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THE DYNAMICS OF A FLEXIBLE ROTOR ON ACTIVE MAGNETIC BEARINGS, ACCOUNTING FOR NON-COAXIALITY OF ITS ELEMENTS

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Abstract. The effect of non-coaxiality of the elements of a compound flexible inhomogeneous rotor on active magnetic bearings arising due to the eccentricity of the coupling clutches on rotor dynamics is studied; recommendations are given for balancing the rotor and for the laws of controlling the electromagnetic suspension.

1. Introduction

Multi-ton compound flexible rotors on active magnetic bearings (AMBs) are becoming more and more used in numerous fields of power engineering; that is why modeling and studying the dynamics of such rotors have become important [1–5]. Such a rotor is a combination of various machines and can have several elements connected by flexible clutches. As a rule, eccentricity of the coupling clutches leads to non-coaxiality – displacement of the geometrical axis of the rotor elements. The present paper is aimed at studying the effect of this phenomenon on the dynamics of a compound flexible rotor on AMBs. Such rotors comply the criteria formulated in [6], determining a complex and unique system. One of the principles of modeling and analyzing the dynamics of such systems is successively determining most important factors causing the phenomenon in question and constructing simplest models of the system dynamics, accounting for these factors. Using the above principle, the effect of stiffness of the coupling clutch and displacement of the geometric axes of the rotor elements on its dynamics is studied; recommendations are given for balancing the rotor and for the laws of controlling the electromagnetic suspension.

2. A mathematical model of the dynamics of a rotor with a flexible clutch

A model of the structure presented in Fig. 1 is studied in the global coordinate system 0xyz. 0z axis coincides with the axis of the rotor bearings. Rotor P1 rests on hinged supports (high-stiffness supports), whereas P2 rests on a hinged support and on an electromagnetic support of effective stiffness c in directions 0x and 0y and effective viscous damping k. Chosen support conditions make it possible to construct a simplest model of the rotor dynamics that allows one to analyze characteristics of the effect of non-coaxiality on its dynamics. The dynamic properties of the rotor are characterized by masses m, m_1 and stiffness c_1 (in directions 0x and 0y). Mass m_1 and stiffness c_1 in this structure make it possible to model bending vibrations of the rotor. Stiffness of elastic coupling between the rotors is equal to c_0 (in two mutually orthogonal directions), in a free state non-coaxiality of the rotors is characterized by value Δ. In the process of rotation of the rotor, non-coaxiality will retain its orientation relative to the rotor and can be characterized by length vector Δ, its direction being determined by rotation

$$\nu_{1,2}^2 = \frac{1}{2}(n^2+1) \mp \frac{1}{2}\sqrt{\left(n^2-1\right)^2 + 4\beta n^2}. \qquad (3)$$

Using the following substitution $u = \bar{u} \cdot \exp(j\varphi)$, $u_1 = \bar{u}_1 \cdot \exp(j\varphi)$, $\delta = \bar{\delta} \cdot \exp(j\varphi_0)$, $\delta_1 = \bar{\delta}_1 \cdot \exp(j\varphi_1)$ for ν=const for complex amplitudes of stationary oscillations $\bar{u}, \Delta\bar{u} = \bar{u}_1 - \bar{u}$ from (2) the following expressions are obtained:

$$\bar{u} = \left(-1 + \bar{\delta}\nu^2 + \bar{\delta}_1 \frac{\beta\nu^2}{n^2-\nu^2}\right) / \left(1 - \nu^2 - \frac{\beta n^2}{n^2-\nu^2} + 2h\nu \cdot j\right), \quad \Delta\bar{u} = \frac{\nu^2}{n^2-\nu^2}(\bar{u} + \bar{\delta}_1). \qquad (4)$$

Figure 2 shows, for the case of n=2, β=0.5, h=0.2, $\delta=\delta_1=0$, the diagrams of the oscillation amplitudes of masses $A = |\bar{u}|$, $B = |\Delta\bar{u}|$ as a function of rotation frequency the rotor, constructed using relations (4). The effect of non-coaxiality is characterized by resonances at critical rotation frequency of the rotor (3), similar to the case of imbalance effect. However, it differs substantially from the effect of imbalance: at low rotation numbers $\nu \approx 0$ non-zero displacements of the rotor are observed, and for ν→∞ mass displacements tend to 0.

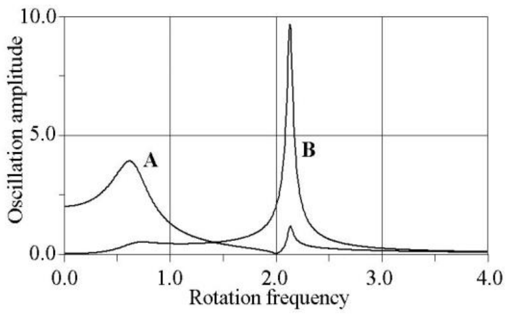

Fig. 2. Amplitudes *A* and *B* as a function of the rotation frequency of the rotor in the presence of non-coaxiality.

To deal with the practically important issue of suppressing the level of arising resonance oscillations, if this level is comparable with the gaps in the stand-by bearings of a rotor on AMBs, one can use rotor balancing in exploitation conditions. In equations (2), the balancing procedure is done by selecting imbalance parameters δ, δ_1, φ_0 и φ_1, such that the rotor displacement level does not exceed a permissible value. If the low-frequency resonance due to the rigid body motion of the rotor causes no problems during the acceleration of the rotor, then, to eliminate high-frequency resonance, it suffices to introduce imbalance ($\delta = 1/\nu_2^2$, $\varphi_0 = 0$, $\delta_1 = 0$). The corresponding resonance curves are shown in Fig. 3.

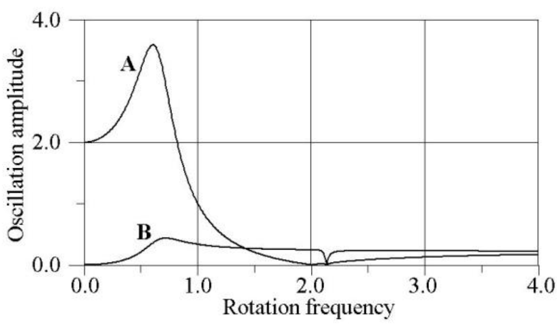

Fig. 3. Amplitudes *A* и *B* as a function of the rotation frequency of the rotor with suppressed high-frequency resonance.

The introduction of the shown balancing weight suppressed the resonance peak, however at higher rotation numbers of the rotor ($\nu>\nu_2$), as is expected, displacements of mass *m* no longer tend to 0 but become stable on the level of the introduced imbalance. If the displacement level exceeds a permissible value both at the low-frequency and high-frequency resonances, the rotor has to be balanced at the two critical rotation frequency of the rotor. In doing so, one must keep in mind that the higher the imbalance, the higher is the level where displacements of the rotor will become stable for high revolution numbers. Hence, the principle of sufficient minimum should be used in the balancing process: the imbalance introduced must be such that displacements of the rotor during acceleration do not exceed the permissible level.

Let *K* be coefficient of suppression of displacement of mass *m* for resonance oscillations $\nu=\nu_1$ in the process of balancing. The requirement of the absence of displacements $\Delta\bar{u}=0$ for $\nu=\nu_2$ will be used as a second condition in the balancing process. Keeping in mind the above conditions of balancing, the following equation system is written for finding complex values of imbalance:

$$-1+\bar{\delta}\nu_1^2+\bar{\delta}_1\frac{\beta\nu_1^2}{n^2-\nu_1^2}=K,\quad -1+\bar{\delta}\nu_2^2+\bar{\delta}_1\frac{\beta\nu_2^2}{n^2-\nu_2^2}+\bar{\delta}_1 2h\nu_2 j=0. \tag{5}$$

It is noteworthy that Fig. 3 corresponds to the solution of system (5) for *K*=0.904. The diagrams of oscillation amplitude A of mass *m* as a function of the rotation frequency of the rotor for various values of parameter *K*, constructed using relations (4) and (5), are presented in Fig. 4.

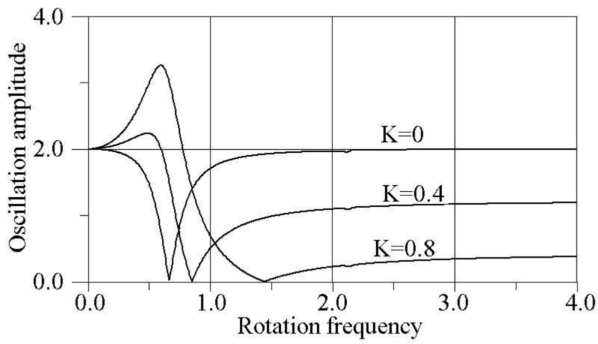

Fig. 4. Amplitude A as a function of the rotation frequency of the rotor for various values of parameter *K*.

By analyzing the relations in Fig. 4 one can determine an optimal value of balancing weights that will keep the displacement of the rotor in the presence of low-frequency resonance oscillations within permissible limits, whereas, at high rotation frequency, displacements of the rotor will be stabilized at a lowest possible level. In this way, introduction of balancing weights may lead to deterioration of the dynamic characteristics of the structure in the service range of the rotation frequency of the rotor. Thus, for digitally controlled rotors on AMBs, non-coaxiality effects of the rotors may be counteracted by using in the controlling system a law according to which an additional harmonic force is formed in the magnets of AMBs during the acceleration of the rotor [2], its value being equal to the non-coaxiality force but of the opposite phase. The parameters of this force (value, orientation relative the rotor), as well as characteristics of the imbalance, are unknown. They may be determined at the starting-setting stage of the system using the methodology presented in [3, 4] and allowing one to identify the unknown parameters by solving the inverse dynamic problem, on condition that the mathematical model of a flexible rotor on AMBs [5] will account for the phenomenon of non-coaxiality of the rotor elements.

4. Conclusions

The conducted studies showed that the effect of non-coaxiality of the rotor elements on its dynamics is similar to the effect of a harmonic force having the oscillation frequency equal to the rotation frequency of the rotor. The level of resonance oscillation of the rotor caused by non-coaxiality of the rotor elements can be suppressed by introducing imbalance. Another way to compensate non-coaxiality is to form, using the controlling system of the electromagnetic suspension, an additional harmonic force of the value equal to the force caused by non-coaxiality but of the opposite phase.

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