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FRACTURE ASSESSMENT DIAGRAM FOR SOLID WITH CIRCULAR CRACK SUBJECTED TO CONCENTRATED FORCES

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Abstract. Structural criterion of fracture is used to evaluate the ultimate load for solid weakened by circular crack deformed by the action of two point forces applied to the crack faces at its center. Estimation of the risk of failure is based on the two-criterion approach under linear elastic conditions.

It is known that the widely used classical criteria of fracture suggested by Griffith and Irwin do not applicable for short unstable cracks as they lead to unlimited critical load. To overcome this difficulty the structural strength criterion was proposed. For the penny shaped crack embedded in an infinite solid and subjected to axisymmetric loading conditions this criterion is expressed as [1]

$$\frac{2}{2ad+d^2}\int_a^{a+d} \sigma_z(r) r\, dr \le \sigma_c, \tag{1}$$

where a – crack radius, (r,z) – cylindrical coordinates, σ_c – tensile strength limit, d – structural fracture parameter of the dimension of length, so called fracture process zone, σ_z – normal stress at the plane $z=0,\ r>a$.

General solution for the tensile stress ahead of the crack is given in [2, 3]

$$\sigma_z(r,0) = \frac{2}{\pi\sqrt{r^2-a^2}} \int_0^a \frac{\rho\sqrt{a^2-\rho^2}}{r^2-\rho^2} p(\rho)\, d\rho. \tag{2}$$

The pressure on crack faces is given by

$$p(\rho) = \frac{P}{2\pi\rho}\delta(\rho), \tag{3}$$

where P – concentrated force, $\delta(\rho)$ – Dirac delta-function. Substituting for $p(\rho)$ from equation (3) into equation (2), we obtain

$$\sigma_z(r,0) = \frac{Pa}{\pi^2 r^2\sqrt{r^2-a^2}}, \quad r>a.$$

Substituting into (1) and evaluating the integral, we find (Fig.1, curve *1*)

$$\frac{p_*}{\sigma_c} = \frac{\pi(1+\eta)}{2(1-\eta)\arccos\eta}, \quad 0 \le \eta \le 1, \tag{4}$$

where $\eta = a/(a+d)$ – dimensionless crack radius, p_* – critical load (stress) $p_* = P_*/(\pi d^2)$. As $\eta \to 0$ (infinitely small crack) we have physically unaffected limit transition to the strength of defect-free material: $p_* / \sigma_c \to 1$. Elementary fracture cell in this case is found to be the circle of radius d. In the case of infinitely large crack ($\eta \to 1$) for the stable cracks it follows $p_* / \sigma_c \to \infty$, so the fracture force increases with crack growth.

Stress intensity factor (SIF) is defined as

$$K_I = \lim_{r \to a} \sqrt{2\pi(r-a)}\ \sigma_z(r,0),$$

so that SIF is $K_I = P / (\pi a)^{3/2}$. Applying the Irwin criterion $K_I \le K_{Ic}$, where K_{Ic} is critical SIF, we have $P_* \to \infty$ as $a \to 0$. Unlimited strength of the solid without a crack demonstrates an insufficiency of Irwin criterion for small (short) cracks. Assuming that fracture occurs if the equality $K_I = K_{Ic}$ is satisfied for Griffith crack (plain strain problem) it can be shown that [4]: $d = (2/\pi)\left(K_{Ic}^{\infty}/\sigma_c\right)^2$, where $K_{Ic}^{\infty} = K_{Ic}\big|_{\eta \to 1}$ is so called «apparent» fracture toughness, or critical SIF for infinitely large crack.

The approximate formula for the tensile stress ahead of the crack has the form $\sigma_z \approx K_I \big/ \sqrt{2\pi(r-a)}$. Substitution of this asymptotic approximation into criterion (1) yields the following expression for SIF (Fig. 1, curve 2)

$$\frac{K_{Ic}}{K_{Ic}^{\infty}} = \frac{(1+\eta)\sqrt{1-\eta}}{\eta\sqrt{2\eta}\,\arccos\eta}. \qquad (5)$$

Equations (4) and (5) in common represent the two-criterion fracture assessment diagram (Fig. 1, curve *3*) which is convenient for the experimental data analysis [4-7].

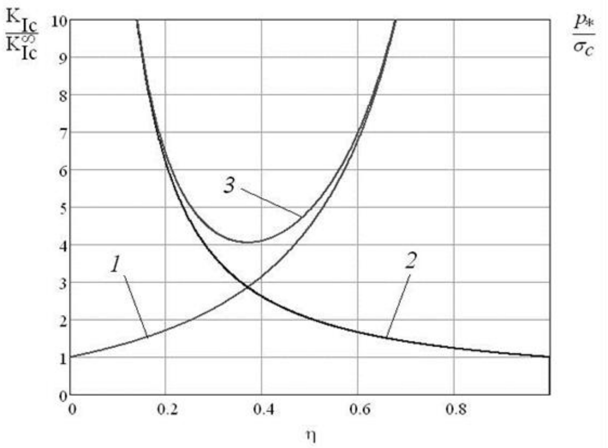

Fig. 1. Fracture assessement diagram.

Critical load according to Griffith criterion may be written through the nondimensional parameter η

$$\frac{p_*}{\sigma_c} = \frac{\pi}{\sqrt{2}}\left(\frac{\eta}{1-\eta}\right)^{3/2},\quad 0 \le \eta \le 1. \qquad (6)$$

Results of calculations using formulas (4) and (6) are presented in Fig. 2. The relative difference in the value of critical load does not exceed 10 % if crack diameter is $2a \ge 18d$.

The Griffith energy balance for an incremental increase in the crack radius a under equilibrium conditions can be expressed in the following way

$$\frac{\partial}{\partial a}(W+U)=0, \quad (7)$$

where W – potential energy supplied by the internal strain energy and external forces, U – work required to create new surfaces. The potential energy is given by [8]

$$W=\frac{2\pi^2(1-\nu^2)}{E}\int_0^a[g(t)]^2\,dt, \quad g(t)=\frac{2}{\pi}\int_0^t\frac{rp(r)}{\sqrt{t^2-r^2}}\,dr.$$

where ν – Poisson's ratio, E – elasticity modulus.

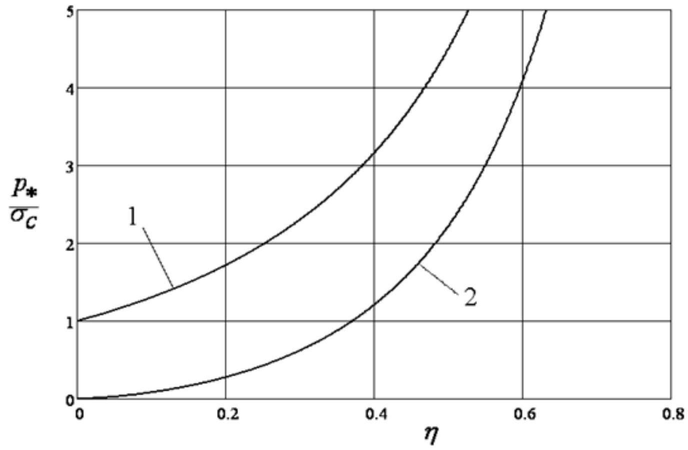

Fig. 2. Critical load versus crack size:
1 – structural criterion (4), *2* – Irwin criterion (6).

For the load (3) we have $g(t)=P/(\pi^2 t)$. Thus,

$$W=\frac{2(1-\nu^2)P^2}{\pi^2 Ea}.$$

Since the formation of a crack requires the creation of two surfaces U becomes

$$U=2\gamma\cdot\pi a^2,$$

where γ is the surface energy of the material.

From the equation (7) we deduce critical force according to Griffith criterion

$$P_*=\sqrt{\frac{2\gamma E}{1-\nu^2}(\pi a)^3}. \quad (8)$$

Fracture toughness is related to constant γ by equation $K_{Ic}=\sqrt{2\gamma E/(1-\nu^2)}$, implying the similarity between Griffith and Irwin approaches in linear fracture mechanics (plain strain). Equation (8) may be expressed as $P_*=K_{Ic}(\pi a)^{3/2}$, which immediately follows from the Irwin criterion.

On recalling the plain strain analogy the critical load may be put in the form [9]

$$P_*=\sqrt{\frac{2\gamma E}{1-\nu^2}\pi l}, \quad (9)$$

where $2l$ is length of the crack and the plate thickness is set equal to unity.

It follows from (8) and (9) that critical force tends to zero as $a,l\to 0$. (It should be noted, that in experiments with glass samples equation (9) has been used for the cracks with a minimal length of $2l \sim 5\ cm$ [9]). Structural criterion does give the finite value of critical load as $a \to 0$: $P_* = \sigma_c \cdot \pi d^2$. The latter result is in agreement with the tensile strength of intact medium.

References

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