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ON PLASTIC FLOW OF SOLIDS FOR STRESS STATES CORRESPONDING TO AN EDGE OF THE COULOMB-TRESCA PRISM

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Abstract. Plastic flow states corresponding to an edge of the Coulomb-Tresca prism in the Haigh-Westergaard three-dimensional space of principal stresses are considered. Constitutive equations are formulated by the generalized associated plastic flow rule due to Koiter. These equations impose the minimal kinematical constraints on plastic strains increments and as it is elucidated are equivalent to three-dimensional equations of the mathematical plasticity proposed by Ishlinskii in 1946. It is then shown that obtained constitutive equations can be formulated as a tensor permutability equation for the stress tensor and the plastic strains tensor increment. A new explicit form of the plastic flow rule for stress states corresponding to an edge of the Coulomb-Tresca prism is obtained and discussed.

1. Introduction

The Coulomb-Tresca yield criterion holds a unique position in mathematical theory of plasticity as the both two-dimensional and then the first three-dimensional theories of perfect plastic flow of metals have been created by this criterion by Saint-Venant and Levy.

A special position of the Coulomb-Tresca yield criterion and the associated plastic flow rule were fully recognized by D.D. Ivlev, who developed a mathematical theory of plasticity on an edge of the Coulomb-Tresca prism in a number of researches started from 1959. In this connection he relied on an early researches of A.Y. Ishlinskii, who proposed the original theory of plasticity in the most general three-dimensional formulation and took into account the necessity of satisfying the two yield conditions as it takes place in the case of “full plasticity”, the condition of incompressibility of plastic flow, and the original tensor commutativity condition for the stress tensor and the tensor of increment of plastic deformations. However the generalized associated flow rule by Koiter was used in further discussions instead of the tensor commutativity condition. Nevertheless, it can be demonstrated that the tensor commutativity condition follows from the generalized associated flow rule and vice versa. Therefore, the Ishlinskii theory of plasticity is fully conforms to modern viewpoints on plastic flow of metals. It should be noted that analytical relations for stresses and increments of displacements in three-dimensional formulations of the Ishlinskii-Ivlev theory of plasticity form regular systems of partial differential equations of *hyperbolic* analytical type.

In the present paper a three-dimensional constitutive equation for perfect plastic solids is obtained by using the trinomial tensor formula by V.V. Novozhilov. The plastic flow of solid corresponds to an edge of the Coulomb-Tresca prism. It is shown that the constitutive

where

$$|\mathbf{s}| = \frac{1}{\sqrt{6}}\sqrt{(s_1 - s_2)^2 + (s_1 - s_3)^2 + (s_2 - s_3)^2},$$

$$\left|d\boldsymbol{\varepsilon}^P\right| = \frac{1}{\sqrt{6}}\sqrt{(d\varepsilon_1^P - d\varepsilon_2^P)^2 + (d\varepsilon_1^P - d\varepsilon_3^P)^2 + (d\varepsilon_2^P - d\varepsilon_3^P)^2}.$$

For the further discussion the Novozhilov trinomial formula needs to be used [6], which is a nonlinear algebraic equation for tensors $\tilde{\mathbf{s}}$ и $\widetilde{d\boldsymbol{\varepsilon}^P}$ under the condition that the principal axes of $d\boldsymbol{\varepsilon}^P$ are at the same time the principal axes of the $\mathbf{s}$ as it takes place in the case of flow on the edge of Coulomb-Tresca prism:

$$\tilde{\mathbf{s}} = \frac{1}{\cos 3\psi}\left\{\cos(2\psi + \vartheta)\,\widetilde{d\boldsymbol{\varepsilon}^P} + \sqrt{3}\sin(\psi - \vartheta)\left[\widetilde{d\boldsymbol{\varepsilon}^P}\cdot\widetilde{d\boldsymbol{\varepsilon}^P} - \frac{2}{3}\mathbf{I}\right]\right\}. \qquad (14)$$

It is known that trinomial tensor equations of the form (14) cannot be inverted, in other words, the equation for $\widetilde{d\boldsymbol{\varepsilon}^P}$ cannot be obtained. The angle variables ϑ, ψ in (14) are the form angles of stress and flow

$$\vartheta = \operatorname{arctg}\frac{2s_2 - s_1 - s_3}{\sqrt{3}(s_1 - s_3)}, \qquad \psi = \operatorname{arctg}\frac{2d\varepsilon_2^P - d\varepsilon_1^P - d\varepsilon_3^P}{\sqrt{3}(d\varepsilon_1^P - d\varepsilon_3^P)}.$$

It should be noticed that the phase ϑ can be determined as angle of inclination of the vector representing in the Haigh-Westergaard space the deviator $\mathbf{s}$ and lying in the deviatoric plane to the corresponding shear axis. The latter is orthogonal to projection on the deviatoric plane of the second coordinate axis of the Haigh-Westergaard space. The phase ϑ is constant for stress states on the edge of Coulomb-Tresca prism. In particular, if the plastic flow is on the edge

$$\sigma_1 = \sigma_2 = \sigma_3 + 2k,$$

then the phase ϑ equals $\frac{\pi}{6}$. Besides, the following equation is valid: $\tilde{\mathbf{s}} = \frac{\sqrt{3}}{2k}\mathbf{s}$.

One can also obtain

$$\mathbf{s} = \frac{2k}{\sqrt{3}\cos 3\psi}\left\{\cos(2\psi + \vartheta)\frac{\sqrt{2}\,\widetilde{d\boldsymbol{\varepsilon}^P}}{\sqrt{\operatorname{tr}(\widetilde{d\boldsymbol{\varepsilon}^P}\cdot\widetilde{d\boldsymbol{\varepsilon}^P})}} + 2\sqrt{3}\sin(\psi - \vartheta)\left[\frac{\widetilde{d\boldsymbol{\varepsilon}^P}\cdot\widetilde{d\boldsymbol{\varepsilon}^P}}{\sqrt{\operatorname{tr}(\widetilde{d\boldsymbol{\varepsilon}^P}\cdot\widetilde{d\boldsymbol{\varepsilon}^P})}} - \frac{1}{3}\mathbf{I}\right]\right\}$$

and

$$\mathbf{s} = \frac{2k}{\sqrt{3}\cos 3\psi}\left\{\cos(2\psi + \frac{\pi}{6})\frac{\sqrt{2}\,\widetilde{d\boldsymbol{\varepsilon}^P}}{\sqrt{\operatorname{tr}(\widetilde{d\boldsymbol{\varepsilon}^P}\cdot\widetilde{d\boldsymbol{\varepsilon}^P})}} + 2\sqrt{3}\sin(\psi - \frac{\pi}{6})\left[\frac{\widetilde{d\boldsymbol{\varepsilon}^P}\cdot\widetilde{d\boldsymbol{\varepsilon}^P}}{\sqrt{\operatorname{tr}(\widetilde{d\boldsymbol{\varepsilon}^P}\cdot\widetilde{d\boldsymbol{\varepsilon}^P})}} - \frac{1}{3}\mathbf{I}\right]\right\} \qquad (15)$$

The aforementioned constitutive equation (15) in combination with the incompressibility equation

$$\operatorname{tr} d\boldsymbol{\varepsilon}^P = 0$$

are equivalent to the associated flow rule. Consequently, the stress tensor deviator for stress states corresponding to the edge of Coulomb-Tresca prism is related to the incremental tensor of plastic deformations by an essentially nonlinear tensor equation, which fully determines the plastic flow in its fully free modes.

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