HEAT DISTRIBUTION WITH STRUCTURE IN SOLID STATES

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Abstract. The concept of the heat stream structure was introduced. It is considered that heat transfer is carried out on N channels each of which has the own heat distribution speed and frequency. The equation describing heat transfer process in a one-dimensional crystal was offered.

1. Introduction

Classical methods of the heat conductivity theory are widely known [1-3]. Nevertheless, there are some problems connected with the decision of non-stationary heat conductivity and thermal stability tasks, which lead to difference between the theoretical and experimental results, for example, at the description of heat conductivity processes in metal constructions, multilayered materials, etc.

Heat distribution in the majority of materials is described by the Fourier's law according to which the thermal stream is proportional to the temperature gradient [4]. The Fourier's law describes heat transfer at the macroscopic level for systems, which are near thermodynamic balance very well. However, the heat distribution, for example, in a one-dimensional crystal is hard to explain by means of the Fourier's law. Besides the Fourier's law describes instant heat distribution in a system at the expense of what on small time intervals deviations are observed.

J. Maxwell [5] was the first who suggested using the Fourier's equation with a damper. Later, V.A. Fok [6] for the first time considered the hyperbolic equation for energy transfer. For quickly proceeding thermal processes, for example heatstrokes, the wave theory version was offered by G.A. Genyev [7]. Cattaneo [8] received the system of equations empirically taking into account delays. However, even such system of hyperbolic equations gives divergences with experimental results.

As a rule, heat distribution is carried out in several mechanisms even in the homogeneous environment. The transferable part of internal energy is considered as a nonequilibrium phonon gas, diffused generally on different mechanisms. So there are not less than two mechanisms of heat distribution in solid states: electronic and hole heat conductivity. It is obviously observed when the light energy passes through glass protections. The part of electromagnetic energy according to the exponential Buger's law is absorbed.

In the work, the new method of the solution of the heat conductivity problem based on probabilistic methods is offered.

2. The description of heat distribution process with structure

In modern classical mechanics there are theories, for example the Cosserat mechanics, where it is considered that the mass point has internal structure at the expense of that it possesses

additional degrees of freedom. By analogy with the mass point, we considered that the thermal stream also has structure.

It was accepted that thermal energy of the stream extends on N various mechanisms, which were called heat distribution channels. On each channel the thermal stream extends with a certain speed c_n. Quantity of heat on each channel also defines a thermal stream structure.

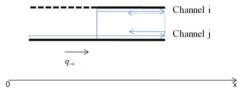


Fig. 1. Thermal stream distribution.

The heat conductivity process was considered on the example of the one-dimension task (see fig. 1). Heat transfer is carried out along axis of X. Heat current density is the local characteristic: $q_{\vec{n}}$ and $q_{\vec{n}}$, extending on and against axis of X respectively on N distribution channels (all 2N-sizes). Then the thermal stream density in the whole system is equal:

$$q(x,t) = \sum_{n=1}^{N} q_n(x,t) = \sum_{n=1}^{N} q_{\vec{n}}(x,t) - q_{\vec{n}}(x,t).$$
 (1)

Transferable part of internal energy U (x, t) equals the sum of energy on each channel:
$$U(x,t) = \sum_{n=1}^{N} [u_{\vec{n}}(x,t) + u_{\vec{n}}(x,t)] = \sum_{n=1}^{N} \frac{1}{c_n} [q_{\vec{n}}(x,t) + q_{\vec{n}}(x,t)], \qquad (2)$$

where c_n – the heat distribution speed on the channel number n.

As energy transfer is irreversible and nonequilibrium process, it is necessary to take into account local temperatures on each channel more precisely. Therefore, at non-stationary heat transfer two physical processes take place: the first is energy transfer from more heated parts to less heated, the second is the energy exchange between various distribution channels, i.e. relaxation process.

It is enough to write down system of 2N continuity equations for the energy linear density on each channel according to the law of energy conservation for the conclusion of the heat transfer equations generally:

$$\frac{d\mathbf{u}_{\vec{\mathbf{n}}}}{dt} = -\frac{\partial \mathbf{q}_{\vec{\mathbf{n}}}}{\partial \mathbf{x}} - \sum_{\mathbf{m}=1}^{\mathbf{N}} [\mathbf{a}_{\overrightarrow{\mathbf{n}}\overrightarrow{\mathbf{m}}} + \mathbf{b}_{\overrightarrow{\mathbf{n}}\overrightarrow{\mathbf{m}}}] \mathbf{q}_{\vec{\mathbf{n}}} + \sum_{\mathbf{m}=1}^{\mathbf{N}} \mathbf{b}_{\overrightarrow{\mathbf{n}}\overrightarrow{\mathbf{m}}} \mathbf{q}_{\overrightarrow{\mathbf{m}}} + \sum_{\mathbf{m}=1}^{\mathbf{N}} \mathbf{a}_{\overleftarrow{\mathbf{n}}\overrightarrow{\mathbf{m}}} \mathbf{q}_{\overleftarrow{\mathbf{m}}}, \tag{3}$$

$$\frac{\mathrm{d}\mathbf{u}_{\overline{\mathbf{n}}}}{\mathrm{d}\mathbf{t}} = \frac{\partial \mathbf{q}_{\overline{\mathbf{n}}}}{\partial \mathbf{x}} - \sum_{\mathbf{m}=1}^{N} [\mathbf{a}_{\overline{\mathbf{n}}\overline{\mathbf{m}}} + \mathbf{b}_{\overline{\mathbf{n}}\overline{\mathbf{m}}}] \mathbf{q}_{\overline{\mathbf{n}}} + \sum_{\mathbf{m}=1}^{N} \mathbf{b}_{\overline{\mathbf{n}}\overline{\mathbf{m}}} \mathbf{q}_{\overline{\mathbf{m}}} + \sum_{\mathbf{m}=1}^{N} \mathbf{a}_{\overline{\mathbf{n}}\overline{\mathbf{m}}} \mathbf{q}_{\overline{\mathbf{m}}}. \tag{4}$$

Here a_{nm} is a reflection speed matrix, i.e. a part of the thermal energy which passed from the channel number n on the channel number m with the distribution direction change, b_{nm} is a transition speed matrix from the channel with number n on the channel number m without distribution direction change and $b_{nn} = 0$.

The equations of the system express conservation law of energy transfer taking into account an internal exchange between channels. The received system of equations permits to describe heat transfer processes.

3. Conclusions

The received system of equations make possible to do the following conclusions:

• In the non-stationary decision heat exchange, heat streams and a field of temperatures can significantly differ from settlement with one stationary coefficient of heat conductivity.

- For thin layers of materials dependence of integrated stationary coefficient of heat conductivity on sample thickness is possible.
- Character of the decision of system depends on structure of the falling heat stream, i.e. more correct task of boundary conditions demands.
- Expediently for the main materials experimentally to investigate their thermal characteristics in the dynamic mode of heat transfer.

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