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ALGORITHM OF COMBINED METHOD OF 3D ANALYSIS FOR THE BOUNDARY PROBLEMS IN INFINITE MEDIUM

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Abstract. Algorithm of the 3D analysis developed for solution of boundary problem by combined method based on incorporating the FEM and Somigliana's integral formula is considered. The algorithm is modified for the case of inhomogeneous medium. Efficiency of software implementations of both algorithms has been tested.

1. Introduction

In many cases of practical importance the analysis of structures with regard to their interaction with infinite base or the surrounding elastic medium is based on numerical solution of three-dimensional exterior boundary problem. Using of the algorithms of the finite element method (FEM) or boundary element method (BEM) has some peculiar properties in infinite domain [1-3]. The FEM analysis is performed in reduced bounded domain of finite size, or special "infinite" elements are considered. Some problems inevitably arise for this approach, such as validation of this size and choice of boundary conditions on the exterior boundary in the first case, or objectionable system of equations to solve in the second case. For the BEM approach unhomogeneous medium is inconvenient, as well as domains with several holes, although these peculiarities take place in practice.

These peculiarities of the boundary problem in infinite domain cause some computational difficulties and increase in computing cost. Various algorithms offer to avoid such problems at the expense of combining different methods and equations. An algorithm dealt with matching of FEM and Somigliana's integral formula – combined method (CM) – is considered.

2. Algorithm of combined method (CM)

Algorithm of 3D analysis for exterior boundary problem by combined method (CM) is considered.

Let's Navier equilibrium equations and boundary conditions in a space region Ω bounded by a surface S are satisfied for an displacement vector $u(\xi)$, $\xi \in \Omega$. Boundary conditions are

$$t_i(\xi) = \sigma_{ij}(\xi) n_i(\xi) = p_i(\xi), \quad \xi \in S_1;$$

$$u_i(\xi) = u_{iS}(\xi), \quad \xi \in S_2; \; S_1 \bigcup S_2 = S, \tag{1}$$

where $n_i(\xi)$ are components of the unit outward normal on the surface S, $\sigma_{ij}(\xi)$ are components of stress tensor, $p_i(\xi)$ and $u_{iS}(\xi)$ are specified components of force and displacement vectors.

A sub-region Ω_0 bounded by a surface S_0 is selected in the semi-infinite space region Ω (see Fig. 1). An iteration process can be performed as follows.

Generally, having (k-1)th approximation

$$u_i(\xi) = u_i^{(k-1)}, \quad \xi \in S_0, \tag{2}$$

we can find k-th approximation $u_i^{(k)}(\xi), \xi \in S_0$ by means of Somigliana's integral formula

$$u_j(\xi) = \int_S (t_i(\eta) G_{ij}(\eta,\xi) - F_{ij}(\eta,\xi) u_i(\eta)) dS(\eta), \tag{3}$$

using $u_i^{(k)}(\xi), \xi \in S_1$ and $t_i^{(k)}(\xi), \xi \in S_2$ determined by the FEM analysis of boundary problem in the sub-region Ω_0 with boundary conditions (1, 2).

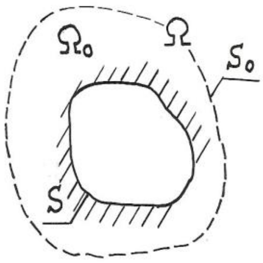

Fig. 1. Computational domain selection in a space region Ω.

The process is to be completed when deficiency of successive approximations $u_i^{(k)}(\xi)$ and $u_i^{(k+1)}(\xi)$ $(\xi \in S)$ achieves the required accuracy ε

$$\max_{\xi \in S} \left| \frac{u_i^{(k+1)}(\xi) - u_i^{(k)}(\xi)}{u_i^{(k)}(\xi)} \right| < \varepsilon, \quad i = 1, 2, 3. \tag{4}$$

To calculate the integral in Eq. 1 for the first term containing $t_i(\eta), \eta \in S$ such approach is used. Nodal forces P_i in the nodes ξ^l of a finite element ΔS incoming in the right-hand side of the FEM equations can be treated as some integral characteristic of surface forces $t_i(\eta)$:

$$\sum_{l=1}^{3} P_i(\xi^l) = \int_{\Delta S} t_i(\xi) dS. \tag{5}$$

So such an approximation formula is used:

$$\int_{\Delta S} t_i(\xi) G_{ij}(\xi,\eta) dS(\xi) \approx \sum_{i=1}^{3} P_i(\xi^l) G_{ij}(\xi^l,\eta). \tag{6}$$

Strictly speaking, this formula (6) is not a quadrature one but it provides reasonable accuracy at the cost of using of the FEM solution as an alternative of numerical differentiation.

Computer numerical implementation of this algorithm has been tested by model problems analysis. There were exterior boundary problems concerned with self-balanced and non-self-balanced external loads applied to a sphere-shaped concavity, such as uniform pressure, two self-balanced point forces and one point force applied symmetrically or non-symmetrically. Also, the FEM analysis of all these problems was performed to compare results. Used meshes had different number of finite elements. Generally, accuracy of the CM solution was the same as accuracy of the FEM one though number of elements of the CM meshes was far less than those for the FEM. Displacements determined by the CM for non-self-balanced external loads include rigid-body components unlike those determined by the FEM. This feature of the CM caused better accuracy of its results in such cases.

Also, Boussinesq problem, i.e. point force applied to a half-subspace, was considered. Here the region Ω has infinite boundary surface S, it means integration over the plane in (3).

The infinite region had been truncated to get a finite computational region Ω_0 (semi-sphere) with circle boundary S_*. Besides radius of sphere for the CM analysis was 5 times less than one for the FEM analysis. The accuracy of the CM and the FEM could be compared because of availability an exact solution. Vertical displacement error was 4-20% for the CM and 30-65% for the FEM (last values for points of boundary surface) unless an additional error of the CM caused by the truncation of the range of integration in (3). Solution of Boussinesq problem is often used for stress-strain analysis of constructions in view of foundation displacements and subsidence. Therefore using of the CM is worthwhile for problems like that.

3. Double-boundary algorithm of combined method (DCM) for inhomogeneous medium

The algorithm of combined method has been modified for inhomogeneous medium on the assumption of that the inhomogeneous region having a substantial influence on the strain-stress state of the considered structure can be bounded by a closed surface L. The remaining region Ω_L can be considered as a homogeneous one. A surface S_0 is selected in the region Ω_L (see Fig. 2) and closed sub-region Ω_0 bounded by surfaces S and S_0 is considered.

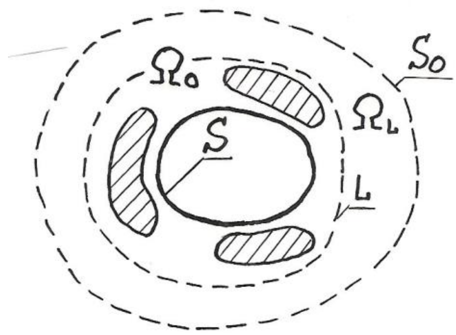

Fig. 2. Computational domain selection in an inhomogeneous region.

The first step in process is establishing of boundary condition $u_i^{(0)}$ at the surface S_0 (usually $u_i^{(0)} = 0$). Than boundary problem in the sub-region Ω_0 can be solved by FEM to define boundary condition (1) on the surface L. The domain of integration in (3) is to be a boundary of homogeneous region, hence it will be the surface L instead of the surface S. Than boundary problem in the sub-region Ω_0 is solved by FEM to define the second approximation $u_i^{(2)}$ on the surface L and so far. It is a double-boundary algorithm of combined method (DCM).

Values of stress and strain using in (3) in the modified algorithm in question are calculated by the FEM while in the original algorithm part of them are defined by (1). Thus partition of computational domain into finite elements is to be more careful to decrease an error.

Computer numerical implementation of the DCM algorithm has been tested by stress-strain analysis of sphere-shaped concavity surrounded by a spherical layer with physical properties different from those in the rest region under the internal uniform pressure. Dimension of the computational domain for the CM analysis was 3 times less than one for the FEM analysis. The accuracy of the CM and the FEM was comparable. Results of both methods are partly underestimated due to insufficient dimension of the computational domain.

Underground non-reinforced caverns subjected to internal pressure or any otherwise loads are often used in construction and mining as gas or oil storage or for waste dumping.

Large-sized cavern is often located in an inhomogeneous medium because of complicated structure of rock formation. Stability analysis of such a cavern is to be made with regard to layered rock structure. Also for land surface subsidence predication and risk analysis of safe upkeep of buildings it's necessary to estimate rock mass deformation caused by underground caverns. That is why ground subsidence and horizontal displacement under building foundation are to be calculated [4].

4. Summary

Based on the results of the analysis performed the practicality of the method under study has been proved. Accuracy of the results obtained is generally the same as accuracy of the FEM' results even though a truncated computational domain and number of finite elements are comparatively inconsiderable.

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