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THE MODEL OF CUTTING PROBLEM SUBJECT TO A COULOMB-MOHR YIELD CONDITION

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Abstract. The rigid-plastic model of cutting problem is analyzed subject to a Coulomb-Mohr yield condition. The fields of stress, of strain and of deformation are investigated. Local continuation of stress field to rigid zones (the blank body and cuttings) is constructed. The existence region of full solution is detected and selection criteria of preferred solution is done. Depending of strain distribution and density changes in cuttings is given.

1. Introduction

The rigid-plastic model of cutting problem subject to Tresca and Mises yield conditions based on the assumption of existence of an isolated slip line (shear plane). In this formulation of the problem, kinematically admissible solutions [1] and [2] are known. The completeness of these solutions is studied in [3]. It is shown that they have significant limitations. A solution minimizing the volume density of energy dissipation in the shear plane with the existence of a statically admissible continuation of the stress field to rigid zones is suggested.

In this paper, the supplement of approach stated in [3] is considered subject to solution of cutting problem with a Coulomb-Mohr yield condition.

2. Problem Definition

We consider the problem of cutting taking into account the compressibility irreversible (Fig. 1). A blank moves from left to right with a velocity *V*. On the isolated slip line a Coulomb-Mohr yield condition is performed [4]: $(1/4)(\sigma_{11}-\sigma_{22})^2+\sigma_{12}^2=(k+\sin\rho(\sigma_{11}+\sigma_{22})/2)^2$, where σ_{ij} – the components of the stress tensor; k – adhesion coefficient; $\rho=\pi/2-2\varphi$ – the angle of internal friction.

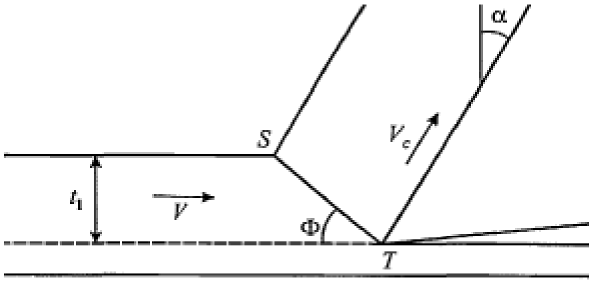

Fig. 1. Chip formation during deformation with single slip line.

The areas *ASM* and *BSN* are subjected to an one-sided pressure q and q'. The minimal angle of the wedge γ_* enduring the stress q is

$$|q| = k\left(1 - e^{2tg\rho(\pi/2-\gamma_*)}\left(1-\sin\rho\right)/\left(1+\sin\rho\right)\right)/\sin\rho, \gamma_* \geq \pi/2,$$

$$|q| = 2k\left(1+\cos\left(\gamma_* - 2\theta\right)\right)/\left(1+\sin^2\rho + 2\sin\rho\cos\left(\gamma_* - 2\theta\right)\right), \gamma_* \leq \pi/2,$$

where $2\theta = \arcsin\left(\sin\rho\cdot\sin\gamma_*\right) + \pi$.

The angles γ_* and $\gamma_*^{'}$ calculated for q and q' determinates the location of Σ and Σ'. At $\gamma \geq \gamma_*$, $\gamma' \geq \gamma_*^{'}$ the continuation near *S* can be constructed (sufficient conditions). When building the continuation in the blank body they take the form:

$$\frac{\left(1+\sin\rho\right)\left(tg\left(\Phi+\lambda-\alpha\right)+tg\rho\right)}{\cos\rho - \sin\rho\cdot tg\left(\Phi+\lambda-\alpha\right)} + 1 \leq \frac{1}{\sin\rho}\left(1 - \frac{1-\sin\rho}{1+\sin\rho}e^{2tg\rho(\Phi-\varphi)}\right), \Phi \leq \varphi,$$
$$\frac{\left(1+\sin\rho\right)\left(tg\left(\Phi+\lambda-\alpha\right)+tg\rho\right)}{\cos\rho - \sin\rho\cdot tg\left(\Phi+\lambda-\alpha\right)} + 1 \leq \frac{2\left(1-\sin\left(\varphi-\Phi-2\theta\right)\right)}{1+\sin^2\rho - 2\sin\rho\cdot\sin\left(\varphi-\Phi-2\theta\right)}, \Phi \geq \varphi, \quad (1)$$

where $2\theta = \arcsin\left(\sin\rho\cdot\cos\left(\varphi-\Phi\right)\right) + \pi$.

Similarly, we obtain sufficient conditions for *NST* area:

$$\left.\begin{array}{l} 1 - \dfrac{\left(1-\sin\rho\right)\left(tg\left(\Phi+\lambda-\alpha\right)+tg\rho\right)}{\cos\rho - \sin\rho\cdot tg\left(\Phi+\lambda-\alpha\right)} \leq \dfrac{1}{\sin\rho}\left(1 - \dfrac{1-\sin\rho}{1+\sin\rho}e^{2tg\rho(\alpha-\Phi+\varphi)}\right), \\ \Phi \geq \alpha+\varphi, \\ 1 - \dfrac{\left(1-\sin\rho\right)\left(tg\left(\Phi+\lambda-\alpha\right)+tg\rho\right)}{\cos\rho - \sin\rho\cdot tg\left(\Phi+\lambda-\alpha\right)} \leq \dfrac{2\left(1-\sin\left(\Phi-\alpha-\varphi-2\theta\right)\right)}{1+\sin^2\rho - 2\sin\rho\cdot\sin\left(\Phi-\alpha-\varphi-2\theta\right)}, \\ \Phi \leq \alpha+\varphi, \end{array}\right\} \quad (2)$$

where $2\theta = \arcsin\left(\sin\rho\cdot\cos\left(\Phi-\alpha-\varphi\right)\right) + \pi$.

We consider the necessary conditions of the existence of a continuation in *MST* (Fig. 3).

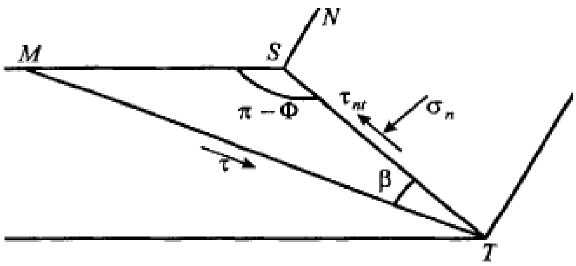

Fig. 3. The construction to rigid zones (necessary conditions of continuation).

Let τ – the shear stress in cross section *MT*. Out of balance of *MST* area $\tau = \left(\cos\beta - \sin\beta\cdot ctg\Phi\right)\left(\tau_{nt}\cdot\cos\beta + \sigma_n\cdot\sin\beta\right)$.

At β=0, $\tau = \tau_{nt}$, therefore for β>0 necessary condition is $d\tau/d\beta\big|_{\beta=0} \leq 0$. Hereof

$$ctg\Phi \geq tg\left(\Phi+\lambda-\alpha\right). \quad (3)$$

Similarly, we obtain necessary conditions for *NST* area:

$$ctg\left(\pi/2+\Phi-\alpha\right) \leq tg\left(\Phi+\lambda-\alpha\right). \quad (4)$$

We construct the continuation in *RTS* near *T* point (Fig. 4) using [2].

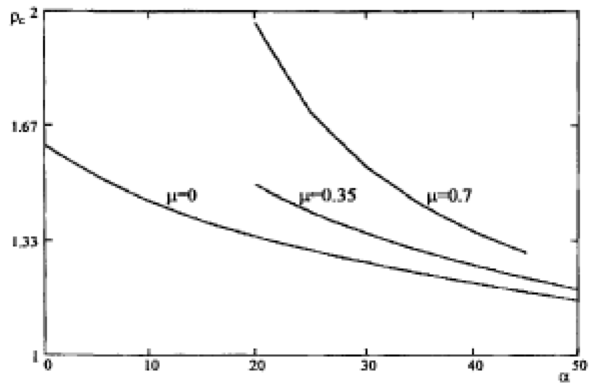

Fig. 7. The material density change in cuttings depending on α.

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