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ANALYSIS OF THE OPENING PROBLEM OF THE COMPOSITE MATERIALS STRUCTURES WITH BOUNDARY INTEGRAL EQUATION METHOD

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Abstract. Almost all the structures in the fields of practical engineering are applied with various kinds of holes, for example, there are manholes in the plane wings. When the structures are subject to loads, the stress concentration will appear around the holes, which will reduce both the static strength and the fatigue strength. In this paper, in view of the orthotropic plate with manholes, the calculation model was established based on complex function, and the analytic solution of hole-edge stress of orthotropic plate with complex hole shape was figured out. By virtue of mapping function, the parametric equation which accurately describes hole boundary was obtained. As a result, based on the established model, under different fiber array, the hole-edge stress of composite material plate with wing manholes was calculated, and stress distributions around the holes under two kinds of fiber array were compared. Thus, this study provides theoretical reference for design of flywheel.

1. Introduction

Composite materials have played a more and more important role in the fields of transportation since the 1990s, which have been successfully introduced into areas of the aerospace, automobiles, train, ship-building and so on. Such wide application promotes the researches on the composite materials structures. At the same time, it also poses some problems to be solved, for example, the stress concentration on the hole edge. This problem has an important impact on structural safety. It is well known that the areas near the holes are the weakest areas of the structures.

Compared with the studies on the hole-edge stress distribution of orthotropic plate with an elliptical holes or circular holes, there are limited studies regarding orthotropic plate with other type holes. The methods of approximate solution, semi-analytic solution and finite element method are usually used to analyze the stress and strength of the composite plate with regular holes, but there are some shortcomings in these three methods. First, it is difficult to describe accurately the boundary conditions. Second, the semi-analytic solution is limited to some special types of holes and cannot solve the problems on boundary conditions of the

The boundary conditions of the orthotropic plane with holes are:

$$\begin{aligned}\varphi(z_1) &= A\ln z_1 + (B_1 + iC_1)z_1 + \varphi_0(z_1),\\ \psi(z_2) &= B\ln z_2 + (B_2 + iC_2)z_2 + \psi_0(z_2),\end{aligned} \qquad (6)$$

where A, B, B_1, B_2, C_1, C_2 are determined by the specific conditions.
Then it is the key issue that the mapping function of the hole edge should be found. According to the Riemann theory, such kind of function exists, and is unique to the different types of holes. The searching process is omitted here since it is very complicated, and the results are directly stated below.

3. Calculation of the stress distribution on the wing manhole edge

There is an anisotropic plate with the target type of holes under the force distributing at the plate edge and functioning at the middle surface. Supposing the size of hole is much smaller than that of plate, moreover, the hole is not close to plate edge. Then the plate can be considered as infinite and thus the influence of the edge can be ignored, which makes the problem simpler.

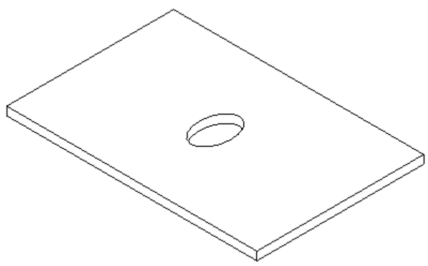

Fig. 1. An orthotropic plate with a wing manhole.

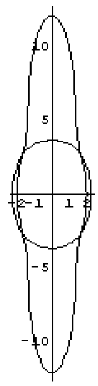

Fig. 2. Hole-edge Stress distribution of an orthotropic plate with a wring manhole under tensile force.

According to the complex variable function theory, by means of Schwarz-Christoffel mapping, the mapping function of wing manhole edge can be figured out. The function which maps the hole boundary in the plane z to a unite circle in the plane ζ is:

$$z = \omega(\zeta) = -\frac{1}{\zeta} - 0.2411\zeta - 0.02\zeta^3 + 0.0045\zeta^5.$$

Then by using the two transforming equations $z_1=x_1+iy_1$, $z_2=x_2+iy_2$, the position of the point in the plane z_1, z_2 can be figured out, which is correspondent to the one in the plane z:

$$z_1 = \omega_1(\zeta) = (1.138 + i1.861s_1)\zeta + (1.138 - i1.861s_1)\frac{1}{\zeta} +$$
$$(-0.03 + i0.03s_1)\zeta^3 + (-0.03 - i0.03s_1)\frac{1}{\zeta^3} + (0.00681 + i0.00681s_1)\zeta^5 + (0.00681 - i0.00681s_1)\frac{1}{\zeta^5},$$
$$z_2 = \omega_2(\zeta) = (1.138 + i1.861s_2)\zeta + (1.138 - i1.861s_2)\frac{1}{\zeta} +$$
$$(-0.03 + i0.03s_2)\zeta^3 + (-0.03 - i0.03s_2)\frac{1}{\zeta^3} + (0.00681 + i0.00681s_2)\zeta^5 + (0.00681 - i0.00681s_2)\frac{1}{\zeta^5}.$$

Now that z_1 and z_2 have been obtained. Through Cauchy integral of the equation (6), $\varphi(z_1)$, $\psi(z_2)$

can be determined. Combining the results with (5), the stress distribution on the hole edge can also be solved.

Esp. 1. The description of the problems is like this:

An orthotropic plate with a wing manhole has the neutral plane as its symmetrical plane, with its fiber arrange parallel to the long axes, and is applied with tensile force in the direction of fiber arrange. The point of force application is far away from the hole edge. The given engineering constants are as follows: E_{11}=135 GPa, E_{22}=9.5 GPa, G_{12}=6.0 GPa, v_{12}=0.3.

Using the methods above, the exact analytical solution of the stress on the hole edge can be obtained. All calculations and stress distribution figures of this paper have been completed with the software Mathematica, which has a powerful ability to deal with the complex variable functions. The stress distribution is represented in Fig. 2 according to the analytical expressions.

As shown in the Fig. 2, the maximum tensile stress appears at the positions of v=90° and 270°, and the maximum compressive stress at v=0° and 180°. The stress concentration factor is 8.43. The stress equals zero at the positions of v=46°, 134°, 226°, and 314°.

Esp. 2. The description is like this:

All is the same as the Esp. 1 but the fibre arrange direction. In this situation, the fibre arrange direction has an angle of 45 degrees with the horizontal axis. With the repeated process in Esp. 1, the graphics of the stress distribution are obtained as follows:

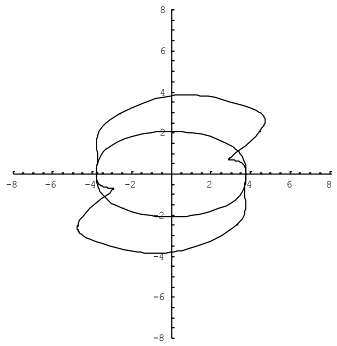

Fig. 3. Hole-edge stress distribution in *x* direction of an orthotropic plate (The angle between machine direction and X-axis is 45) with wing manhole under tensile force.

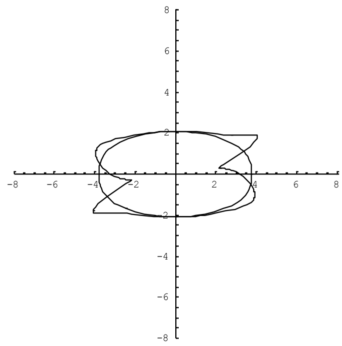

Fig. 4. Hole-edge stress distribution in *y* direction of an orthotropic plate (The angle between machine direction and X-axis is 45) with wing manhole under tensile force.

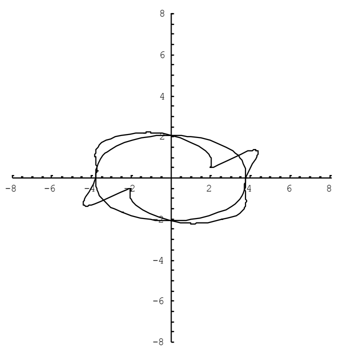

Fig. 5. Hole-edge stress distribution in *xy* direction of an orthotropic plate (The angle between machine direction and X-axis is 45) with wing manhole under tensile force.

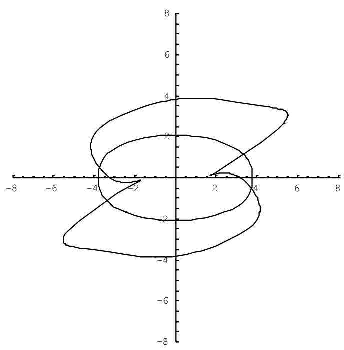

Fig. 6. Circumferential stress distribution around the hole of an orthotropic plate (The angle between machine direction and X-axis is 45) with wing manhole under tensile force.

As shown in the Fig. 3 to Fig. 6, the stress in the direction of x axis plays an important role in determining the circular stress distribution of the hole edge. The stress concentration factor under the load non-parallel to the direction of the fibre is 2.995, relatively which is much lower than that parallel to the fibre.

4. Conclusion

The calculation model was established and the analytic solution of the stress on the hole edge with irregular shapes of holes in the orthotropic materials structures was figured out. By using the mapping function, the parametric equations of the hole edge was obtianed. On the basis of the established models, by taking two different cases for example, a lot of valuable conclusions were drawn for the practical engineering.

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References

[1] B. Grüber, W. Hufenbach, L. Kroll // *Composites Science and Technology* **67(7)** (2007) 1439.
[2] V.G. Ukadgaonker, D.K.N. Rao // *Composite Structures* **49(1)** (2000) 41.
[3] H. Engels, D. Zakharov, W. Becker // *Archive of Applied Mechanics* **9(71)** (2001) 601.
[4] W.J. Yen, C. Hwu // *International Journal of Solids and Structures* **27(13)** (1991) 1705.
[5] S.Ş. Ömer // *Composites Science and Technology* **65(11)** (2005) 1780.
[6] M.N.M. Allam, A.M. Zenkour, H.F. El-Mekawy // *Composite Structures* **79(1)** (2007) 1.
[7] W. Hufenbach, M. Schäfer, A.S. Herrmann // *Ingenieur-Archiv* **60(8)** (1990) 507.
[8] P.S. Theocaris // *Engineering Fracture Mechanics* **38(1)** (1991) 37.
[9] C. Li, *Calculation of the Stress along Hole Boundary in the Composite Plate with Any Type of Holes* (The Association of German Engineers Publishing House, 2003). (In German).