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STUDY OF VISCOELASTIC PARAMETER INFLUENCE ON DYNAMIC RESPONSE IN POROVISCOELASTIC PRISMATIC SOLID

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Abstract. Dynamic behavior of poroviscoelastic solids is studied in this paper. Poroviscoelastic problem formulation is based on Biot's theory of poroelasticity. In order to describe viscous properties of skeleton by means of the correspondence principle such classical viscoelastic models are used: Kelvin-Voigt model, standard linear solid and model with weakly singular kernel. A numerical modelling of wave propagation is done by means of boundary element approach. Boundary element method (BEM) and boundary integral equation (BIE) method are applied to solving three-dimensional boundary-value problems. Solution is obtained in Laplace domain. Numerical inversion of Laplace transform is based on Durbin's method with variable integration step and Runge-Kutta relying method. Results of numerical experiments are given.

1. Introduction

Wave propagation in dispersed media is a great interest of many disciplines. Mechanics of advanced materials, such as poro-, visco- or poroviscoelastic materials, is relevant to such disciplines as geophysics, geo- and biomechanics, seismology, physical chemistry, petroleum engineering etc.

Study of wave propagation processes in saturated porous continua began from the works of Y. I. Frenkel (1944) and M. Biot (1941) [1]. The similarity of the transient responses of poroelastic and viscoelastic media was first mentioned by Simon et al. (1984) [2]. Common state of the art can be found in works of R. de Boer (2000), M. Schanz (2001) [3]. The implementation of the solid viscoelastic effects in the theory of poroelasticity was first introduced by Biot [4]. Resent years the dynamic interaction analyses involving poroelastic/poroviscoelastic media is extensively studied in literature. The governing equation for saturated poroviscoelastic media by introducing the Kelvin–Voigt (one-dimensional solution) were developed by Shanz and Cheng [5].

Because the complexity of the inertial viscosity and mechanical coupling in porous media most transient response problems can only be solved via numerical methods. There are two major approaches to dynamic processes modeled by means of BEM: solving BIE system directly in time domain or in Laplace or Fourier domain followed by the respective transform inversion [6]. So, the Laplace transform is one of main methods in dealing with transient response of porous media. Classical formulations for BIE method with their discretized realization and traditional BEM are successful approaches for solving three-dimensional problem. Extension of BIE method and BEM and problems, where material models are

The equilibrium and instantaneous values of the relaxation function associated with material modules are connected as follows:

$$\omega^2 K_\infty = K_0 ,\ \omega^2 G_\infty = G_0 \tag{7}$$

Equilibrium and instantaneous values are denoted by «∞» and «0» respectively.

3. Boundary-element approach

Boundary of the solid is approximated by a set of quadrangular and triangular 8-node biquadratic elements. Triangular elements are considered as singular quadrangular elements. The Cartesian coordinates of an arbitrary point of the element are expressed through the coordinates of the nodal points of this element, using shape functions of the local coordinates. Shape functions are quadratic polynomials of interpolation. Unknown boundary fields are integrated by interpolation node values. Gaussian quadrature and hierarchic integrating algorithm are used for integration over the boundary elements, what allows reaching the necessary accuracy. Boundary-element schemes are based on the mixed approximation of boundary functions. Bilinear elements are used to approximate boundary displacements, constant elements – for traction approximation. After applying collocation method on the set of boundary functions approximation nodes, system of linear equations is obtained.

Fundamental and singular solutions are considered in term of singularity isolation. Since it is possible to isolate singularities of BIE kernels [3], so regularized BIE system is considered with the help of Green-Betti-Somigliana formulae in order to use described BE discretization [10]. The problem is solved in Laplace domain. Durbin's method [11] with variable integrating step (8) and Runge-Kutta nodes based CQM for numerical inversion of Laplace transform are used to obtain solution in time domain. Substituting basic linear multistep method for Runge-Kutta method referred to as Butcher tableau [12].

4. Numerical example

The problem of poroviscoelastic prismatic solid, clamped at one end and subjected to a Heaviside type load $t_3 = 1N/m^2$ at another end, is considered. The length of the solid is 9 *m*. Poroelastic material is Berea sandstone. Material constants are: $K = 4{,}8 \cdot 10^9\ N/m^2$, $G = 7{,}2 \cdot 10^9\ N/m^2$, $\rho = 2458\,kg/m^3$, $\phi = 0{,}19$, $K_s = 3{,}6 \cdot 1010\,N/m^2$, $\rho_f = 1000\,kg/m^3$, $K_f = 3{,}3 \cdot 10^9\ N/m^2$ $k = 1{,}9 \cdot 10^{-10}$ $m^4/(N \cdot s)$, $\nu = 0$.

The displacements and pore pressure are obtained at point *A*, which is situated on the axis x_3 in 1.5 *m*. from loaded end, see Fig. 1. For transient response investigation in inner point by means of BEM solid is divided on two subdomains with imaginary boundary (marked line on Fig. 2) laid in 1.5 *m*. far from loaded end. Boundary element mesh for current numerical solutions on each subdomain is consists of 408 and 168 elements respectively.

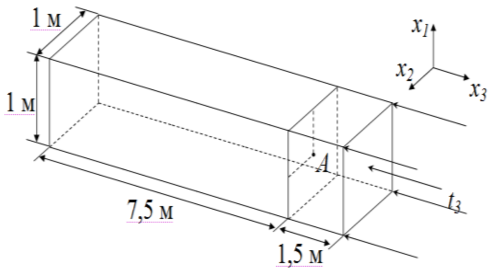

Fig. 1. Problem statement.

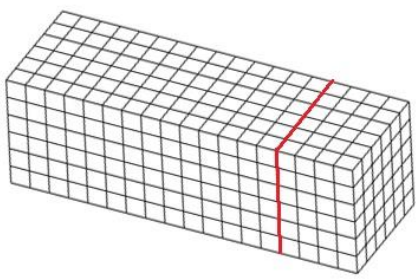

Fig. 2. Boundary-element mesh.

5. Conclusions

The comparison of dynamic responses with different values of viscoelastic parameter for each model is presented. Also Fig. 4 and Fig. 5 show the similarity of dynamic response behavior in the cases of Kelvin-Voigt and standard linear solid models.

An effect of inner displacements wave field restructuring is demonstrated in the case of standard linear solid model, when properties of viscoelastic material were changing from instantaneous to equilibrium. Amplitude and period of displacement response function increased. Such effect was partly described with the help of numerical methods before by V.G. Bazhenov and L.A. Igumnov [8].

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