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SIMULATION OF A COMPRESSIONAL SLOW WAVE IN PARTIALLY SATURATED POROELASTIC 1-D COLUMN

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Abstract. We simulate wave propagation in a partially saturated porous medium, where the feature is the presence of a slow wave. The pores are filled with a wetting fluid and a non-wetting fluid, and the model, based on a Biot-type three-phase theory. In the present paper, the solution of a finite one dimensional column with Neumann and Dirichlet boundary conditions are presented. The solution is obtained in the Laplace domain and the time-step method is chosen to obtain the time domain solution. The material data of Massillion sandstone are used for calculations. The column response to the dynamic loading is examined in terms of displacement, pore water pressure, pore air pressure. By neglecting the viscosity of the fluid, assuming very large permeabilities, the second compressional wave are identified.

1. Introduction

Research of wave propagation processes in saturated porous continua began from the works of Y.I. Frenkel (1944) and M. Biot (1956). L.Y. Kosachevsky in 1959 showed that Frenkel's and Biot's models rely on the same stress-strain relations but the latter is of a greater generality. After that waves in saturated porous continua have been studied by C. Zwikker & C.W. Kosten, J. Geertsma & D.C. Smith, P.P. Zolotaryov, V.N. Nikolayevsky, V.P. Stepanov, L.M. Doroginitskaya, C. McCann & D.M. McCann, S.H. Yakubov, A.A. Gubaydullin etc., but two classical Biot's works are considered the most influential. Common state of the art can be found in works of R. de Boer (2000), M. Schanz (2001, 2009), V.N. Nikolayevsky (2005). Works of T.I. Belyankova, V.V. Kalinchuk, S. Diebels, W. Ehlers contain an overview of Biot's theory as well as other approaches to porous continua modeling.

Biot's model is based on the description of how two phases – porous elastic skeleton and gas or liquid filler – interact [1]. Historically Biot's theory was the first model to predict all three possible types of waves in porous continuum: fast shear wave, fast and slow compression waves. Both fast waves are in their nature close to the ones of elastic continuum, and slow compression wave presence is the principal difference between elastic and poroelastic continua. This wave is caused by transfer of pore filler particles with respect to the skeleton. Ignoring the slow wave leads to serious inaccuracies in estimation of fast waves damping. We study a simple macroscopical three-phase model describing wave propagation in partially saturated porous media. The model consists of a continuous non-wetting phase and a continuous wetting phase and is an extension of classical biphasic (Biot-type) models.

2. Analytical solution

Consider a problem of a poroelastic column of length l with one clamped end and a Heaviside-shaped in time load $f(t) = S_0 H(t)$ to another end. The governing equations of partially saturated poroelasticity in the Laplace for one-dimensional case are three scalar coupled ordinary differential equations [2]

$$(K+\frac{G}{3})\bar{u}_{y,yy}-(\rho-\beta S_w\rho_w-\gamma S_a\rho_a)s^2\bar{u}_y-(\alpha-\beta)S_w\bar{p}^w_{,y}+(\alpha-\gamma)S_a\bar{p}^a_{,y}=0, \quad (1)$$

$$-(\alpha-\beta)S_w s\bar{u}_{y,y}-(\zeta S_{aa}S_w+S_u)s\bar{p}^a+\frac{\beta S_w}{\rho_w s}\bar{p}^w_{,yy}-(\zeta S_{ww}S_w+\frac{\varphi}{K_w}S_w-S_u)s\bar{p}^w=0, \quad (2)$$

$$-(\alpha-\gamma)S_a s\bar{u}_{y,y}-(\zeta S_{ww}+S_u)s\bar{p}^w+\frac{\gamma S_a}{\rho_a s}\bar{p}^a_{,yy}-(\zeta S_{aa}+\frac{\phi}{K_a}S_a-S_u)s\bar{p}^a=0. \quad (3)$$

The solution for the stress boundary condition $\hat{u}_y(y=0)=0$, $\hat{\sigma}_y(y=l)=-S_0$, $\hat{p}^w(y=l)=0$, $\hat{p}^a(y=l)=0$ is

$$\hat{u}_y=\frac{S_0 s^{-1}}{(K+\frac{4}{3}G)(t_1\lambda_1+t_2\lambda_2+t_3\lambda_3)}\sum_{i=1}^{3}\left[\frac{e^{-\lambda_i s(y+l)}-e^{\lambda_i s(y-l)}}{1+e^{-2\lambda_i sl}}t_i\right] \quad (4)$$

$$\hat{p}^w=\frac{S_0}{(K+\frac{4}{3}G)(t_1\lambda_1+t_2\lambda_2+t_3\lambda_3)}\sum_{i=1}^{3}\left[\frac{e^{-\lambda_i s(y+l)}-e^{\lambda_i s(y-l)}}{1+e^{-2\lambda_i sl}}t_i a_i\right] \quad (5)$$

$$\hat{p}^a=\frac{S_0}{(K+\frac{4}{3}G)(t_1\lambda_1+t_2\lambda_2+t_3\lambda_3)}\sum_{i=1}^{3}\left[\frac{e^{-\lambda_i s(y+l)}-e^{\lambda_i s(y-l)}}{1+e^{-2\lambda_i sl}}t_i b_i\right] \quad (6)$$

The time domain results are calculated with the time-step method. The time-step method for numerical Laplace transform inversion is based on the integration theorem and dedicated to the calculation of the original function integral [3, 4].

In order to employ time-step method for getting the original function $f(t)$ such as $f(0)=0$ from known image $\bar{f}(s)$ we need to apply the integration theorem to its derivative:

$$f(t)=\int_0^t f'(\tau)d\tau=\frac{1}{2\pi i}\lim_{R\to\infty}\int_{c-iR}^{c+iR}\bar{f}(s)s\int_0^t e^{s(\tau)}d\tau ds=... \quad (7)$$

Following the theorem, the integral can be found as:

$$f(0)=0,\quad f(n\Delta t)=\sum_{k=1}^{n}\omega_k(\Delta t),\quad n=1,...N, \quad (8)$$

$$\omega_n(\Delta t)=\frac{R^{-n}}{L}\sum_{l=0}^{L-1}\bar{f}(s)se^{-in\varphi},\quad s=\frac{\gamma(z)}{\Delta t},\quad z=Re^{i\varphi},\quad \varphi=2\pi\tfrac{l}{L}. \quad (9)$$

Here R is the radius of the analyticity region for $\bar{f}(\gamma(z)/\Delta t)$ and $\gamma(z)$ is the characteristic function for the linear multistep method applied to the Cauchy problem arising within the integral evaluation. Backward differentiation (BDF) based methods of order ≤ 6 are applicable in scope of this solution scheme. For BDF-2 we have $\gamma(z)=3/2-2z+z^2/2$.

3. Results

Assuming the value $S_0=1N/m^2$, the pore pressures p^w, p^a at the clamped end and displacement u_y at the another end are calculated with varying saturation degree. The pore size distribution index ϑ is set to 1.5, the residual water saturation S_{rw} is set to 0, and the air entry saturation S_{ra} is set to 1. The column is made of Massilon sandstone with the following properties: $K=1.02\cdot10^9\,N/m^2$, $G=1.44\cdot10^9\,N/m^2$, $\phi=0.23$, $\rho_s=2650kg/m^3$,

$\rho_w = 997 kg/m^3$, $\rho_a = 1.10 kg/m^3$, $K_s = 3.55 \cdot 10^{10} N/m^2$, $K_w = 2.25 \cdot 10^9 N/m^2$, $K_a = 1.10 \cdot 10^5 N/m^2$, $\kappa = 2.5 \cdot 10^{-12} m^2$, $\eta_w = 1.0 \cdot 10^{-3} Ns/m^2$, $\eta_a = 1.8 \cdot 10^{-5} Ns/m^2$. To capture the slow wave, an column with length of $l = 9m$ is used and pore pressure $p^w(t, y = 7m)$ is calculated. To elevate the viscosity of the two fluids, two arbitrarily permeabilities, κ_w and κ_a are chosen in the calculation. The parameter of water saturation is set to be $S_w = 0.99$ in calculation.

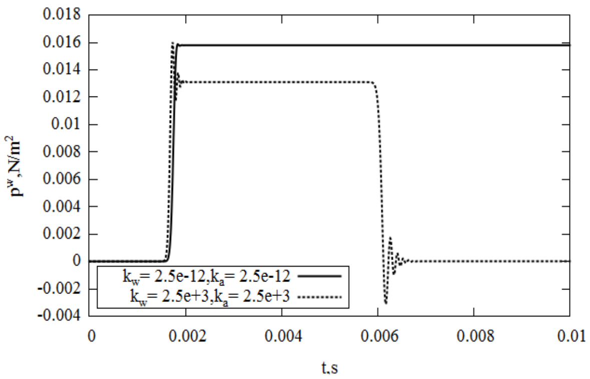

Fig. 1. Pore water pressure.

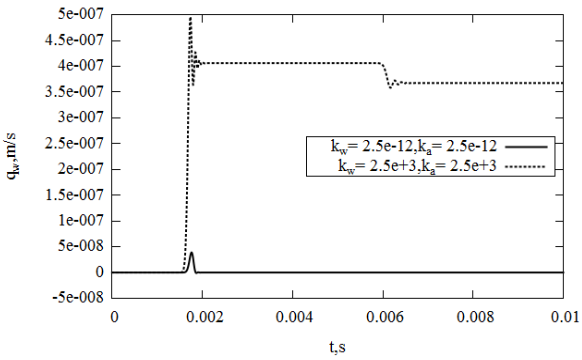

Fig. 2. Pore water flux.

Figures 1-2 demonstrate the effect of slow wave appearance in porous continuum for large permeability coefficient value. For pore water pressure and flux, there are two step jumps by calculating with the large permeabilities. The arrival of the first wave at 2 m is observed that causes the first step jump. The second wave, arriving at later time, is of negative amplitude and cancels the first wave. When calculating with real permeabilities, only the arrival of the fast wave is observed.

4. Conclusions

Based on the simple macroscopical three-phase model, an analytical solution in the Laplace domain for a partially saturated poroelastic one dimensional column has been studied. The time domain solutions are obtained by with the time-step method. The solutions are calculated for different water saturation. For nearly saturated case, the partially saturated solution come close to the saturated solution. When decreasing water saturation, the displacements become much larger, pore water pressure decrease to very small values, and the pore air pressure first increase and then decrease.

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References

[1] M.A. Biot // *Journal of the Acoustical Society of America* **28(2)** (1956) 168.
[2] P. Li, M. Schanz // *Geophysical Journal International* **184** (2011) 1341.
[4] C. Lubich // *Numerische Mathematik* **52** (1988) 129.
[5] C. Lubich // *Numerische Mathematik* **52** (1988) 413.