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PREDICTION OF SHORT FATIGUE CRACK PROPAGATION ON THE BASE OF NON-LOCAL FRACTURE CRITERION

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Abstract. Models of short fatigue crack propagation, taking into account the non-monotonic crack growth rate and predicting an existence of one or several threshold stress intensity factors, are considered. The models are formulated on the base of Leonov-Panasyuk-Dugdale formalism with using the non-local fracture criterion. A comparison of the obtained results with experimental data are given and discussed.

1. Introduction

The problem of short fatigue cracks has received considerable attention due to inability of linear elastic fracture mechanics for the correct description of short cracks anomalous behavior. The short fatigue cracks demonstrate non-monotonic behavior including acceleration, deceleration to crack arrest, or deceleration followed by acceleration. The long fatigue cracks do not propagate at levels below the threshold stress intensity factor range ΔK_{th}, whereas it is known that short cracks grow below ΔK_{th} [1].

The paper proposes a model of short fatigue crack, which describes the deceleration stage below ΔK_{th} and acceleration stage above ΔK_{th}. The condition of the crack growth is obtained on the base of a non-local fracture criterion [2] in combination with Leonov-Panasyuk-Dugdale crack model [3, 4]. The obtained analytical evaluating the threshold stress intensity factor ΔK_{th} is in a good agreement with experimental data

2. Non-local fracture criteria

The correct analysis of the short fatigue crack behavior leads to a necessity to take into account the microstructure of material. In this case, the elementary act of failure is supposed to cover some representative volume of material (grain, structural element) instead of one material point, and the process of failure is determined by the cumulative stress-strain state of representative volume as a whole.

The non-local failure condition initially was proposed by Wieghardt [5]. The first practical application and revealing of averaging area size dependence on micro-structure of material was done by Neuber [6]. Original physical interpretation and modifications of the criterion were offered by Novozhilov [7]. The application of non-local failure criterion to the analysis of short fatigue cracks propagation in the form of d_*-concept was proposed by Sähn [8] and developed in [9-11].

In general, the non-local measure of stress-strain state $\overline{B}$ is defined by the equation [2, 11]:

The length of the cohesive zone s can be defined from the results of substitution the expression (5) into the non-local condition of crack growth (4):

$$\frac{s}{a}=1-\cos\frac{\pi}{2}\left(\frac{\sigma_0}{q}-\frac{\sigma_Y}{q}\sqrt{\frac{2d_*}{a}}\right). \tag{6}$$

Displacements of crack faces in the case of plane stress state for Leonov-Panasyuk-Dugdale crack are defined by the relation [3, 4] :

$$u_y\big|_{y=0}=\frac{2\sigma_0}{E}\sqrt{a^2-x^2}+\frac{q}{\pi E}\left[(x-l)\Gamma(a,x,l)-(x-l)\Gamma(a,x,-l)-4\sqrt{a^2-x^2}\arccos\frac{l}{a}\right], \tag{7}$$

where $\Gamma(a,x,l)=\ln\dfrac{a^2-xl-\sqrt{(a^2-x^2)(a^2-l^2)}}{a^2-xl+\sqrt{(a^2-x^2)(a^2-l^2)}}$, $l=a-s$.

The crack tip opening displacement δ is defined on the base (7) and (6) as:

$$\delta=2u_y\big|_{\substack{y=0\\x=l}}=-\frac{8q}{\pi E}\left(l\ln\frac{l}{a}-\frac{\pi}{2}\sqrt{a^2-l^2}\frac{\sigma_Y}{q}\sqrt{\frac{2d_*}{a}}\right). \tag{8}$$

Considering macrocrack with $d_*/a<<1$ we obtain from (8) the classical expression for the crack opening $\delta=-\dfrac{8q}{\pi E}l\ln\dfrac{l}{a}$ as in the original model of Leonov-Panasyuk-Dugdale.

4. Description of short crack propagation based on the cohesive zone length Δ*s*

It is assumed that the rate of short fatigue cracks is defined in an analogy with Paris' equation by the power-type dependence from the range of the cohesive zone length Δs:

$$\frac{da}{dN}=C_1|\Delta s|^{m_1}, \tag{9}$$

where $\Delta s=s_{\max}-s_{\min}$, C_1 and m_1 are material constants.

The Fig. 2a shows dependence of the dimensionless rate of crack propagation defined by (9) and (6) on the dimensionless crack length a/d_* for $m_1=2$. There are two characteristic parts of curve: the first part is area of monotonic decreasing crack rate to zero. The zero value corresponds to the threshold stress intensity ΔK_{th}. The second part is area of monotonic increasing crack rate. The same character of diagram is observed in experiments for short fatigue cracks [1].

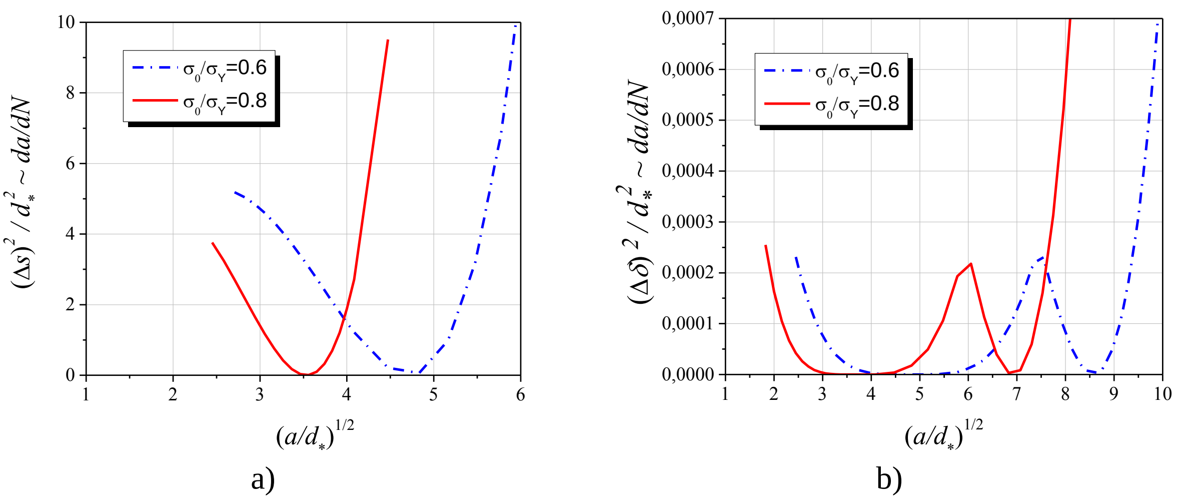

Fig. 2. Dimensionless rate of crack growth vs dimensionless crack length for different crack models a) by equation (9) and b) by equation (11) under two loading levels.

The threshold stress intensity ΔK_{th} is defined from the condition $\Delta s = 0$ (that equivalently to $da/dN = 0$) with help of the expression (6) for the loading case of $R = \sigma_{min}/\sigma_{max} = 0$ as follows:

$$K_{th} = \sigma_Y \sqrt{8\pi d_*} \,. \tag{10}$$

The calculated on the base of (10) the value of $K_{th} = 9.4\,\mathrm{MPa}\sqrt{\mathrm{m}}$ for steel 45 with yields stress $\sigma_Y = 245\,\mathrm{MPa}$ and grain size $d_* = 59$ μm [13] is in a good agreement with observed in experiments $K_{th} = 8\,\mathrm{MPa}\sqrt{\mathrm{m}}$ [14].

5. Description of short crack propagation based on crack tip opening Δδ

It is assumed that the rate of short fatigue cracks is defined by the power-type dependence of crack growth rate on the range of the crack tip opening Δδ:

$$\frac{da}{dN} = C_2 \left|\Delta\delta\right|^{m_2} , \tag{11}$$

where $\Delta\delta = \delta_{max} - \delta_{min}$, C_2 and m_2 are material constants.

The Fig. 2b shows dependence of the dimensionless rate of crack propagation defined by (11) and (8) on the dimensionless crack length a/d_* for $m_2 = 2$. The model (11) predicts two threshold stress intensity factor ΔK_{th} (that is observed in some experiments [1]). The value of minimum ΔK_{th} coincides with the prediction of ΔK_{th} by the model (9).

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