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STRAIN FIELD AND ENERGY DISSIPATION AT A CRACK TIP UNDER AXISYMMETRIC STRAIN CONDITIONS

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Abstract. Plastic flow at the crack tip is considered using the ideal rigid-plastic body theory under axisymmetric strain conditions. The material at the crack tip is treated as a body consisting of an elastic outer region and a rigid-plastic inner region. It is shown that distributions of the energy dissipation and strains at the crack tip are defined by conformable expressions under plane strain conditions at the limiting case.

1. Introduction

A problem of determination of the strain field and energy dissipation in the neighborhood of a crack tip is of a practical importance to strength analysis and fracture strength of constructional elements. The solution of this problem is given for spatial case near the discontinuity surface of displacement velocity in [1]. The plastic flow at the crack tip under plane strain conditions is given in papers [2, 3]. The generalization of these results under axisymmetric strain condition is suggested bellow.

2. Steady-state crack propagation in an elastic-plastic material under axisymmetric strain conditions

The material at the crack tip is composite (Fig. 1) [4, 5]: the outer part (I) of the area surrounding the crack tip is elastic; the inner part (II) with boundary *ACDBEFGA* is rigid-plastic, where the strains have finite values and are determined by finite strain tensors.

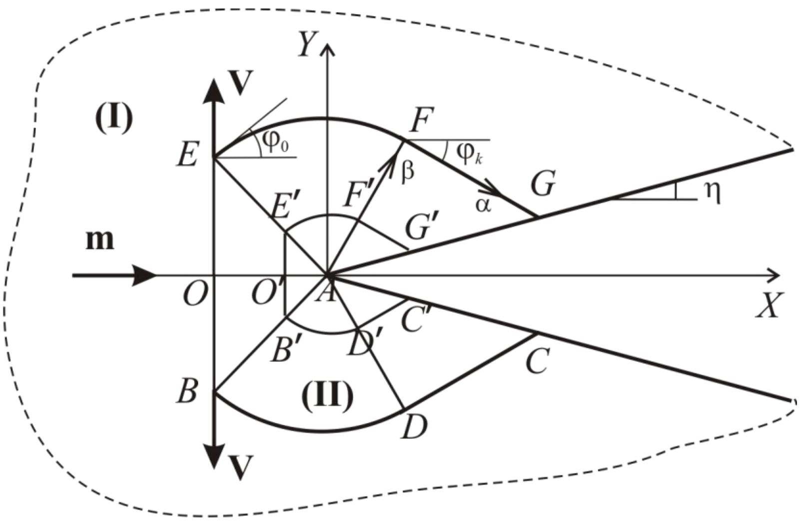

Fig. 1. Plastic flow at the crack tip: (I) – elastic region, (II) – rigid-plastic region.

The following assumptions are introduced:

– the free surface of the crack at the rigid-plastic region is straightforward;

– the plastic flow is steady-state; the size of the rigid-plastic region (II) does not change with time and sufficiently small, it is defined by line length *OA*; the coordinate origin is at the crack tip (point *A*) that is located on radius *r* distance from symmetry axis;

– the material of the outer region moves to the rigid-plastic region (II) along axis *X* with velocity $m > 0$ (*m* is crack propagation velocity).

The deformation of the material particles at the crack tip is related with deformation along its motion trajectory which consists of two parts (fig. 2): the straight segment L_1 in uniform strain field (in the triangle *EAB*) and the curvilinear segment L_2 (in the slip-line fan *EAF*). Further, the particle deformation along line L_1 is disregarded [3].

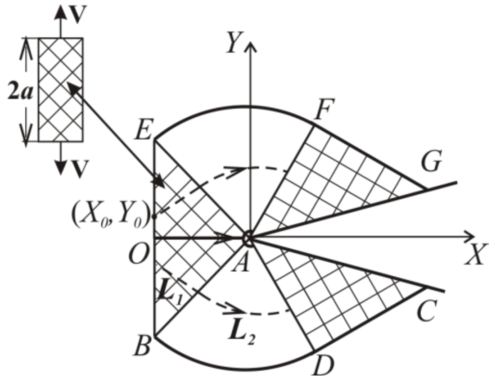

Fig. 2. Particle path and stress-strain state at the crack tip: the hatched areas are the regions of the uniform strain field.

3. The communication of the stress field and the displacement velocity under the plane strain and axisymmetric strain conditions at the slip-line fun center

The stress and the displacement velocity fields with Tresca-Saint Venant yield condition under axisymmetric strain conditions is determined by the following relationships along the slip-lines [6]:

$$\begin{cases} d\left(\dfrac{\sigma}{k} - 2\varphi\right) - (\sin\varphi + \cos\varphi)\dfrac{dS_\alpha}{r} = 0 \quad \text{along } \alpha\text{-line}, \\ d\left(\dfrac{\sigma}{k} + 2\varphi\right) + (\sin\varphi + \cos\varphi)\dfrac{dS_\beta}{r} = 0 \quad \text{along } \beta\text{-line}, \end{cases}$$
$$\begin{cases} V_r\cos\varphi + V_z\sin\varphi + \dfrac{V_r}{2r}dS_\alpha = 0 \quad \text{along } \alpha\text{-line}, \\ V_r\sin\varphi - V_z\cos\varphi - \dfrac{V_r}{2r}dS_\beta = 0 \quad \text{along } \beta\text{-line}, \end{cases} \tag{1}$$

where dS_α, dS_β are elements of *α*, *β*-lines, *φ* is the slope of the slip line α to the *r* axis, $\sigma = \frac{1}{2}(\sigma_1 + \sigma_2)$, *k* is the yield stress.

It is clear that the integrals $\int\limits_{L_2}(\sin\varphi + \cos\varphi)\dfrac{dS_\alpha}{r}$, $\int\limits_{L_2}(\sin\varphi + \cos\varphi)\dfrac{dS_\beta}{r}$, $\int\limits_{L_2}\dfrac{V_r}{2r}dS_\alpha$, $\int\limits_{L_2}\dfrac{V_z}{2r}dS_\beta$ tend to zero under $Y_0 \to 0$, since the arc length L_2 tends to zero also. Then the equations are transformed to the relations of Hencky and Geiringer under plane strain condition:

$$\begin{cases} \dfrac{\sigma}{2k} - \varphi = \xi & \text{along } \alpha\text{-line}, \\ \dfrac{\sigma}{2k} + \varphi = \psi & \text{along } \beta\text{-line}, \end{cases} \qquad \begin{cases} du - v d\varphi = 0 & \text{along } \alpha\text{-line}, \\ dv + u d\varphi = 0 & \text{along } \beta\text{-line}; \end{cases} \tag{2}$$

Therefore, the relation for the energy dissipation accumulation along the limit path at the slip-line fun center under axisymmetric strain condition is obtained from equations (2) as

$$\frac{W}{2k} = -\int_{\varphi_0}^{\varphi_k} \left(\frac{\partial v}{\partial \varphi} + u \right) \frac{d\varphi}{u};$$

and the strains distribution:

$$e = \frac{-\left(1 - \exp(2b)\right)^2}{4\exp(2b)}, \quad g = \frac{-\left(1 - \exp(4b)\right)}{4\exp(2b)}, \quad b = tg\left(\frac{\eta}{2}\right),$$

$$E_1 = e + g, \quad E_2 = e - g, \quad e_0 = g_0 = 0,$$

where η is half crack angle (fig. 1). These results are similar to the relations under plane strain conditions [2, 3].

4. Conclusions

Distributions of strain field and energy dissipation at the crack tip under axisymmetric strain condition for the limit state are coinciding with similar fields under plane strain condition.

References

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