THE MODEL OF CUTTING PROBLEM SUBJECT TO A COULOMB-MOHR YIELD CONDITION

Received: June 10, 2015

A.N. Anisimov¹, Yu.G. Egorova^{2*}, A.I. Khromov²

¹Amur State University of Humanities and Pedagogy,

Kirova st. 17-2, Komsomolsk-na-Amure, Khabarovsk Region, 681000, Russian

²Komsomolsk-na-Amure State Technical University,

Lenin pr. 27, Komsomolsk-na-Amure, Khabarovsk Region, 681000, Russian

*e-mail: egorova_jg@mail.ru

Abstract. The rigid-plastic model of cutting problem is analyzed subject to a Coulomb-Mohr yield condition. The fields of stress, of strain and of deformation are investigated. Local continuation of stress field to rigid zones (the blank body and cuttings) is constructed. The existence region of full solution is detected and selection criteria of preferred solution is done. Depending of strain distribution and density changes in cuttings is given.

1. Introduction

The rigid-plastic model of cutting problem subject to Tresca and Mises yield conditions based on the assumption of existence of an isolated slip line (shear plane). In this formulation of the problem, kinematically admissible solutions [1] and [2] are known. The completeness of these solutions is studied in [3]. It is shown that they have significant limitations. A solution minimizing the volume density of energy dissipation in the shear plane with the existence of a statically admissible continuation of the stress field to rigid zones is suggested.

In this paper, the supplement of approach stated in [3] is considered subject to solution of cutting problem with a Coulomb-Mohr yield condition.

2. Problem Definition

We consider the problem of cutting taking into account the compressibility irreversible (Fig. 1). A blank moves from left to right with a velocity V. On the isolated slip line a Coulomb-Mohr yield condition is performed [4]: $(1/4)(\sigma_{11}-\sigma_{22})^2+\sigma_{12}^2=(k+\sin\rho(\sigma_{11}+\sigma_{22})/2)^2$, where σ_{ij} – the components of the stress tensor; k – adhesion coefficient; $\rho=\pi/2-2\varphi$ – the angle of internal friction.

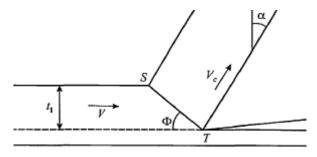


Fig. 1. Chip formation during deformation with single slip line.

3. Cutting Force Detecting

The equilibrium equations in the axes t, n, associated with the slip line ST, have the form $\tau_{nt} \cdot ST = F \cos(\Phi + \lambda - \alpha)$, $\sigma_n \cdot ST = F \sin(\Phi + \lambda - \alpha)$,

where τ_{nt} – shear stress on ST; F – the full power acting from the cutter; Φ – the angle of the shear plane; λ – the friction angle associated with the friction coefficient μ by correlation $\mu = tg\lambda$; α – the front angle of the cutter; σ_n – normal stress on ST.

As far as on ST $\tau_{nt}\cos\rho = k + \sigma_n\sin\rho$, so $F = ST \cdot k/\cos(\Phi + \lambda - \alpha - \rho)$. The cutting force $F_c = kt_1\cos(\lambda - \alpha)/(\sin\Phi\cos(\Phi + \lambda - \alpha - \rho))$ – the horizontal component of $F(t_1$ – the thickness of the layer being cut off).

Velocity Field Detecting. At the left and at the right from ST (signs \ll -» and \ll +» respectively) the relations are performed:

 $V_{t-} = V \cos \Phi$, $V_{n-} = V \sin \Phi$, $V_{t+} = V_c \sin(\alpha - \Phi)$, $V_{n+} = V_c \cos(\alpha - \Phi)$, where V_c – the velocity of cuttings.

As far as on slip line *ST* the value of $[V_n \cos \rho - V_t \sin \rho]$ is uninterrupted, so $V_c = V \sin(\Phi - \rho) / \cos(\alpha - \Phi + \rho)$.

Strain Field Detecting. As a measure of strains the Almansi finite strain tensor $E_{ij} = 1/2(\delta_{ij} - x_{k,i}^0 x_{k,j}^0)$ is accepted $(\delta_{ij} - \text{Kronecker delta}, x_i^0 \text{ if } x_i^0 - \text{Lagrangian and Euler coordinates of particle}).$

The Almansi tensor components during the transition by a particle of material through ST assuming that the material below ST is not deformed are defined as [5]: $E_{ij}^{0+} = \mathcal{S}_{ij} - (W_1 t_i + W_2 n_i) n_j$, $W_1 = [V_t]/(G+V_{n+})$, $W_2 = [V_n]/(G+V_{n+})$, where W_1 and W_2 - respectively the volume densities of energy of shear strain and volume strain per unit of k; $[V_t]$ in $[V_n]$ - respectively the gaps of the tangential and normal components of velocity; G - the normal velocity of line ST moving.

We take for strain characteristics the main values of the Almansi tensor $E_{1,2}$. Then the change of material density after the strain is $\rho_c = \sqrt{(1-2E_1)(1-2E_2)}\rho_c^0$, where ρ_c^0 – starting density.

Constructing of Continuation of the Stress Field to Rigid Zones. According [3], we consider the construction of local continuation of the stress field near *ST* in the blank body and cuttings. The general method of the construction of the continuation was proposed in [6]; the central idea of the method was stated in [7].

We define the localization of free surfaces Σ and Σ' near S (Fig. 2). On areas AS and SB with the angles $\pi/2-\varphi$ and φ by ST the compressive stresses acts:

$$|q| = (1 + \sin \rho) \cdot (\sigma_{11} + \sigma_{22}) / 2 + k, |q'| = (1 - \sin \rho) \cdot (\sigma_{11} + \sigma_{22}) / 2 - k.$$

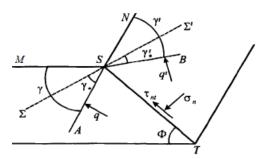


Fig. 2. The construction to rigid zones (sufficient conditions of continuation).

The areas ASM and BSN are subjected to an one-sided pressure q and q'. The minimal angle of the wedge γ_* enduring the stress q is

$$\begin{aligned} & \left| q \right| = k \left(1 - e^{2tg\rho(\pi/2 - \gamma_*)} \left(1 - \sin\rho \right) / \left(1 + \sin\rho \right) \right) / \sin\rho, \gamma_* \ge \pi/2, \\ & \left| q \right| = 2k \left(1 + \cos\left(\gamma_* - 2\theta\right) \right) / \left(1 + \sin^2\rho + 2\sin\rho\cos\left(\gamma_* - 2\theta\right) \right), \gamma_* \le \pi/2, \\ & \text{where } 2\theta = \arcsin\left(\sin\rho \cdot \sin\gamma_*\right) + \pi. \end{aligned}$$

The angles γ_* and γ_* calculated for q and q' determinates the location of Σ and Σ' . At $\gamma \geq \gamma_*$, $\gamma' \geq \gamma_*$ the continuation near S can be constructed (sufficient conditions). When building the continuation in the blank body they take the form:

$$\frac{(1+\sin\rho)\left(tg\left(\Phi+\lambda-\alpha\right)+tg\rho\right)}{\cos\rho-\sin\rho\cdot tg\left(\Phi+\lambda-\alpha\right)}+1\leq \frac{1}{\sin\rho}\left(1-\frac{1-\sin\rho}{1+\sin\rho}e^{2tg\rho(\Phi-\varphi)}\right), \Phi\leq\varphi,
\frac{(1+\sin\rho)\left(tg\left(\Phi+\lambda-\alpha\right)+tg\rho\right)}{\cos\rho-\sin\rho\cdot tg\left(\Phi+\lambda-\alpha\right)}+1\leq \frac{2\left(1-\sin\left(\varphi-\Phi-2\theta\right)\right)}{1+\sin^2\rho-2\sin\rho\cdot\sin\left(\varphi-\Phi-2\theta\right)}, \Phi\geq\varphi, \tag{1}$$

where $2\theta = \arcsin(\sin \rho \cdot \cos(\varphi - \Phi)) + \pi$.

Similarly, we obtain sufficient conditions for NST area:

$$1 - \frac{(1 - \sin \rho) \left(tg \left(\Phi + \lambda - \alpha \right) + tg \rho \right)}{\cos \rho - \sin \rho \cdot tg \left(\Phi + \lambda - \alpha \right)} \le \frac{1}{\sin \rho} \left(1 - \frac{1 - \sin \rho}{1 + \sin \rho} e^{2tg \rho (\alpha - \Phi + \varphi)} \right),$$

$$\Phi \ge \alpha + \varphi,$$

$$1 - \frac{(1 - \sin \rho) \left(tg \left(\Phi + \lambda - \alpha \right) + tg \rho \right)}{\cos \rho - \sin \rho \cdot tg \left(\Phi + \lambda - \alpha \right)} \le \frac{2 \left(1 - \sin \left(\Phi - \alpha - \varphi - 2\theta \right) \right)}{1 + \sin^2 \rho - 2\sin \rho \cdot \sin \left(\Phi - \alpha - \varphi - 2\theta \right)},$$

$$\Phi \le \alpha + \varphi,$$

$$(2)$$

where $2\theta = \arcsin(\sin \rho \cdot \cos(\Phi - \alpha - \varphi)) + \pi$.

We consider the necessary conditions of the existence of a continuation in MST (Fig. 3).

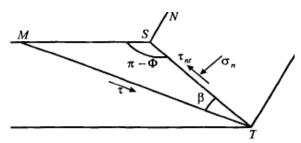


Fig. 3. The construction to rigid zones (necessary conditions of continuation).

Let τ – the shear stress in cross section MT. Out of balance of MST area $\tau = (\cos \beta - \sin \beta \cdot ctg\Phi)(\tau_{nt} \cdot \cos \beta + \sigma_n \cdot \sin \beta)$.

At
$$\beta$$
=0, $\tau = \tau_{nt}$, therefore for β >0 necessary condition is $d\tau/d\beta|_{\beta=0} \le 0$. Hereof $ctg\Phi \ge tg(\Phi + \lambda - \alpha)$. (3)

Similarly, we obtain necessary conditions for *NST* area:

$$ctg\left(\pi/2 + \Phi - \alpha\right) \le tg\left(\Phi + \lambda - \alpha\right). \tag{4}$$

We construct the continuation in RTS near T point (Fig. 4) using [2].

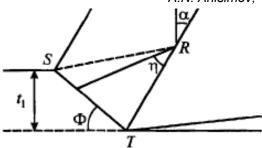


Fig. 4. The slip lines for the material in the semi-plastic state.

Let the material near the contact area is in the semi-plastic state. The plastic area is the triangle *RST* of isogonal lines. Since $\mu = \cos(\rho + 2\eta)/(1 + \sin(\rho + 2\eta))$, the continuation in *NST* exists at

$$\Phi \le \alpha + \pi/4 - \lambda - 3\rho/2. \tag{5}$$

The Area of Existence of Complete Solution. For completeness of solution the Φ angle must satisfy all inequalities (1)-(5). In Fig. 5 the dependence $W(\Phi) = |W_1(\Phi)| + |W_2(\Phi)|$ is shown for $\rho = 10^\circ$. The area of existence of complete solution is shaded.

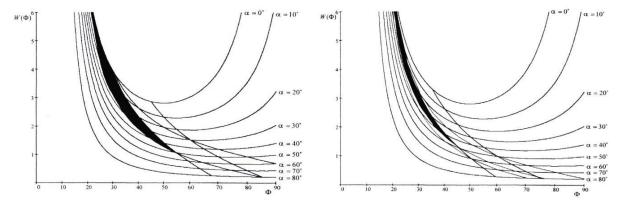


Fig. 5. The area of existence of complete solution: μ =0 (left); μ =0,35 (right).

The Preferred Solution. Let W has a minimal value in the area of existence of complete solution. In this case Σ matches the free surface of the material and Φ detects from (1) in the form of equalities. In Figs. 6, 7 is represented the strain distribution with minimal W and density change ρ_c in cuttings.

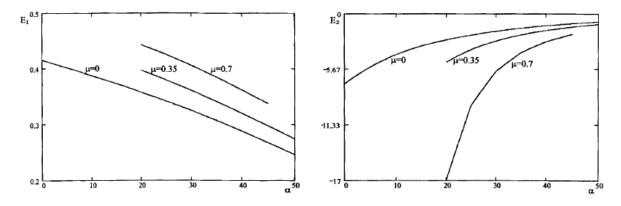


Fig. 6. The particles strain distribution in cuttings depending on α ($\rho = 10^{\circ}$).

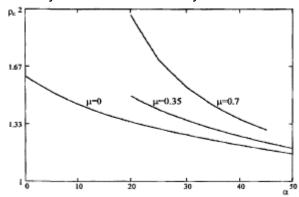


Fig. 7. The material density change in cuttings depending on α .

References

- [1] H. Ernst, M.E. Merchant // Transactions ASM 29 (1941) 299.
- [2] E.H. Lee, B.W. Shaffer // Journal of Applied Mechanics 73 (1954) 405.
- [3] Yu.G. Egorova, S.A. Kaverzina, A.I. Khromov // Doklady Physics 47(8) (2002) 616.
- [4] V.V. Sokolovsky, *Theory of Plasticity* (Moscow, 1969). (In Russian).
- [5] A.N. Anisimov, A.I. Khromov // PMTF **51(2)** (2010) 176. (In Russian).
- [6] J.F.W. Bishop // Journal of the Mechanics and Physics of Solids 2(1) (1953) 43.
- [7] A.I. Khromov, *Deformation and Fracture of Rigid-Plastic Solids* (Dal'nauka, Vladivostok, 1996). (In Russian).