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ANALYZING THICK LAYERED PLATES UNDER THEIR OWN WEIGHT BY THE METHOD OF INITIAL FUNCTIONS

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Abstract. Taking into consideration an own weight when analyzing massive solids is rather important problem of the theory of elasticity and structural mechanics. Thick layered plates are often used as elements of various constructions. The method of initial functions (MIF) is very suitable for their analyzing. In this article the MIF algorithm for taking into account the stress from own weight is extended to analyze layered plates.

1. Introduction

The MIF, founders of which were Russian scientists A.I. Lur'e [1] and V.Z. Vlasov [2], was widely used to analyze thick plates including layered ones under static and dynamic loads on their two faces [3-7]. Despite its drawback consisting in the fact that on the side faces it can satisfy specific boundary conditions which can be treated as analogues of free supported or clamped sides this method is used to analyze layered structures [8-11]. It should note that all investigations with the MIF didn't take into consideration the own weight of the structure. But it can make a significant contribution to the value of the stresses acting in the plate.

In this article a partial solution of the nonhomogeneous system of differential equations of the theory of elasticity is built. This solution is then included in the MIF algorithm to analyze layered plates.

The equations of equilibrium of the theory of elasticity in displacements in a three-dimensional Cartesian coordinate system $Oxyz$ with taking into account an own weight acting along the axis $Oxyz$ may be written in the operator form as

$$
\begin{aligned}
&\left[2(\nu-1)\partial_x^2+(2\nu-1)\left(\partial_y^2+\partial_z^2\right)\right]u-\partial_x\partial_y v-\partial_x\partial_z w=0,\\
&-\partial_x\partial_y u+\left[2(\nu-1)\partial_y^2+(2\nu-1)\left(\partial_x^2+\partial_z^2\right)\right]v-\partial_y\partial_z w=0,\\
&-\partial_x\partial_z u-\partial_y\partial_z v+\left[(2\nu-1)\left(\partial_x^2+\partial_y^2\right)+2(\nu-1)\partial_z^2\right]w=-\rho g.
\end{aligned}
\tag{1}
$$

Here $u=u(x,y,z)$, $v=v(x,y,z)$ and $w=w(x,y,z)$ are unknown functions representing the displacements of a plate, E and ν are respectively Young's modulus and Poisson's ratio of the plate material, ρ is its density, g is an acceleration of gravity and ∂_x, ∂_y and ∂_z are differential operators with respect to x, y and z variables.

A solution of (1) can be found as a sum of the solution of homogeneous system plus any partial solution of the nonhomogeneous system

$$
\mathbf{W}=\mathbf{W}^{\text{hom}}+\mathbf{W}^{\text{nonhom}}. \tag{2}
$$

Here $\mathbf{W}=\{u,v,w\}$ is a vector-column of displacements, $\mathbf{W}^{\mathrm{hom}}$ is a vector of general solution of the homogeneous system and $\mathbf{W}^{\mathrm{nonhom}}$ is a partial solution of the nonhomogeneous system. Using the Hooke's law and the Cauchy relations the stresses can be expressed through the displacements. So, the vector $\mathbf{U}=\{u,v,w,\sigma_z,\tau_{yz},\tau_{xz},\sigma_x,\sigma_y,\tau_{xy}\}$ of displacements and stresses can be determined.

2. A MIF solution of the homogeneous system

The general solution of the homogeneous equation is found using the MIF:

$$\mathbf{U}=\mathbf{L}\mathbf{U}^0. \tag{3}$$

Here $\mathbf{U}^0=\{u^0,v^0,w^0,\sigma_z^0,\tau_{yz}^0,\tau_{xz}^0\}$ is a vector of initial functions determined on the plane $z=0$, $\mathbf{L}=\left[L_{ij}\left(\partial_x,\partial_y,E,\nu,z\right)\right]$ ($i=1,\ldots,9$, $j=1,\ldots,6$) is a matrix of the MIF operators which are received in a closed form [12, 13].

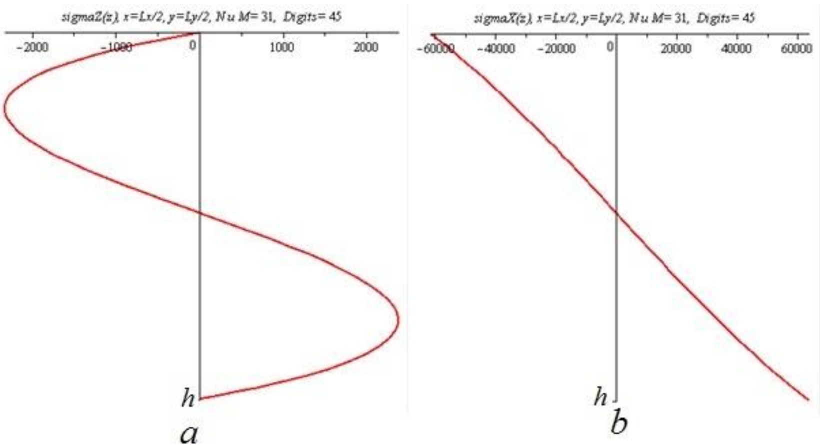

Fig. 1. The stresses σ_z (*a*) and σ_x (*b*) in the center of the one-layered plate.

If the initial functions can be presented by expansions in trigonometric series

$$u^0=\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}q_{mn}^1 c_m s_n,\ v^0=\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}q_{mn}^2 s_m c_n,\ u^0=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}q_{mn}^3 s_m s_n,$$
$$\sigma_z^0=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}q_{mn}^4 s_m s_n,\ \tau_{yz}^0=\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}q_{mn}^5 s_m c_n,\ \tau_{xz}^0=\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}q_{mn}^6 c_m s_n. \tag{4}$$

Here q_{mn}^j are unknown coefficients ($s_m=\sin(\alpha_m x)$, $c_m=\cos(\alpha_m x)$, $s_n=\sin(\beta_n y)$, $c_n=\cos(\beta_n y)$, $\alpha_m=m\pi/A$, $\beta_n=n\pi/B$, *m* and *n* are integers).

The components of the displacement and stress vector $\mathbf{U}$ according (3) and (4) will also be obtained as trigonometric series

$$u=\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\sum_{j=1}^{6}q_{mn}^j\tilde{L}_{1j}^{mn}c_m s_n,\ v=\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\sum_{j=1}^{6}q_{mn}^j\tilde{L}_{2j}^{mn}s_m c_n,\ w=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{j=1}^{6}q_{mn}^j\tilde{L}_{3j}^{mn}s_m s_n,$$
$$\sigma_z=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{j=1}^{6}q_{mn}^j\tilde{L}_{4j}^{mn}s_m s_n,\ \tau_{yz}=\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\sum_{j=1}^{6}q_{mn}^j\tilde{L}_{5j}^{mn}s_m c_n,\ \tau_{xz}=\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\sum_{j=1}^{6}q_{mn}^j\tilde{L}_{6j}^{mn}c_m s_n, \tag{5}$$
$$\sigma_x=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{j=1}^{6}q_{mn}^j\tilde{L}_{7j}^{mn}s_m s_n,\ \sigma_v=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{j=1}^{6}q_{mn}^j\tilde{L}_{8j}^{mn}s_m s_n,\ \tau_{xy}=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{j=1}^{6}q_{mn}^j\tilde{L}_{9j}^{mn}c_m c_n.$$

Here $\tilde{L}_{kj}^{mn}$, $k=1,\ldots,9$ are the values of the MIF operators on the products of the corresponding trigonometric functions.

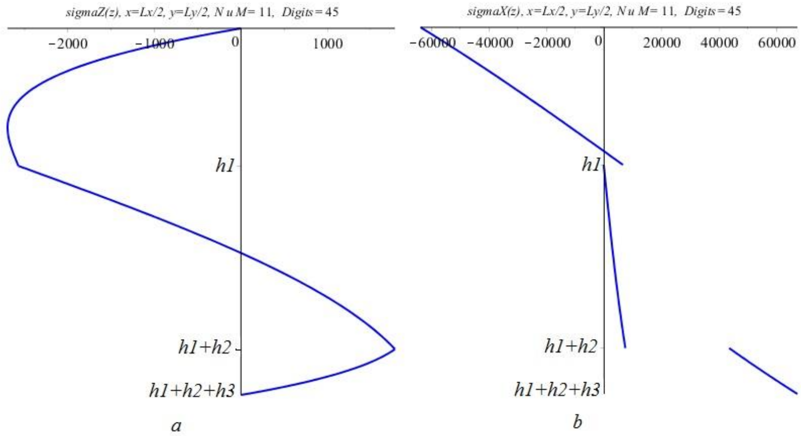

Fig. 2. The stresses σ_z (*a*) and σ_x (*b*) in the center of the three-layered plate.

The solution (5) can be treated as the solution of a plate with dimensions $A \times B$ and thickness h with boundary conditions on the two faces $z = 0, h$ from a class of functions represented by Fourier series and the boundary conditions on the faces $x = 0, A$ and $y = 0, B$ which correspond to the selected representation (4) of the initial functions as a trigonometric series. Thus the solution of the homogeneous system which can satisfy the arbitrary boundary conditions on the two opposite sides $z = 0, h$ of the plate is constructed.

3. The partial solution of the nonhomogeneous system

The partial solution which may be used together the MIF cannot be arbitrary. It must match the boundary conditions specified on the initial plane: three unspecified components must equal to zero on the initial plane. A solution all components of which are zero at the initial plane is universal and is suitable for all boundary conditions. Such a solution is

$$u = v = 0, w = -\frac{\rho g (1+\nu)(1-2\nu)}{2E(1-\nu)} z^2,$$
$$\sigma_z = -\rho g z, \sigma_z = \sigma_z = -\frac{\rho g \nu}{1-\nu} z, \tau_{xy} = \tau_{xz} = \tau_{yz} = 0. \quad (6)$$

To use this solution in the MIF algorithm it should expand it into Fourier series on the plane $z = h$ and then add this expansion to the boundary conditions on this plane with the opposite sign.

Note that when analyzing a layered plate the displacements in the solution (6) obtained in the local coordinate system of a layer should be written in the global coordinate system of the entire plate.

4. Numerical results

The analyses of thick square in plan ($A = 3m, B = 3m, h = 1m$) isotropic ($E = 3 \cdot 10^9 Pa, \nu = 0.2, \rho = 2500 kg/m^3$) plate under its own weight is performed.

Figure 1 shows the graphs of the stresses σ_z and σ_x. It is seen that horizontal layers in the lower half of the plate are pressed against each other whereas in the upper part of the plate such compression is not observed. The graph of the stress σ_x corresponds to the plate bended.

The results of analyzing of the three-layered plate ($A = 3m, B = 3m, h = 1m$, $h_1 = 0.375m$, $h_2 = 0.5m$, $h_3 = 0.125m$, $E_1 = 2.2 \cdot 10^9 Pa$, $E_2 = 0.5 \cdot 10^9 Pa$, $E_3 = 2.2 \cdot 10^9 Pa$, $\nu_1 = 0.2$, $\nu_2 = 0.1$, $\nu_3 = 0.2$, $\rho_1 = 2500 kg/m^3$, $\rho_2 = 1500 kg/m^3$, $\rho_3 = 2500 kg/m^3$) is presented in Fig. 2.

5. Conclusion

The general solution (2) of the inhomogeneous system of the differential equations of the theory of elasticity is built. The approach proposed allows analyzing the stress and strain state of thick isotropic plates taking into account their own weight. It should note that analyzing a layered plate with its own weight using the MIF is performed for the first time.

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