. (1)-(15) that trends are almost similar in nature with significant difference in their magnitude. As disturbance travel through the medium, they are characterized by sudden changes, which results in an inconsistent/non uniform pattern.

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