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EFFECT OF IMPERFECT BONDING ON THE AXISYMMETRIC STRESSES IN BURIED THICK ORTHOTROPIC CYLINDRICAL SHELLS DUE TO INCIDENT LONGITUDINAL WAVE (P-WAVE)

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Abstract. This paper is concerned with the estimation of axisymmetric stresses induced in an imperfectly bonded thick orthotropic buried pipeline due to seismic excitation (p-wave) travelling in the surrounding medium. The shell has been assumed to be imperfectly bonded to the surrounding medium. Particular attention has been focused on the analysis of the effect of stiffness and damping of the surrounding ground on the axial and hoop stresses. A thick shell theory formulation has been used and only axisymmetric behaviour of the shell has been investigated. The relative influence of the variation of orthotropic parameters on the stresses in the shell has also been studied for different angles of incidence of the p-wave. Results have been obtained for different soil conditions - hard (rocky), medium hard and soft. It is observed that both the stiffness and damping parameters in axial as well as radial direction reduce the axial stresses of the shell, significantly. On the other hand, the stiffness and damping parameters in radial direction reduce the hoop stresses of the shell significantly, while those in the axial direction give no variation in the hoop stresses.

Keywords: thick orthotropic cylindrical shell; seismic excitation; imperfect bonding.

1. Introduction

A characteristic feature that differentiates pipeline/lifeline from other structures is that it extends parallel to the ground surface over a distance, which is larger than its other dimensions. The analysis and design of pipelines which are under earthquake induced motion is different from that of buildings, railroads and other structures installed above the ground. In case of above ground structures, it is customary to assume a coherent ground motion over the entire foundation plane. In contrast, incoherent ground motion is of considerable importance for the seismic response of buried pipelines. These reports lead to the conclusion that the damages can be divided into three different categories:

1. Landslide, tectonic upliftment and liquefaction of soil etc.
2. Damage due to surface faulting, and
3. Damages due to seismic ground shaking

The work presented here in this paper is mainly concerned with the response of pipelines due to seismic ground shaking only. Hence, only those articles and papers have been reviewed which are related to the failure of pipelines due to seismic excitation. For the seismic/dynamic response analysis of the pipelines, various methods have been proposed in the past .An account

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of these works are available in some review articles [1-6]. In majority of the works beam theory was used to model the pipeline. Ariman et.al [7, 8] noted that two of most frequently observed failure modes in buried pipelines were buckling and fracture which couldn't be explained by using simple beam theory. To explain the nature of failures, Muleski et al. [9] presented a shell model and used the Flugge's shell theory, modelling the pipeline as thin isotropic elastic cylindrical shell in viscoelastic medium. Dutta et al. [10, 11] have investigated the axisymmetric dynamic response of a buried pipe due to an incident compressional wave. Pipe has been modelled as a thin cylindrical shell of linear homogeneous isotropic material embedded in a linearly isotropic and homogeneous elastic medium of infinite extent. All the work reported however, were concerned with isotropic and homogeneous shells/ pipelines only. During the second U.S. National Conference (1979) on Earthquake engineering, Ariman and Muleski [12] emphasized the need for analyzing the dynamic behaviour of pipes made of orthotropic materials. They popular in pipeline systems and were slowly replacing cast iron and steel. Cole et al. [13] have performed a finite element analysis of buried RPM pipes. In the past, Upadhyay et al. [14, 15] have presented a number of papers concerned with the dynamic response of buried orthotropic shells or pipes. They studied the axisymmetric dynamic response of buried orthotropic cylindrical shells due to p-wave and s-wave excitation and concluded that the shell response is significantly influenced by orthotropy parameters. In these papers, the shell is assumed to be perfectly bonded to the surrounding medium of infinite extent. But in practice, this assumption is never true. Rakesh et al. [16-21]studied the non axisymmetric seismic behaviour of pipeline, assuming an imperfect bond with the continuum. However, in these papers the shell was assumed to be made of isotropic materials. Keeping in mind, the advantages of using orthotropic materials, Dwivedi et al. [22] presented papers emphasizing the effect of imperfect bond on the axisymmetric dynamic response of buried shells made up of orthotropic materials subjected to p-wave and s-wave excitation. In these papers, the author has applied a thick shell theory formulation including the effects of shear deformation and rotary inertia. Further Singh et al. [23] compared the effect of imperfect bond on the dynamic response of buried orthotropic shells as obtained from thick and thin shell theories. Lee et al. [24] have studied the response characteristics of strains in a buried pipeline section, axial relative displacement, and transverse relative displacement. Chen et al. [25] have studied the deformations induced in buried pipelines under permafrost regions. In both of these papers, a finite element model was used. But, here a mathematical approach based on equations and formulations has been presented.

It is widely accepted that the presence of fluid significantly affect the dynamic response of the shell. Therefore, a fluid filled shell must be investigated for the safer design of the pipeline/lifeline. Keeping this in mind Singh and Dwivedi et al. [26] presented papers to describe the dynamic response of the fluid filled buried pipelines. Dwivedi et. al. [27-30] analysis the dynamic response of buried pipeline under anisotropic soil conditions.

However, all these papers are related with the axial and radial displacements produced in the shell due to seismic excitation and no attention has been given to the stresses induced in the shell. But as we know in order to predict the failure of the given shell material, the knowledge of axial and hoop stresses induced in the shell is necessary. Realizing this, Rao et al. [31] have studied the axisymmetric stresses induced in buried thin orthotropic cylindrical shells due to p-wave loading. But in this paper, the shell was assumed to be perfectly bonded to the surrounding medium.

Therefore, the present work is mainly concerned with study of the effect of imperfect bonding on the axial and hoop stresses induced in thick orthotropic cylindrical shells excited by p-wave. The variation in axial and hoop stresses when the pipelines are buried in different soil conditions and under different angles of incidence has also been studied. It is found that, both the stiffness and damping parameters in axial as well as radial direction reduce the axial stresses of the shell, significantly. On the other hand, the stiffness and damping parameters in radial direction reduce the hoop stresses of the shell significantly while those in the axial direction give a slight increase in the hoop stresses.

2. Formulations and Governing Equations

2.1. Equations of motion of the shell. In the theory of shells, it is customary to employ a coordinate system formed by two axes in the middle surface of the shell and the third axis normal to the middle surface. So, a cylindrical polar coordinate system (r,θ,x) is defined (Fig. 1) such that x coincides with the axis of the shell and, in addition, z is measured normal to the shell middle surface

$$z=r,\quad \frac{-h}{2}\leq z\leq\frac{h}{2} \tag{1}$$

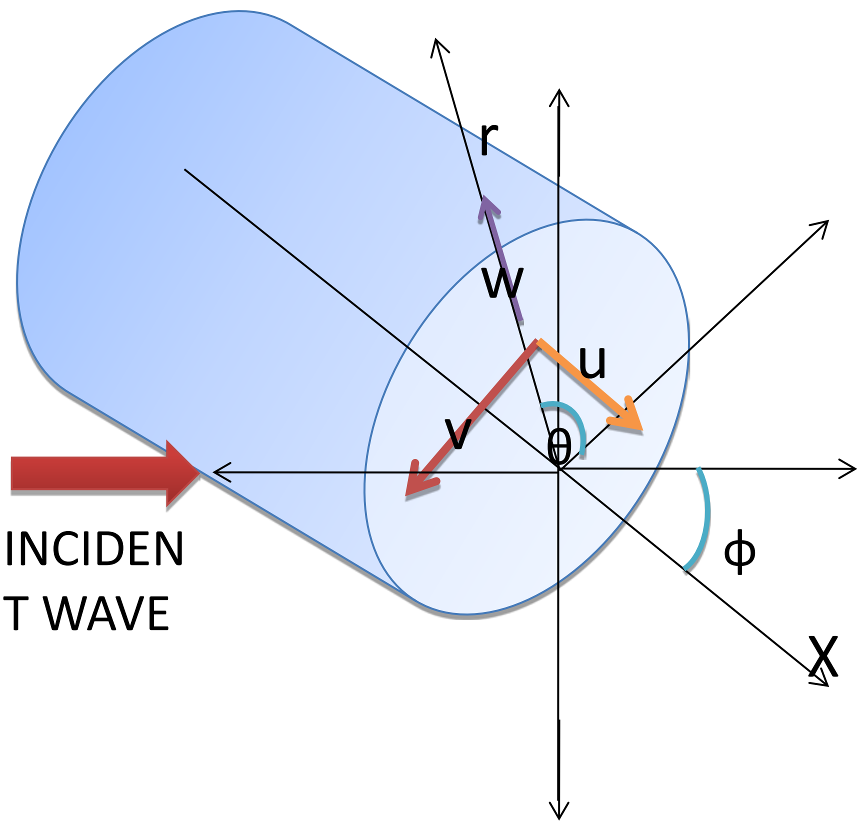

Fig. 1. Geometry of the problem.

Equations governing the axisymmetric motion of a thick orthotropic cylindrical shell may be written as:

$$\left[-\frac{1}{R^2}\left(E_\theta+\frac{D_\theta}{R^2}\right)+G_x\frac{\partial^2}{\partial x^2}-\rho'\frac{\partial^2}{\partial t^2}\right]w+\left[G_x\frac{\partial}{\partial x}\right]\psi_x-\left[\frac{E_\nu}{R}\frac{\partial}{\partial x}\right]u+p_1^*=0 \tag{2}$$

$$\left[-G_x\frac{\partial}{\partial x}\right]W+\left[D_x\frac{\partial^2}{\partial x^2}-G_xI'\frac{\partial^2}{\partial t^2}\right]\psi_x+\left[\frac{D_x}{R}\frac{\partial^2}{\partial x^2}-\frac{I'}{R}\frac{\partial^2}{\partial t^2}\right]u+p_2^*=0 \tag{3}$$

$$\left[\frac{E_\nu}{R}\frac{\partial}{\partial x}-\frac{D_x}{R}\frac{\partial}{\partial x}\right]w+\left[\frac{D_x}{R}\frac{\partial^2}{\partial x^2}-\frac{I'}{R}\frac{\partial^2}{\partial t^2}\right]\psi_x+\left[E_x.\frac{\partial^2}{\partial x^2}-\rho'\frac{\partial^2}{\partial t^2}\right]u+p_3^*=0, \tag{4}$$

in which, t is time; u and w are, respectively ,the displacement components of the shell middle surface in the axial and radial directions. The stiffness parameters and other constants appearing in equations (1-3) are:

$$E_x=hE_{x1},\ D_x=IE_{x1},E_\theta=hE_{\theta1},D_\theta=\ IE_{\theta1},\rho'=h\rho_0,I'=\rho_0 I,I=\frac{h^3}{12},$$

$$E_\nu=hE_{\nu1},k=\frac{\pi}{\sqrt{12}},$$

where ρ_0 is the mass density of the shell material, and $E_{x1}, E_{\theta 1}, E_{v1}$ and G_{x1} are the independent elastic moduli of the shell material and k is the shear correction factor.

The stress –strain relation of the shell material is given by:

$$\sigma_{xx}=E_{x1}\,\varepsilon_{xx}+E_{v1}\,\varepsilon_{\theta\theta}$$
$$\sigma_{\theta\theta}=E_{v1}\,\varepsilon_{xx}+E_{\theta 1}\,\varepsilon_{\theta\theta}$$
$$\sigma_{rx}=G_{x1}(2\,\varepsilon_{rx}) \tag{5}$$

The traction components p_i^* appearing in equations (2-4) are given by:

$$p_1^*=\left(1+\frac{z}{R}\right)\sigma_{rr}\Big|_{h/2}^{-h/2} \tag{6}$$

$$p_2^*=Z\left(1+\frac{z}{R}\right)\sigma_{rx}\Big|_{h/2}^{-h/2} \tag{7}$$

$$p_3^*=\left(1+\frac{z}{R}\right)\sigma_{rx}\Big|_{h/2}^{-h/2}, \tag{8}$$

where σ_{ij} denotes the stresses with their usual meanings and traction components are the stresses σ_{ij} at $z=\pm h/2$.

2.2. General Equations of motion in the medium. For any disturbance propagating in the medium, displacement $\underset{\sim}{d}(r,\theta,x,t)$, at any point satisfies the elastic equation of motion:

$$c_1^2\,\nabla(\nabla.d)-c_2^2\,\nabla\wedge\nabla\wedge d=\frac{\partial^2 d}{\partial t^2}, \tag{9}$$

where

$$c_1=\sqrt{\frac{\lambda+2\mu}{\rho_m}}\ \text{and}\ c_2=\sqrt{\frac{\mu}{\rho_m}}$$

are respectively the longitudinal and shear wave speeds, which depend upon Lame’s constants λ and μ, and density of the medium.

$$d=\nabla\phi+\nabla\overline{\psi}$$

Substituting this representation of d, we have:

$$\Delta\left[(\lambda+2u)\nabla^2\phi-\rho_m\frac{\partial^2\phi}{\partial t^2}\right]+\left[\mu\nabla^2\overline{\psi}-\rho_m\frac{\partial^2\overline{\psi}}{\partial t^2}\right] \tag{10}$$

where ϕ and $\overline{\psi}$ are the scalar and vector potentials respectively .

First bracket of equation, when equated to zero furnishes the scalar wave equation:

$$\nabla^2\phi=\frac{1}{c_1^2}\frac{\partial^2}{\partial t^2} \tag{11}$$

and the second bracket, similarly, yields the vector wave equation as:

$$\mu\nabla^2\overline{\psi}-\rho_m\frac{\partial^2\overline{\psi}}{\partial t^2}=0 \tag{12}$$

When the vector potential $\overline{\psi}$ is expressed in terms of its components in the cylindrical polar co-ordinate system, equation (10) gives three equations as:

$$\nabla^2\psi_r-\frac{\psi_r}{r^2}-\frac{2}{r^2}\frac{\partial\psi_\theta}{\partial\theta}=\frac{1}{c_2^2}\frac{\partial^2\psi_r}{\partial t^2} \tag{13}$$

$$\nabla^2\psi_\theta-\frac{\psi_\theta}{r^2}+\frac{2}{r^2}\frac{\partial\psi_r}{\partial\theta}=\frac{1}{c_2^2}\frac{\partial^2\psi_\theta}{\partial t^2} \tag{14}$$

$$\nabla^2\psi_x=\frac{1}{c_2^2}\frac{\partial^2\psi_x}{\partial t^2}, \tag{15}$$

where ψ_r, ψ_θ and ψ_x are respectively, the radial, tangential and axial component of $\overline{\psi}$.

In view of equation (12), the components of d in r, θ and x directions are written as:

$$d_r = \frac{\partial \phi}{\partial r} + \frac{1}{r}\frac{\partial \psi_x}{\partial \theta} - \frac{\partial \psi_\theta}{\partial x} \tag{16}$$

$$d_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} + \frac{\partial \psi_r}{\partial x} - \frac{\partial \psi_x}{\partial r} \tag{17}$$

$$d_x = \frac{\partial \phi}{\partial x} + \frac{1}{r}\frac{\partial(\psi_\theta . r)}{\partial r} - \frac{1}{r}\frac{\partial \psi_r}{\partial \theta} \tag{18}$$

However if the motion is assumed to be axisymmetric then equations (11 –13) reduce to the following equations:

$$\nabla^2 \psi_r - \frac{\psi_r}{r^2} = \frac{1}{c_2^2}\frac{\partial^2 \psi_r}{\partial t^2} \tag{19}$$

$$\nabla^2 \psi_\theta - \frac{\psi_\theta}{r^2} = \frac{1}{c_2^2}\frac{\partial^2 \psi_\theta}{\partial t^2} \tag{20}$$

$$\nabla^2 \psi_x = \frac{1}{c_2^2}\frac{\partial^2 \psi_x}{\partial t^2} \tag{21}$$

Accordingly, the displacement components given by equations (13-15), for axisymmetric case reduce to the following equations:

$$d_r = \frac{\partial \phi}{\partial r} - \frac{\partial \psi_\theta}{\partial x} \tag{22}$$

$$d_\theta = 0 \tag{23}$$

$$d_x = \frac{\partial \phi}{\partial x} + \frac{1}{r}\frac{\partial(\psi_\theta . r)}{\partial r} \tag{24}$$

2.3. Nature of Excitation. The excitation on the shell is taken in the form of a longitudinal wave of wavelength $\Lambda = \frac{2\pi}{\zeta}$ and speed $c = \frac{\omega}{\zeta}$ moving along the axis of the shell in the surrounding medium. In case of a plane longitudinal wave moving at an angle Φ with the shell axis, c becomes the apparent wave speed of such a wave along the shell axis given by $c = c_1 / \cos\Phi$.

2.4. Solution of wave equation. The first step in solution of wave equation would be to get the values of ϕ and ψ_θ by solving the differential equations (9) and (17), respectively, and substituting these values into equations (19) and (21) to get displacement components. Since we are looking for the solution for an axisymmetric case, ϕ and ψ_θ will be functions of r and x only. Further, because the nature of excitation is a longitudinal wave travelling with speed c along the axis of the shell, for a steady state solution, ϕ and ψ_θ can be assumed in the form:

$$\phi = f(r)e^{i\zeta(x-ct)} \tag{25}$$

$$\psi_\theta = g(r)e^{i\zeta(x-ct)} \tag{26}$$

When we substitute this form of ϕ and ψ_θ in the partial differential equations (9) and (17), these equations reduce to ordinary differential equations.

$$\frac{d^2 f}{dr^2} + \frac{1}{r}\frac{df}{dr} - \left(\zeta^2 - k_1^2\right) f(r) = 0 \tag{27}$$

$$\frac{d^2 g}{dr^2} + \frac{1}{r}\frac{dg}{dr} - \left(\left(\zeta^2 - k_2^2\right) + \frac{1}{r^2}\right) f(r) = 0, \tag{28}$$

where $k_1^2 = \left(\frac{\zeta c}{c_1}\right)^2$ and $k_2^2 = \left(\frac{\zeta c}{c_2}\right)^2$.

Now equations (26) and (27) are the standard form of differential equations for which the solutions can be written as:

$$f(r) = A_1 I_0\left(\sqrt{\zeta^2 - k_1^2}\, .r\right) + A_2 K_0\left(\sqrt{\zeta^2 - k_1^2}\, .r\right) \tag{29}$$

and

$$g(r) = B_1 I_1\left(\sqrt{\zeta^2 - k_2^2}\, .r\right) + B_2 K_1\left(\sqrt{\zeta^2 - k_2^2}\, .r\right), \tag{30}$$

where k_1 and k_2 are wave numbers. A_1, A_2, B_1 and B_2 are constants of integration; I_n and K_n ($n=0,1$) are the modified Bessel functions of the first and second kind, respectively.

Equations (22) and (23) combined with equations (27) and (28) give the solution of wave equation as,

$$\phi(r) = \left[A_1 I_0\left(\sqrt{\zeta^2 - k_1^2}\,.r\right) + A_2 K_0\left(\sqrt{\zeta^2 - k_1^2}\,.r\right)\right] e^{i\zeta(x-ct)} \quad (31)$$

$$\psi_\theta(r) = \left[B_1 I_1\left(\sqrt{\zeta^2 - k_2^2}\,.r\right) + B_2 K_1\left(\sqrt{\zeta^2 - k_2^2}\,.r\right)\right] e^{i\zeta(x-ct)} \quad (32)$$

2.5. Displacement field due to incident P-wave. The longitudinal wave moving along the shell creates an incident displacement field which can be obtained by substituting the values of ϕ and ψ_θ in equations (19) and (21). The constants of integration involved in the expressions of ϕ and ψ_θ are evaluated by using the appropriate boundary conditions. Since the displacement cannot become infinite along the x-axis (as $r \to 0$), the constants A_2 and B_2 in equations (29) and (30) must be zero because the Bessel's functions $K_0(r)$ and $K_1(r)$ tend to infinity as $r \to 0$. Hence, equations (29) and (30) reduce to:

$$\phi^{(i)} = \left[A_1 I_0\left(\sqrt{\zeta^2 - k_1^2}\,.r\right)\right] e^{i\zeta(x-ct)} \quad (33)$$

$$\psi_\theta^i = B_1 I_1\left(\sqrt{\zeta^2 - k_2^2}.\,r\right) e^{i\zeta(x-ct)} \quad (34)$$

Superscript (i) indicates that fields are that due to incident. For incident case ψ_θ would be zero, hence the equations (22) and (24) further reduce to as follows:

$$d_r^{(i)} = \frac{\partial \phi^{(i)}}{\partial r} \quad (35)$$

and

$$d_x^{(i)} = \frac{\partial \phi^{(i)}}{\partial x} \quad (36)$$

Substituting value of ϕ^i from (34) into these equations we obtain:

$$d_r^{(i)} = A_1 \sqrt{\zeta^2 - k_1^2} I_1\left(\sqrt{\zeta^2 - k_1^2}.\,r\right) e^{i\zeta(x-ct)} \quad (37)$$

and

$$d_x^s = \left[A_1 (i\zeta) I_0\left(\sqrt{\zeta^2 - k_2^2}\,.r\right)\right] e^{i\zeta(x-ct)} \quad (38)$$

Or alternatively:

$$d_r^i = d_0\, \overline{C_1} I_1\left(\overline{C_1}\frac{r}{R}\right) e^{i\zeta(x-ct)} \quad (39)$$

and

$$d_x^i = i\, d_0 R\, \zeta\, \overline{C_1}\, I_0\left(\overline{C_1}\frac{r}{R}\right) e^{i\zeta(x-ct)}, \quad (40)$$

where $\overline{C_1} = \beta\sqrt{1 - \left(\frac{c}{c_1}\right)^2}$ and $\beta = \zeta R = 2\pi\frac{R}{\Lambda}$, d_0 is a constant, depending on the incident wave intensity and having the dimension of length.

2.6. Displacement field due to scattered P-wave. The total displacement 'd' at any point in the medium consists of two parts: the incident field d^i created due to the wave along the shell and the scattered field d^s which is discussed as follows:

For the scattered wave, the radiative boundary condition is that, as $r \to \infty$ the disturbance must die out. But the Bessel's functions I_0 and I_1 tend to infinity as $r \to \infty$ and, therefore, to take care of the radiative boundary conditions constants

A_1 and B_1 in equations (29) and (30) should be made zero. The scalar potential ϕ and ψ_θ are thus written as:

$$\phi^{(s)} = A_2 K_0\left(\sqrt{\zeta^2 - k_1^2}.r\right) e^{i\zeta(x-ct)} \tag{41}$$

and

$$\psi_\theta^s = B_2 K_1\left(\sqrt{\zeta^2 - k_2^2}.r\right) e^{i\zeta(x-ct)} \tag{42}$$

The superscript (s) is used to identify the quantities related to scattered field.

Substituting $\phi^{(s)}$ and ψ_θ^s in equations (19) and (21), the radial and axial components of d^s come out as:

$$d_r^s = -\left[A_2\sqrt{\zeta^2 - k_1^2}K_1\left(\sqrt{\zeta^2 - k_1^2}\,.r\right) + B_2 i\zeta\, K_1\left(\sqrt{\zeta^2 - k_2^2}\,.r\right)\right] e^{i\zeta(x-ct)} \tag{43}$$

and

$$d_x^s = \left[A_2 i\zeta K_0\left(\sqrt{\zeta^2 - k_1^2}\,.r\right) - B_2\sqrt{\zeta^2 - k_2^2}\,K_0\left(\sqrt{\zeta^2 - k_2^2}\,.r\right)\right] e^{i\zeta(x-ct)} \tag{44}$$

or,

$$d_r^s = -\left[A_2\frac{\overline{C_1}}{R}\,K_1\left(\overline{C_1}\frac{r}{R}\right) + i\zeta B_2\,K_1\left(\overline{C_2}\frac{r}{R}\right)\right] e^{i\zeta(x-ct)} \tag{45}$$

and

$$d_x^s = \left[A_2 i\zeta K_0\left(\overline{C_1}\frac{r}{R}\right) - B_2\frac{\overline{C_2}}{R}\,K_0\left(\overline{C_2}\frac{r}{R}\right)\right] e^{i\zeta(x-ct)}, \tag{46}$$

where

$$\overline{C_2} = \beta\sqrt{1 - \left(\frac{c}{c_2}\right)^2}$$

In the above equations (44) and (45) the constants A_2 and B_2 are unknown which are obtained by applying the appropriate boundary conditions.

2.7. Solution of Wave Equations. For obtaining the shell response, the mid-plane displacement components are assumed to be of the form:

$$u = \overline{u}_0 e^{i\xi(x-ct)}$$
$$w = \overline{w}_0 e^{i\xi(x-ct)} \tag{47}$$

2.8. Shell boundary conditions. An imperfect bond between the shell and the soil was considered and it was assumed that this bond, which joined the shell and the continuum together, was thin, elastic and inertia-less. This implied that the stresses at the shell soil interface were continuous. To take elasticity of the bond into account, the normal and shear stresses in the bond were assumed to be proportional to the relative normal and tangential displacements between the shell and the continuum, i.e.

$$\sigma_{rr}^*|_{r=R+h/2} = \left[S_r + Z_r\frac{\partial}{\partial t}\right]\left[d_r^i + d_r^s - d_r\right]\Big|_{r=R+h/2} \tag{48}$$

$$\sigma_{rx}^*|_{r=R+h/2} = \left[S_x + Z_x\frac{\partial}{\partial t}\right]\left[d_x^i + d_x^s - d_x\right]\Big|_{r=R+h/2}, \tag{49}$$

where σ_{rr}^* and σ_{rx}^* are the normal and shear stresses respectively at the outer surface of the shell due to the motion of the surrounding medium. S_r and Z_r are the stiffness and damping coefficients of the bond in the radial direction and S_x and Z_x are the stiffness and damping coefficients of the bond in the axial direction respectively.

2.9. Final response equations of the shell. The final response equation in the matrix form is given by:

$$[A]\,\{M\} = \{F\}, \tag{50}$$

where

$$\{M\} = \left[U, W, \frac{A_2}{Rd_0}, \frac{B_2}{Rd_0}\right]^T, \tag{51}$$

where $U=\frac{\bar{u}_0}{d_0}$ and $W=\frac{\bar{w}_0}{d_0}$ and $[A]$ is a 5X5 matrix and $\{F\}$ is a 5x1 matrix. The elements of these matrices are as following:

$A_{11}= -i\beta\bar{h}\frac{\eta_2}{\eta_3}$

$A_{12} = 2ik^2\beta$

$A_{13} = -\bar{h}\left[\left(1+\frac{\bar{h}^2}{12}\right)\eta_1/\eta_3 + \beta^2k^2\right] + \Omega^2 + \left(1-\frac{\bar{h}}{2}\right)\frac{\tau_f^2\bar{\mu}_f\,\bar{c}^2\beta^2 I_0(\alpha_3)}{\delta_f\, I_1(\alpha_3)}$

$A_{14} = \left(1+\frac{\bar{h}}{2}\right)\bar{\mu}\left[\tau^2\gamma^2\left\{K_0(\alpha_1)+\frac{K_1(\alpha_1)}{\alpha_1}\right\} - \frac{(\tau^2-2)\gamma^2}{\alpha_1}K_1(\alpha_1) - \beta^2(\tau^2-2)K_0(\alpha_1)\right]$

$A_{15} = -\left(1+\frac{\bar{h}}{2}\right)\bar{\mu}\left[i\tau^2\beta\,\delta\left(K_0(\alpha_2)+\frac{\gamma K_1(\alpha_2)}{\delta\alpha_1}\right) - \frac{i\beta(\tau^2-2)\gamma^2}{\alpha_1}K_1(\alpha_1) \quad - i(\tau^2-2)\beta\,\delta K_0(\alpha_2)\right]$

$A_{21}= \frac{\bar{h}}{12}\left[-\frac{\bar{h}\beta^2}{\eta_3} + \Omega^2\right]$

$A_{22} = \left[-\frac{\bar{h}^2\beta^2}{6\eta_3} - 2k^2 + \frac{\bar{h}}{6}\Omega^2\right]$

$A_{23} = -i\bar{h}\beta K^2$

$A_{24} = \bar{h}\left(1+\frac{\bar{h}}{2}\right)\mu[-\,i\,\beta\,\gamma K_1(\alpha_1)]$

$A_{25} = \frac{\bar{h}}{2}\left(1+\frac{\bar{h}}{2}\right)\bar{\mu}\left[(\beta^2+\delta^2)K_1(\alpha_2)\right]$

$A_{31} = -\frac{\bar{h}\beta^2}{\eta_3} + \Omega^2$

$A_{32} = \frac{\bar{h}}{6}A_{31}$

$A_{33} = -A_{11}$

$A_{34} = \left(1+\frac{\bar{h}}{2}\right)\mu[-\,2i\,\beta\,\gamma K_1(\alpha_1)]$

$A_{35} = \frac{2}{\bar{h}}A_{25}$

$A_{41} = A_{42} = 1$

$A_{43} = 0$

$A_{44}= \frac{\zeta_x\Gamma_x}{\Gamma_x - i\beta\bar{c}\zeta_x}\{-2i\beta\gamma K_1(\alpha_1)\} - i\beta K_0(\alpha_1)$

$A_{45} = \frac{\zeta_x\Gamma_x}{\Gamma_x - i\beta\bar{c}\zeta_x}\left\{(\beta^2+\delta^2)K_1(\alpha_2)\right\} + \delta K_0(\alpha_2)$

$A_{51} = A_{52} = 0$

$A_{53} = 1$

$A_{54} = \frac{\zeta_r\Gamma_r}{\Gamma_r - i\beta\bar{c}\zeta_r}\left[\tau^2\gamma^2\left\{K_0(\alpha_1)+\frac{K_1(\alpha_1)}{\alpha_1}\right\} - \frac{(\tau^2-2)\gamma^2}{\alpha_1}K_1(\alpha_1) - (\tau^2\text{-}2)\beta^2K_0(\alpha_1)\right] + \gamma K_1(\alpha_1)$

$A_{55} = \frac{\zeta_r\Gamma_r}{\Gamma_r - i\beta\bar{c}\zeta_r}\left[i\beta\delta\tau^2\left\{K_0(\alpha_2)+\frac{\gamma K_1(\alpha_2)}{\delta\alpha_1}\right\} - \frac{i\beta\gamma(\tau^2-2)}{\alpha_1}K_1(\alpha_2) - i\beta\delta(\tau^2-2)K_0(\alpha_2)\right] + i\beta K_1(\alpha_2)$

$F_1 = \left(1+\frac{\bar{h}}{2}\right)\bar{\mu}\left[\left\{(\tau^2\epsilon^2 - 2\beta^2)I_0(\alpha_1) + \frac{2}{\alpha_1}\gamma^2 I_1(\alpha_1)\right\}\right]$

$F_2 = -\frac{\bar{h}}{2}\left(1+\frac{\bar{h}}{2}\right)\bar{\mu}[\,2i\,\gamma\,\beta\, I_1(\alpha_1)]$

$F_3 = \frac{2}{\bar{h}}F_2^1$

$F_4 = \frac{\zeta_x \Gamma_x}{\Gamma_x - i\beta \bar{c}\, \zeta_x} \frac{\bar{h}}{2} [-2\, i\, \gamma\, \beta\, I_1(\alpha_1)] + i\beta I_0(\alpha_1)$

$F_5 = \frac{\zeta_r \Gamma_r}{\Gamma_r - i\beta \bar{c} \zeta_r} \left[\left\{ \left(\tau^2 \epsilon^2 - 2\beta^2\right) I_0(\alpha_1) + \frac{2}{\alpha_1} \gamma^2 I_1(\alpha_1) \right\} \right] + \gamma I_1(\alpha_1)$

$\Omega^2 = \rho_0 hR/G_{x1}\omega^2 = \beta^2 \overline{h}\overline{c}^2 \tau^2 \overline{\rho}\, \overline{\mu}$ is the non-dimensionalized frequency of the shell.

$\overline{h} = \frac{h}{R}$

$\overline{\rho} = \frac{\rho_0}{\rho_m}$

$\overline{\mu} = \frac{\mu}{G_{x1}}$

$\overline{c} = \frac{c}{c_1}$

$\tau^2 = \frac{c_1^2}{c_2^2} = \frac{2(1-\upsilon_m)}{(1-2\upsilon_m)} \alpha_1 = \left(1 + \frac{\bar{h}}{2}\right) \gamma$

$\gamma = \bar{C_1} = \beta \sqrt{1 - \left(\frac{c}{c_1}\right)^2}$

$\alpha_2 = \left(1 + \frac{\bar{h}}{2}\right) \delta$

$\delta = \bar{C_2} = \beta \sqrt{1 - \left(\frac{c}{c_2}\right)^2}$

$\varepsilon = \overline{c}\, \beta$

$\eta_1 = \frac{E_{\theta 1}}{E_{x1}}, \eta_2 = \frac{E_{\nu 1}}{E_{x1}}, \eta_3 = \frac{G_{x1}}{E_{x1}},$

where η_1, η_2 and η_3 are the non-dimensionalized orthotropic parameters of the shell. ζ_x and Γ_x are the non-dimensionalized stiffness and damping coefficients of the bond in the axial direction and ζ_r and Γ_r those in the radial direction.

2.10. Calculations of Stresses. The axial and hoop stresses of the shell are given by equations (4) as:

$$\sigma_{xx} = E_{x1}\, \varepsilon_{xx} + E_{\nu 1}\, \varepsilon_{\theta\theta}$$
$$\sigma_{\theta\theta} = E_{\nu 1}\, \varepsilon_{xx} + E_{\theta 1}\, \varepsilon_{\theta\theta} \quad (52)$$

We also have strain displacement relations for the shell as:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} \text{ and } \varepsilon_{\theta\theta} = \frac{w}{R+z} \quad (53)$$

Further, for obtaining the axial and hoop stresses, the mid-plane displacement components and slope are assumed to be of the form, given below:

$$u = \overline{u}(r)\, e^{i\zeta(x-ct)} = [\overline{u}_0 + (r - R)\psi_x] e^{i\zeta(x-ct)}$$
$$w = \overline{w}(r) e^{i\zeta(x-ct)} = \overline{w}_0 e^{i\zeta(x-ct)}$$
$$\psi_x = \overline{\psi_x} e^{i\zeta(x-ct)} \quad (54)$$

Therefore, we get:

$$\sigma_{xx} = \left[(\overline{u}_0 i\, \zeta + z\, \overline{\psi_x}\, i\, \zeta)\, E_{x1} + E_{\nu 1} \frac{\overline{w}_0}{R+z} \right] e^{i\zeta(x-ct)} \quad (55)$$
$$\sigma_{\theta\theta} = \left[(\overline{u}_0 i\, \zeta + z\, \overline{\psi_x}\, i\, \zeta) E_{\nu 1} + E_{\theta 1} \frac{\overline{w}_0}{R+z} \right] e^{i\zeta(x-ct)} \quad (56)$$

According to thin shell theory, we have:

$$\psi_x = -\frac{\partial w}{\partial x} \text{ or } \overline{\psi}_x = -i\zeta\, \overline{w}_0$$

Also we have:

$$\overline{w}_0 = W\, d_0 \text{ and } \overline{u}_0 = U\, d_0$$

Using these relations and above stress equations:

$$\sigma_{xx} = E_{x1} d_0 [U\, i\, \zeta - z\, W\, (i\, \zeta)^2] e^{i\, \zeta (x - c\, t)} + E_{v1} d_0 \left[\frac{W}{R+z}\right] e^{i\, \zeta (x - c\, t)} \tag{57}$$

$$\sigma_{\theta\theta} = E_{v1} d_0 [U\, i\, \zeta - z\, W\, (i\, \zeta)^2] e^{i\, \zeta (x - c\, t)} + E_{\theta 1} d_0 \left[\frac{W}{R+z}\right] e^{i\, \zeta (x - c\, t)} \tag{58}$$

We have stress resultants N_{xx} and $N_{\theta\theta}$ given by:

$$N_{xx} = \int_{\left(-\frac{h}{2}\right)}^{\frac{h}{2}} \sigma_{xx} \left(1 + \frac{z}{R}\right) dz \tag{59}$$

and

$$N_{\theta\theta} = \int_{\left(-\frac{h}{2}\right)}^{\frac{h}{2}} \sigma_{\theta\theta}\, dz \tag{60}$$

Combining equations (57), (58), (59) and (60) we get:

$$N_{xx} = d_0 \left[E_{x1} U\, i\, \zeta\, h - E_{x1} W\, (i\, \zeta)^2 \frac{h^3}{12R} + E_{v1} W\, \frac{h}{R}\right] e^{i\, \zeta (x - c\, t)} \tag{61}$$

and

$$N_{\theta\theta} = d_0 \left[E_{v1} U\, i\, \zeta\, h + E_{\theta 1} \frac{W}{R} h \left(1 + \frac{h^2}{12R^2}\right)\right] e^{i\, \zeta (x - c\, t)} \tag{62}$$

We define S_{xx} and $S_{\theta\theta}$ (the non dimensional form of axial and hoop stress) as:

$$S_{xx} = \frac{N_{xx}/d_0}{E_{X1}}$$

$$S_{xx} = U\, i\, \beta\, \bar{h} + \frac{\bar{h}^2}{12} \beta^2 W + \eta_2 W \bar{h} \tag{63}$$

and

$$S_{\theta\theta} = \frac{N_{\theta\theta}/d_0}{E_{X1}}$$

$$S_{\theta\theta} = \eta_2 U\, i\, \beta \bar{h} + \eta_1 W \left(\bar{h} + \frac{\bar{h}^3}{12}\right) \tag{64}$$

3. Results and Discussions

Results have been presented here mainly from the point of view of bringing out the effects of different parameters on the axial and hoop stresses of the shell under different soil conditions and different angles of wave incidence Φ. For this, only one bond parameter has been varied at a time, keeping the other parameters fixed. The effects of the soil condition and the angles of wave incidence have been shown by changing $\bar{\mu}$ and Φ values, respectively. Effects of the variation of orthotropic parameters η_1, η_2, η_3 have been shown for different cases. Some parameters which have been kept constant throughout are $\bar{h}=0.05, \bar{\rho}=0.3$ and $\nu_m=0.25$. The bond parameters are taken as zero i.e.,$\zeta_x = \zeta_r = \Gamma_x = \Gamma_r = 0$ that represents the perfect bond condition between the shell and surrounding soil. Unless mentioned η_1, η_2, η_3 values have been kept at $\eta_1 = 0.5$, $\eta_2 = 0.05$, $\eta_3 = 0.02$. Results have been given here only for two angles of incidence (Φ=5° and 60°) though the results were obtained for other angles also. Corresponding to soft, medium hard and very rocky conditions, $\bar{\mu}$ has been taken to be 0.01, 0.1, 1.0 respectively.

Figures 2 – 4 show the effects of variation of ζ_x on the axial stress S_{xx} induced in the shell. Figures 2, 3 & 4 give the results ,respectively for soft, medium and hard soil conditions for grazing angle of incidence, Φ=5°. Figures 5-7 show the effect of the variation of ζ_x on the hoop stress for the varying soil condition. Figure 2 indicates that, for soft soil, the axial stress S_{xx} induced in the shell decreases as the value of ζ_x is increased. It is due to the fact that with the higher values of ζ_x, the axial stiffness of layer is decreased and hence a low value of S_{xx} is obtained. It is very well known from Figures 3&4 that same trend is observed for medium hard and hard soil condition. However, the changes in ζ_x does not show any dominant effect variation of hoop stress.

From Figures 8-11 it is observed that non-zero values of ζ_r and Γ_r, reduce the hoop stress of the shell quit significantly, but they give rise to the values of axial stress. Similarly, the non-zero values of ζ_x and Γ_x enhance the value of the hoop stress of the thick cylindrical shell but they reduce the axial stress. Further, it is observed that the increase in the hoop or axial stress is not as significant as the reduction in their values.

It is also obvious from the Figures that the values of stress considering a loose contact (imperfect bond) between shell and medium are lower than the values of stresses that induced in thick shell when there is a perfect bond between shell and medium. It is due to the fact that there is always some energy losing due to the damping, when imperfect bond is considered.

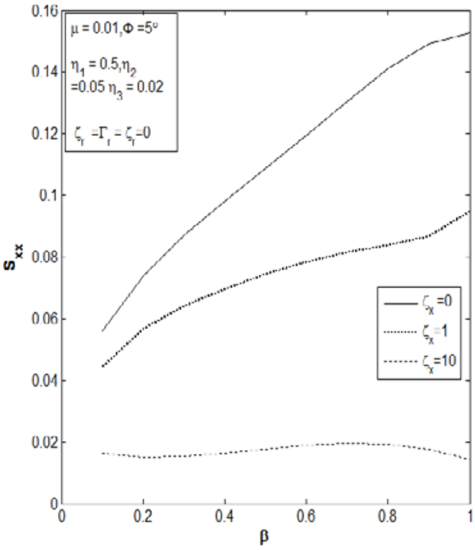

Fig. 2. Axial stress (S_{xx}) versus wavelength parameter (β) for $\overline{\mu} = 0.01$ and φ=5°with ζ_x as a parameter.

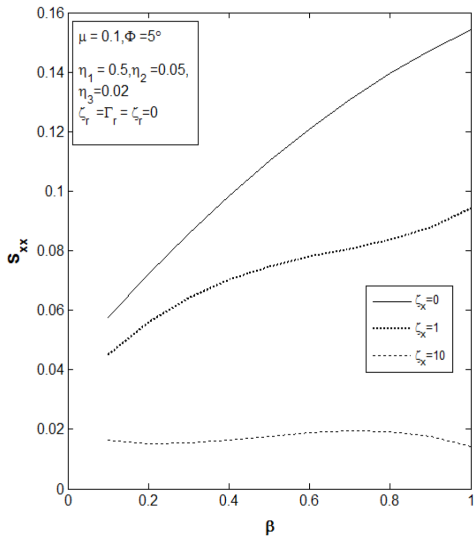

Fig. 3. Axial stress (S_{xx}) versus wavelength parameter (β) for $\overline{\mu} = 0.1$ and φ=5°with ζ_x as a parameter.

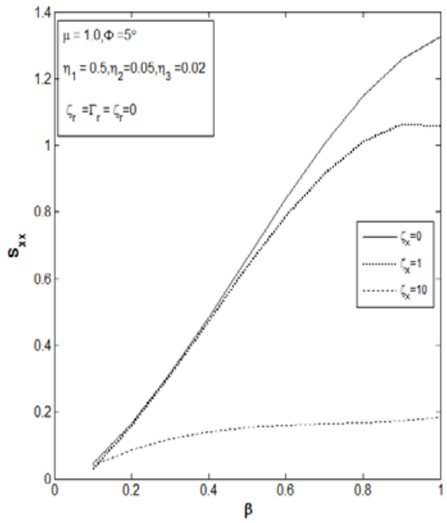

Fig. 4. Axial stress (S_{xx}) versus wavelength parameter (β) for $\overline{\mu} = 1$ and φ=5°with ζ_x as a parameter.

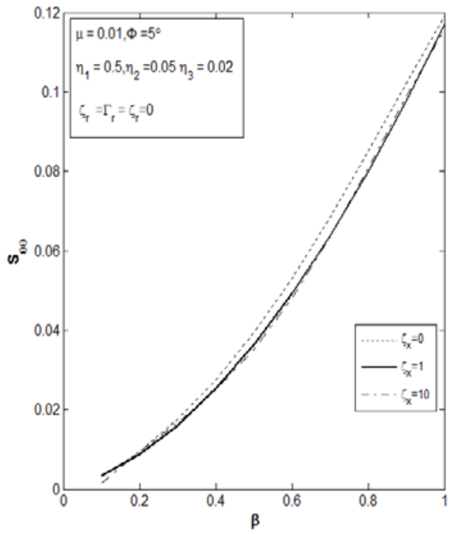

Fig. 5. Hoop stress ($S_{\theta\theta}$) versus wavelength parameter (β) for $\overline{\mu} = 0.01$ and φ=5°with ζ_x as a parameter.

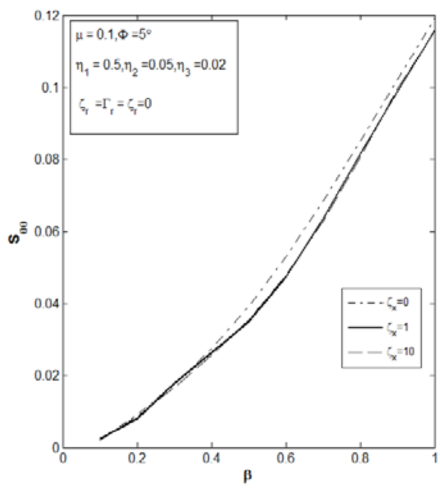

Fig. 6. Hoop stress ($S_{\theta\theta}$) versus wavelength parameter (β) for $\overline{\mu} = 0.1$ and φ=5°with ζ_x as a parameter.

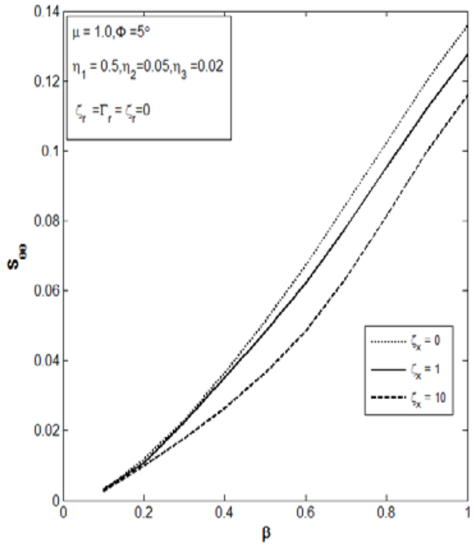

Fig. 7. Hoop stress ($S_{\theta\theta}$) versus wavelength parameter (β) for $\overline{\mu} = 1$ and φ=5°with ζ_x as a parameter.

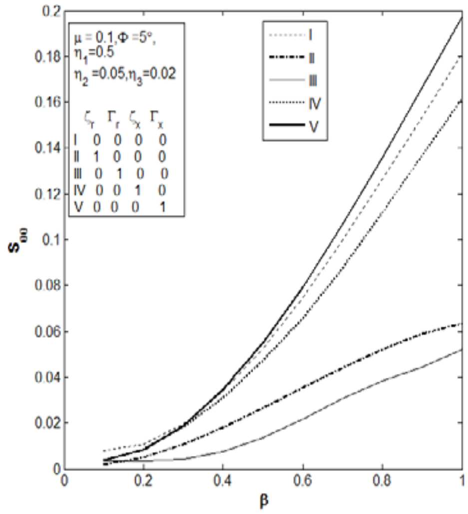

Fig. 8. Comparative effect of $\zeta_x, \zeta_r, \Gamma_x, \Gamma_r$ on hoop stress (S_{xx}) for $\overline{\mu} = 0.1$ and φ=5°.

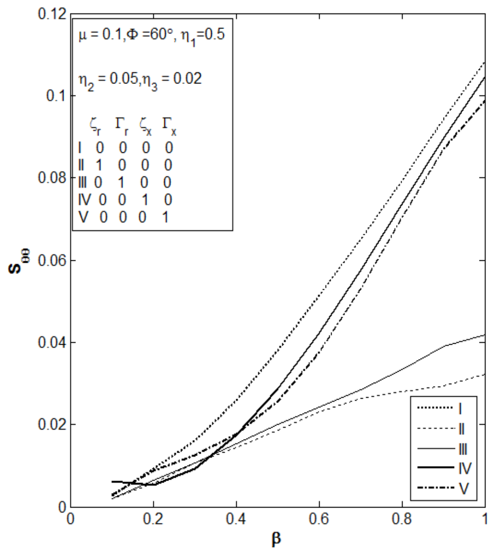

Fig. 9. Comparative effect of $\zeta_x, \zeta_r, \Gamma_x, \Gamma_r$ on hoop stress ($S_{\theta\theta}$) for $\overline{\mu} = 0.1$ and φ = 60°.

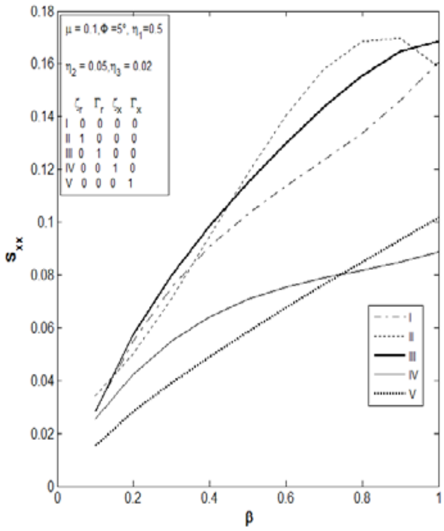

Fig. 10. Comparative effect of $\zeta_x, \zeta_r, \Gamma_x$, Γ_r on axial stress (S_{xx}) for $\bar{\mu} = 0.1$ and φ = 5°.

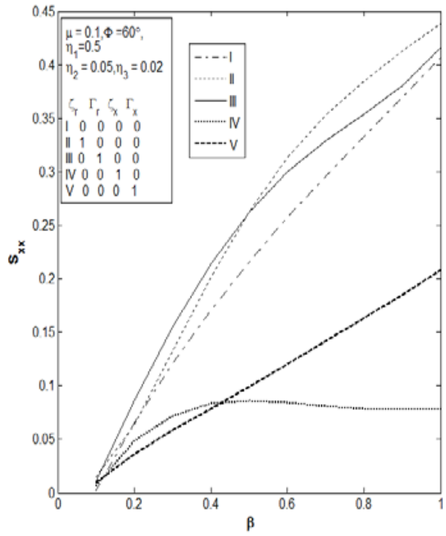

Fig. 11. Comparative effect of $\zeta_x, \zeta_r, \Gamma_x$, Γ_r on axial stress ($S_{\theta\theta}$) for $\bar{\mu} = 0.1$ and φ = 60°.

5. Conclusions

Based on the results presented in the previous chapter, following conclusions about the seismic response of imperfectly bonded buried orthotropic pipelines can be drawn:

1. Bond parameters ζ_x, ζ_r, Γ_x and Γ_r influence the shell response significantly. Axial bond parameter ζ_x, Γ_x influences the axial stress of the shell and radial bond parameters ζ_r and Γ_r, influence the hoop stress to a great content. But, axial bond parameters do not influence the radial stress significantly and vice versa.

2. Generally, consideration of an imperfect bond gives lower values of shell stresses. However, an imperfect bond assumed in one direction may give slightly higher values of the stresses in the other direction.

3. In general, a loose contact (i.e. imperfect bond) between the shell and the soil gives lower values of stresses as compared to the values for a perfectly bonded shell. Therefore, by assuming a perfect bond one gets a conservative and safe estimate of the shell stresses.

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