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NONLINEAR LONGITUDINAL MAGNETOELASTIC WAVES IN A ROD WITH ACCOUNT OF DAMAGE IN ITS MATERIAL

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Abstract. In this paper we study the propagation of longitudinal magnetoelastic waves in a rod with damage. It is shown that for a stationary magnetic field the system of equations of magnetoelasticity can be reduced to one evolution equation with respect to the function of longitudinal deformation. The equation comprises variants of generalized unperturbed Burgers equations, when the medium does not have conductivity. For these equations, solutions have been found in the form of stationary shock waves. The connection between the main parameters (amplitude, width of the front) of the shock wave and the parameters of the system have been established. The influence of the damage parameters and the elastic nonlinearity of the material on the width of the front of the shock wave is determined. The evolutionary equation of magnetoelasticity has been investigated by an approximate method, when the medium is conductive. The influence of the conductivity parameters of the medium and material damage on the amplitudes of the first and second harmonics of the decomposition has been analyzed.

Keywords: longitudinal deformation; nonlinearly elastic rod; material damage; magnetic field; evolutionary equation; asymptotic solution.

1. Introduction

The mechanics of the damaged continuum is intensively developing, starting with the fundamental works of L.M. Kachanov, generalized in the monograph [1], and Yu.N. Rabotnov, generalized in the monograph [2]. The value of these first works, now recognized as classical, consists in the possibility of applying a single scheme for representing damage to describe processes in elastic and elastoplastic bodies.

Damage is usually understood as a reduction in the elastic response of the body due to a reduction in the effective area which transfers internal forces from one part of the body to the other part, which in its turn is caused by the appearance and development of a diffuse field of microdefects (microcracks in elasticity, dislocations in plasticity, micropores – with creep, surface microcracks – with fatigue) [3].

Damage can not be measured directly like speed, force or temperature. The degradation of the mechanical properties of the body, can be detected by analyzing the response of the body to various external influences. According to experimental practice, the presence of a damage field in materials can be indirectly detected and partly quantified through a decrease in the velocity of the ultrasonic signal [4], a decrease in the Young's modulus (“modulus defect”) [5],

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a decrease in density ("loosening") [6], a change in hardness [7], a drop in the stress amplitude during a cyclic test [8], creep acceleration in the third stage [9].

In the traditional calculations, as the measure of damage in the deformation development process is taken the scalar damping parameter $\psi(x,t)$, which characterizes the relative density of microdefects uniformly scattered in a unit volume. This parameter is zero when there is no damage, and is close to one at the time of destruction.

The process of accumulation of damages in the material of the construction under study is calculated by successively solving the kinetic equation of damage at each stage of loading. Investigation of the process of accumulation of damages in the structural element continues until the parameter $\psi(x,t)$ reaches a given limit value close to one.

As a rule, in the mechanics of a deformable solid, the dynamics problems are considered separately from the problems of damage accumulation. When developing such methods, it is customary to postulate in advance that the velocity of an elastic wave is a given function of the damage, and then experimentally determine the proportionality coefficients. The phase velocity of the wave and its damping are usually considered power functions of frequency and linear damage functions [10]. With undoubted advantages (simplicity), this approach has a number of shortcomings, like any approach that does not rely on mathematical models of processes and systems.

The authors of works [11–13] consider the problem to be self-consistent, including, in addition to the equation of development of damage, the dynamic equation of the theory of elasticity.

Let us note that for a number of problems it is necessary to take into account that many elements of the construction work under conditions of interaction with an external magnetic field, which affects the formation and propagation of elastic waves [14, 15].

2. System of magnetoelasticity and evolutionary equation

We will further consider the propagation of longitudinal waves in a homogeneous, nonlinearly elastic rod located in an external magnetic field, and also take into account the presence of damage in the material of the rod. To describe the propagation of longitudinal waves in a rod, we choose the classical nonlinear Bernoulli equation [16].

The system of magnetoelasticity equations for a rod, taking into account the presence of damage, looks as following:

$$\frac{\partial^2 u}{\partial t^2} - c_0^2\left(1 + \frac{\alpha_0}{E}\frac{\partial u}{\partial x}\right)\frac{\partial^2 u}{\partial x^2} + \frac{H_0}{4\pi\rho}\frac{\partial h}{\partial x} - \beta_1\frac{\partial \psi}{\partial x} = 0, \tag{1}$$

$$\frac{\partial h}{\partial t} + H_0\frac{\partial^2 u}{\partial x \partial t} - \frac{c^2}{4\pi\sigma}\frac{\partial^2 h}{\partial x^2} = 0, \tag{2}$$

$$\frac{\partial \psi}{\partial t} + \alpha\psi - \beta_2 E\frac{\partial u}{\partial x} = 0, \tag{3}$$

where $u(x,t)$ – displacements of particles on the median line of the rod, $h(x,t)$ – intensity of the magnetic field, $\psi(x,t)$ – the already mentioned scalar function characterizing the relative density of microdefects uniformly scattered in a unit volume, $c_0 = \sqrt{E/\rho}$ – velocity of propagation of a longitudinal wave in the absence of a magnetic field, E – Young's modulus, ρ – density of the material, α_0 – coefficient of elastic nonlinearity ($\alpha_0 < 0$ for the majority of the materials and alloys, $\alpha_0 > 0$ for the majority of composites), H_0 – intensity of external magnetic field, c – light speed in vacuum, σ – electrical conductivity, $\alpha = 1/\tau_*$, β_1, β_2 –

constant parameters, which characterize the material damage and the connection of cyclic processes with the processes of damage accumulation, τ_* – relaxation time.

In composing the system of equations (1) – (3) it is taken into account that the total magnetic field consists of its constant value and perturbations appearing as a result of interaction with the field of deformations:

$\mathbf{H} = H_0\mathbf{n} + \mathbf{h}$,

where $\mathbf{n}$ – vector of normal, $\mathbf{h}$ – small perturbations of the magnetic field. It is also assumed that an external constant magnetic field with an intensity H_0 perpendicular to the direction of propagation of the wave. In this case, for longitudinal waves in the rod, the following relations are satisfied:

$\mathbf{u} = (u_1,0,0) \equiv u(x,t)$, $\mathbf{h} = (0,0,h_3) \equiv h(x,t)$, $\mathbf{H} = (0,0,H_0 + h)$.

If the magnetic field is a stationary one ($\partial h/\partial t = 0$), the system of equations (1) – (3) can be reduced to a single equation with respect to displacement:

$$\left(1+\frac{H_0^2\sigma}{c^2\rho\alpha}\right)\frac{\partial^2 u}{\partial t^2} - c_0^2\left(1+\frac{\beta_1\beta_2 E}{\alpha c_0^2}\right)\frac{\partial^2 u}{\partial x^2} + \frac{H_0^2\sigma}{c^2\rho}\frac{\partial u}{\partial t} + \frac{1}{\alpha}\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial t^2} - c_0^2\frac{\partial^2 u}{\partial x^2}\right) - \\ - \frac{c_0^2\alpha_0}{E}\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} - \frac{c_0^2\alpha_0}{\alpha E}\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}\right) = 0 \tag{4}$$

For the convenience of analyzing the dependencies, we introduce dimensionless quantities for the longitudinal displacement, coordinate, and time, respectively $U = u/u_0$, $z = x/X$, $\tau = t/T$. The equation (4) has the following form:

$$\frac{\partial^2 U}{\partial \tau^2} - \frac{\partial^2 U}{\partial z^2} + a_1\frac{\partial U}{\partial \tau} + \frac{\partial}{\partial \tau}\left(\frac{\partial^2 U}{\partial \tau^2} - a_2\frac{\partial^2 U}{\partial z^2}\right) - a_3\frac{\partial U}{\partial z}\frac{\partial^2 U}{\partial z^2} - a_4\frac{\partial}{\partial \tau}\left(\frac{\partial U}{\partial z}\frac{\partial^2 U}{\partial z^2}\right) = 0, \tag{5}$$

where a_i – dimensionless complexes have the form:

$$a_1 = \frac{H_0^2\sigma}{c^2\rho\alpha}\frac{1}{\left(1+\dfrac{H_0^2\sigma}{c^2\rho\alpha}\right)^2},\quad a_2 = \left(1+\frac{H_0^2\sigma}{c^2\rho\alpha}\right)\frac{1}{1+\dfrac{\beta_1\beta_2 E}{\alpha c_0^2}},$$

$$a_3 = \frac{\alpha_0 u_0 \alpha}{c_0 E}\left(\frac{1+\dfrac{H_0^2\sigma}{c^2\rho\alpha}}{1+\dfrac{\beta_1\beta_2 E}{\alpha c_0^2}}\right)^{3/2},\quad a_4 = \frac{\alpha_0 u_0 \alpha}{c_0 E}\frac{\left(1+\dfrac{H_0^2\sigma}{c^2\rho\alpha}\right)^{5/2}}{\left(1+\dfrac{\beta_1\beta_2 E}{\alpha c_0^2}\right)^{3/2}},$$

and as characteristic values of length and time are taken respectively:

$$X = \frac{c_0}{\alpha}\frac{\left(1+\dfrac{\beta_1\beta_2 E}{\alpha c_0^2}\right)^{1/2}}{\left(1+\dfrac{H_0^2\sigma}{c^2\rho\alpha}\right)^{3/2}},\quad T = \frac{1}{\alpha\left(1+\dfrac{H_0^2\sigma}{c^2\rho\alpha}\right)}.$$

It is seen that in equation (5) there are two types of dissipation. One type of dissipation is due to the presence of defects in the material (third-order derivatives), the other by the magnetic field (the first-order derivative), since we are considering a conducting medium.

We unite the parameters of the initial problem in dimensionless parameter complexes

$\dfrac{H_0^2\sigma}{c^2\rho\alpha} = m_1$, $\dfrac{\beta_1\beta_2 E}{\alpha c_0^2} = m_2$, $\dfrac{\alpha_0 u_0 \alpha}{c_0 E} = m_3$,

m_1 characterizes the conductivity of the field ($m_1 > 0$), m_2 – material damage (damage parameter, $-1 \le m_2 \le 1$), m_3 – elastic nonlinearity of the material ($m_3 < 0$), then the coefficients of equation (5) are essentially simplified.

In equation (5) we introduce a moving coordinate system $\xi = z - c\tau$, $\theta = \varepsilon\tau$, where c – velocity of the waves previously unknown, ε – small parameter ($\varepsilon << 1$).The choice of variables is explained by the fact that the perturbation propagating with velocity c along the axis z, slowly evolving in time due to nonlinearity, dispersion and dissipation. We assume that in equation (5) all nonlinear and dissipative terms are small quantities of order ε.

In the first approximation on ε, we obtain the evolutionary equation with respect to the function of longitudinal deformation $W = \dfrac{\partial U}{\partial \xi}$:

$$\frac{\partial W}{\partial \theta} + \frac{a_1}{2\varepsilon} W + \frac{(1-a_2)}{2\varepsilon}\frac{\partial^2 W}{\partial \xi^2} + \frac{a_3}{2\varepsilon} W \frac{\partial W}{\partial \xi} - \frac{a_4}{2\varepsilon}\frac{\partial}{\partial \xi}\left(W \frac{\partial W}{\partial \xi}\right) = 0,$$

it is taken in account here that velocity $c = 1$, which can be identified from the equation of zeroth approximation. Let us rewrite the resulting equation in the form:

$$\frac{\partial W}{\partial \theta} + b_1 W + b_2 \frac{\partial^2 W}{\partial \xi^2} + b_3 W \frac{\partial W}{\partial \xi} - b_4 \frac{\partial}{\partial \xi}\left(W \frac{\partial W}{\partial \xi}\right) = 0, \tag{6}$$

where $b_1 = \dfrac{m_1}{2\varepsilon(1+m_1)^2}$, $b_2 = \dfrac{m_2 - m_1}{2\varepsilon(1+m_2)}$, $b_3 = \dfrac{m_3}{2\varepsilon}\left(\dfrac{1+m_1}{1+m_2}\right)^{3/2}$, $b_4 = \dfrac{m_3(1+m_1)^{5/2}}{2\varepsilon(1+m_2)^{3/2}}$.

Obviously, that we can not neglect the latter term in (6). Both nonlinear terms in the equation are almost equally significant. Equation (6) can be classified as a perturbed Burgers equation [17] with additional nonlinearity.

We consider a particular case of equation (6). When $m_1 \to 0$ we get the equation, which belong to the generalized undisturbed Burgers equations:

$$\frac{\partial W}{\partial \theta} + b_2 \frac{\partial^2 W}{\partial \xi^2} + b_3\left(W \frac{\partial W}{\partial \xi} - \frac{\partial}{\partial \xi}\left(W \frac{\partial W}{\partial \xi}\right)\right) = 0 \tag{7}$$

where $b_2 = \dfrac{m_2}{2\varepsilon(1+m_2)}$, $b_3 = \dfrac{m_3}{2\varepsilon(1+m_2)^{3/2}}$.

In contrast to the classical Burgers equation, there is another type of nonlinearity here, both nonlinearities appear equally. Equation (7) has a stationary solution in the form of a shock wave. This is possible due to the presence in the equation of dissipative and non-linear terms, which balance each other. Taking into account the boundary conditions [18, 19] (the function at infinity has different values), we find the solution of the ordinary differential equation and its derivative

$$\chi = \frac{2}{b_3(W_2 - W_1)}\left[(b_3 W_2 - b_2)\ln(W_2 - W) - (b_3 W_1 - b_2)\ln(W - W_1)\right], \tag{8}$$

$$\frac{dW}{d\chi} = \frac{b_3}{2}\frac{(W_2 - W)(W - W_1)}{b_2 - b_3 W}, \tag{9}$$

where $W = W(\chi)$, $\chi = \xi - v\theta$ – traveling coordinate, W_1, W_2 – limit values of the function, $W_2 > W_1$. The velocity of the shock wave is given by $v = b_3(W_1 + W_2)/2$. The profile of the stationary shock wave $W(\chi)$ and the graph of the derivative (9) are depicted in Fig. 1. The graph of the derivative is mapped into the upper half-plane and displaced along the ordinate axis by W_1. From physical considerations and analysis of the function (8), it follows that $-1 < m_2 \le 1$. The front of the shock wave and its width behave ambiguously, depending on the change in the parameters of the damage and the elastic nonlinearity of the material.

When m_2 changes from -1 to $m_{2(1)}$ the width of the front of the shock wave decreases ($-\frac{2}{3\sqrt{3}A} < m_3 < 0$). When m_2 changes from $m_{2(1)}$ to $m_{2(2)}$ the wave profile does not satisfy the boundary conditions. With a further increase in the parameter, within the limits of admissible values, the width of the wave front increases (Figure 2). Values $m_{2(1)}$ and $m_{2(2)}$ can be determined from the condition that the derivative (9) is negative on the whole interval.

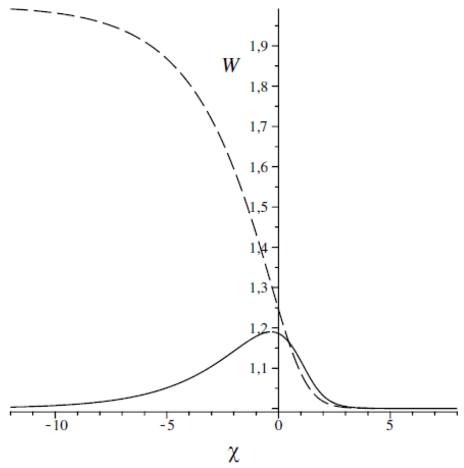

Fig. 1. Dependencies $W(\chi)$ (dashed line) and $W'(\chi)$ (solid line) at fixed values of the parameters.

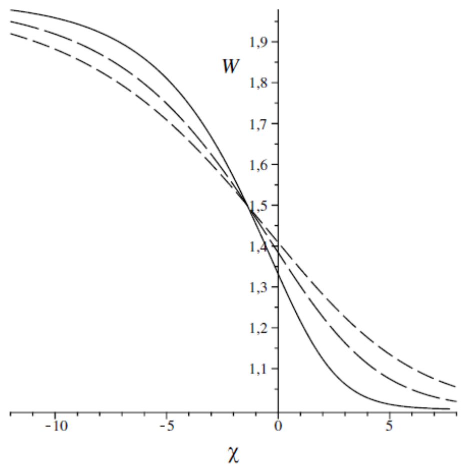

Fig. 2. Dependency $W(\chi)$ at the changing of the parameter m_2, $m_2^{(1)} < m_2^{(2)} < m_2^{(3)}$ ($m_2^{(1)}$ – solid line, $m_2^{(2)}$ – long dashed line, $m_2^{(3)}$ – short dashed line).

A qualitative analysis of equation (7) yields an estimate for the parameters of the shock wave:

$$\frac{Ab_3}{b_2}\left(\frac{\Delta}{2}-1\right) = \text{const} \quad \text{or} \quad \frac{Am_3}{m_2\sqrt{1+m_2}}\left(\frac{\Delta}{2}-1\right) = \text{const},$$

where $A = W_2 - W_1$ – amplitude of the shock wave, Δ – width of the front of the shock wave (Fig. 3). From the figure it can be seen that the function $\Delta(m_2)$ has a maximum or minimum point at value $m_2 = -2/3 = m_2^*$ (dashed with spaces in Fig. 3) depending on values of parameter m_3. At $\Delta = 0$ the front of the wave is vertical, i.e. the solution has the jump character. The width of the front of the shock wave can both increase and decrease with increasing friction due to material damage.

Values $m_{2(1)}$ and $m_{2(2)}$ can be identified from the cubical equation:

$$m_2^3 + m_2^2 - A^2 m_3^2 = 0,$$

which has one real positive root and either two real negative roots, or two complex conjugate with the negative real part. The equation has three real roots ($D > 0$) when $-\frac{2}{3\sqrt{3A}} < m_3 < 0$. The negative roots of the equation are the sought values of m_2. The positive root $m_{2(3)}$ does not satisfy the condition for the existence of a shock wave in the system under consideration.

When $m_3 < -\frac{2}{3\sqrt{3A}}$ a shock wave satisfying the boundary conditions exists for any values of m_2 from the given half-interval. When m_2 changes from -1 to $-2/3$ the width of the front of the shock wave decreases. When m_2 changes from $-2/3$ to 1 width of the front of the shock wave increases. When $m_3 = -\frac{2}{3\sqrt{3A}}$ and $m_2 = -2/3$ the jump of the shock wave happens.

For hypothetical materials, as well as for some composites, the elastic nonlinearity parameter is positive ($m_3 > 0$). The graph of the dependence of the width of the front of the shock wave on the parameter m_2, In this case, is shown in Fig. 3 as dot-dash. When $m_2 = m_{2(3)}$ – the width of the front is zero ($\Delta = 0$). When $m_2 > m_{2(3)}$ the wave profile does not satisfy the boundary conditions.

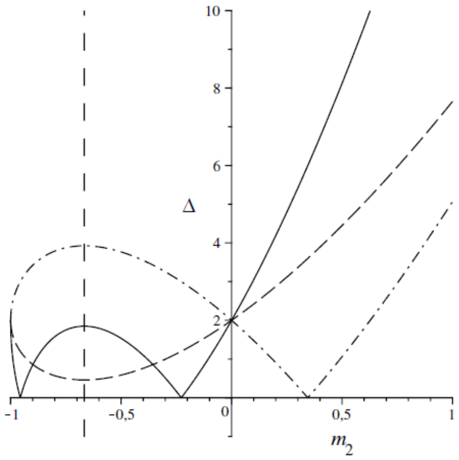

Fig. 3. Dependence of the front width of the shock wave on the parameter m_2 (when $-\frac{2}{3\sqrt{3A}} < m_3 < 0$ – solid line, when $m_3 < -\frac{2}{3\sqrt{3A}}$ – dashed line, when $m_3 > 0$ – dash-dotted line).

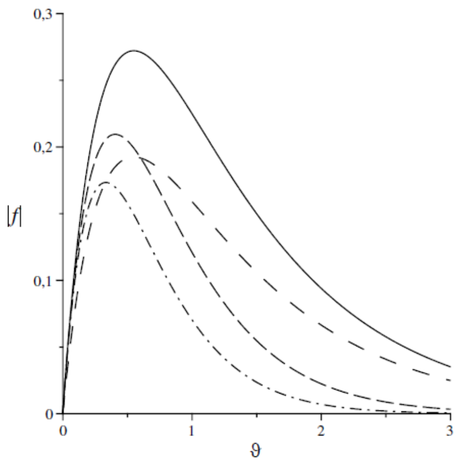

Fig. 4. Dependency $\left|f\left(\tilde{\vartheta}\right)\right|$ for changing the parameter $\tilde{\chi}_1$ from 0 to 2, $\chi_2 \neq 0$ ($\tilde{\chi}_1 = 0$ – solid line, $\tilde{\chi}_1 = 1$ – dashed line, $\tilde{\chi}_1 \to 2$ – dash-dotted line), when $\tilde{\chi}_1 \to 0$ and $\chi_2 \to 0$ – dashed line with spaces.

If simultaneously with the absence of conductivity there are no defects in the material ($m_2 \to 0$), then equation (7) has the form:

$$\frac{\partial W}{\partial \theta} + b_3\left(W\frac{\partial W}{\partial \xi} - \frac{\partial}{\partial \xi}\left(W\frac{\partial W}{\partial \xi}\right)\right) = 0, \; (b_3 = \frac{m_3}{2\varepsilon}). \quad (10)$$

The equation has a solution in the form of a stationary shock wave:

$$\chi = \frac{2}{(W_2 - W_1)}\left[W_2 \ln(W_2 - W) - W_1 \ln(W - W_1)\right],$$

whose width of the front is a constant ($\Delta = \text{const}$), that is, it does not depend on the amplitude of the wave and the initial parameters of the system. In this case, the interaction of heterogeneous nonlinearities leads to the appearance of a shock wave. Linearization of equation (10) with respect to small perturbations ($W = W_0 + \tilde{W}(\xi, \theta)$)

$$\frac{\partial \tilde{W}}{\partial \theta} + b_3\left(\frac{\partial \tilde{W}}{\partial \xi} - \frac{\partial^2 \tilde{W}}{\partial \xi^2}\right)W_0 = 0,$$

shows that the second nonlinear term introduces damping. This attenuation contributes to the appearance of a stationary shock wave in the system. Equation (10) has the Burgers equation property, in spite of the fact that there are obviously no dissipative terms in this equation.

In equation (7), in comparison with equation (10), dissipation is present. If the dissipative term enters the equation with the minus sign, then the increase of b_2 leads to a spreading of the wave profile, if - with the sign is plus, then the increase of leads to a steepening of the wave profile.

Let us analyze the asymptotic solution of equation (6). For that we represent the function W in the form of series $W = \varepsilon W_1 + \varepsilon^2 W_2 + \ldots$, where ε – small parameter ($\varepsilon << 1$). As an initial condition, we choose a sinusoidal profile of unit amplitude $W\big|_{\theta=0} = \sin(k\xi)$. Substituting the expansion in (6), we obtain in the first and second approximations:

$$\frac{\partial W_1}{\partial \theta} + b_1 W_1 + b_2 \frac{\partial^2 W_1}{\partial \xi^2} = 0, \quad W_1\big|_{\theta=0} = \sin(k\xi), \tag{11}$$

$$\frac{\partial W_2}{\partial \theta} + b_1 W_2 + b_2 \frac{\partial^2 W_2}{\partial \xi^2} = -b_3 W_1 \frac{\partial W_1}{\partial \xi} + b_4 \frac{\partial}{\partial \xi}\left(W_1 \frac{\partial W_1}{\partial \xi}\right), \quad W_2\big|_{\theta=0} = 0. \tag{12}$$

Solving the Cauchy problem (11) and (12), we find:

$$W_1 = e^{(k^2 b_2 - b_1)\theta} \sin(k\xi),$$

$$W_2 = \frac{e^{2(k^2 b_2 - b_1)\theta}\left(e^{(2k^2 b_2 + b_1)\theta} - 1\right)}{2b_1 + 4k^2 b_2}\left(2k^2 b_4 \cos(2k\xi) - k b_3 \sin(2k\xi)\right).$$

The ratio of the amplitude of the second harmonic to the amplitude of the first is expressed by the formula:

$$f = \frac{\sqrt{1 + \chi_2^2}}{(2 + \chi_1)} e^{(1-\chi_1)\vartheta}\left(e^{(2+\chi_1)\vartheta} - 1\right), \tag{13}$$

where $f = \dfrac{A_2 b_2}{A_1 |b_4|}$, $\vartheta = \theta k^2 b_2$, $\chi_1 = \dfrac{b_1}{k^2 b_2}$, $\chi_2 = \dfrac{b_3}{2k b_4}$.

The parameter m_2 varies within $-1 < m_2 < m_1$. The graph of the dependency (13) for different values of $\tilde{\chi}_1 = -\chi_1$ is shown in the Figure 4. The analysis of the function f shows us, that its maximum is attained at the point $\tilde{\vartheta} = \dfrac{1}{\tilde{\chi}_1 - 2} \ln\left(\dfrac{\tilde{\chi}_1 + 1}{3}\right)$. From the graph it is clear that with the growth of $\tilde{\chi}_1$ the amplitude of the curve goes down, but the maximum of the function shifts to the left. The change in parameter χ_2 influences the change in the amplitude of the curve. When $m_1 \to \infty$: $\tilde{\chi}_1 \to 0$ и $\chi_2 \to 0$, in the Fig. 4 the corresponding curve $f(\tilde{\vartheta})$

id depicted with the dashed line with spaces. When $m_1 \to 0$: $\tilde{\chi}_1 \to 0$, $\chi_2 \to 1/(2k)$. When $m_1 < m_2 < 1$ the ratio of the amplitudes of the second and first harmonics is not limited from above. The amplitude of the second harmonic grows faster than the amplitude of the first harmonic.

Considering the function of amplitudes ratio as a function of the parameter m_1, we get, that the function $F(m_1)$ ($F = A_2/A_1$) has a minimum point at which the maximum of the curve $F(\theta)$ reaches the minimum value. The maximum of amplitudes ratio $F(\theta)$ equals to $F_{\max} = -\frac{\sqrt{b_3^2 + 4b_4^2 k^2}}{6kb_2} e^{\frac{k^2 b_2 - b_1}{2k^2 b_2 + b_1} \ln\left(\frac{k^2 b_2 - b_1}{3k^2 b_2}\right)}$. Dynamics of the coordinates of the maximum point when the parameter m_1 changes is shown in the Figure 5. It can be seen from the Fig. 5 that when the conductivity parameter increases from zero, the amplitude of the curve $F(\theta)$ first falls, then grows, tending at infinity to $F = \frac{\sqrt{3}}{9} \frac{|m_3|}{\sqrt{1+m_2}} \left(\frac{5}{2} + m_1 + m_2\right) \sqrt{m_1}$. The abscissa of the maximum point (the dashed line in Figure 5) with increasing parameter of m_1 decreases, in the limit tending to zero.

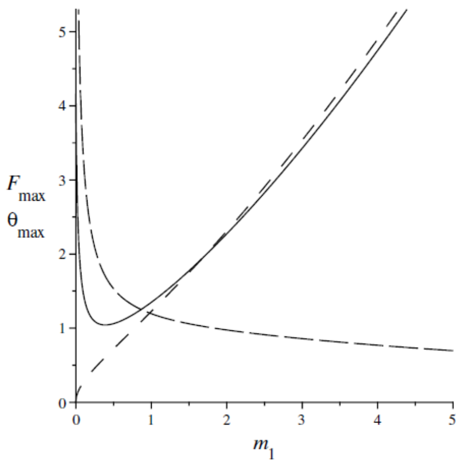

Fig. 5. Dependencies $F_{\max}(m_1)$ (solid line) and $\theta_{\max}(m_1)$ (dashed line), asymptote – dashed line with spaces.

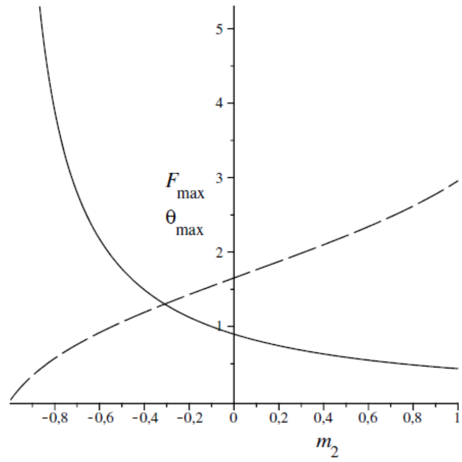

Fig. 6. Dependencies $F_{\max}(m_2)$ (solid line) and $\theta_{\max}(m_2)$ (dashed line).

The dynamics of the coordinates of the maximum point for a fixed value of the parameter m_1 and varying of parameter m_2 is shown in the Fig. 6. As the damage parameter increases from -1 to m_1, the amplitude of the curve $F(\theta)$ decreases, the maximum of the curve shifts to the right. The analysis carried out is valid for any values of the elastic nonlinearity parameter of the material.

3. Conclusions

In the work it has been shown that if the medium does not have a conductivity, the evolution equation of magnetoelasticity is a generalized Burgers equation. The critical value of the parameter of the elastic nonlinearity of the material influencing the width of the front of the shock wave when the material damage parameter is changed has been found. An increase in the damage parameter can both increase or decrease the width of the front of the shock wave. The extremum point of the front width function has been determined as a dependency on the damage parameter.

It has been also noted in the paper that if, together with the absence of conductivity, there are no defects in the material, the initial equation reduces to the equation of a simple wave with one more, additional, nonlinear term. The equation has a solution in the form of a stationary shock wave of constant width. An analysis of the equation in the linear approximation with respect to small perturbations shows that the additional nonlinear term is analogous to the dissipative term in the Burgers equation.

In the case of a conducting medium, the evolutionary equation of magnetoelasticity is investigated by an approximate method. The intervals of the damage parameter have been found within limits of amplitudes of harmonics. It has been shown that the presence of conductivity can lead to both a decrease and an increase in the amplitude of the second harmonic and to a shift in its maximum, depending on the ratios of the remaining parameters of the initial system.

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