

DO ROTATIONAL WAVES REALLY EXIST?

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Abstract. A review of theoretical representations and experimental data on rotational motions of elements of a continuous medium is given. Description of wave rotational motions is impossible in the framework of the classical theory of elasticity, but it is allowed within the scope of generalized continua models (the Cosserat microspolar model, moment and gradient models, etc.), as well as in the framework of A.V. Vikulin's model, in which the stresses are described by a symmetric tensor. Physical, geological and geophysical data are given that confirm both rotational approaches to describing the blocks and plates composing a geomedium and the existence of rotational waves.

Keywords: microstructured media; the Cosserat theory; rotational waves; geodynamics.

1. Introduction

One of the main ways of describing the deformation of solids is the classical theory of elasticity considering a medium as a continuum of material points possessing, in general, three translational degrees of freedom. However, yet in the middle of the 19th century, works began to appear with deviations from the canons of the classical continuum. So, in 1839, J. Mac Cullagh's work [1] was published, which was devoted to construction of an elastic medium model capable describing both the observed reflection and refraction. In the Mac Cullagh continuum, the strain energy depends on the rotational components of the strain. The concept of "couple stresses" was introduced in their works in 1862 by A. Clebsch [2] and in 1874 by G. Kirchhoff [3]. The importance of taking into account couple stresses was also mentioned by W. Voigt in 1887 in Ref. [4], and in 1891 P. Duhem [5] introduced the rotational measure of deformation.

2. The Cosserat model

In 1909, the brothers Eugène and François Cosserat generalized and developed works of G. Kirchhoff, A. Clebsch, P. Duhem and W. Voigt. In their paper [6] they laid the theoretical foundations of one of the first (and at this moment, perhaps, the best known) continuum models of an elastic medium that cannot be described within the scope of the classical theory of elasticity [6]. The Cosserat medium named in their honor consists of solid non-deformable body-particles having three translational and three rotational degrees of freedom.

In the three-dimensional case, the linear dynamic equations of the Cosserat continuum can be written in the vector form:

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} - (\lambda + 2\mu) \text{grad div } \mathbf{u} + (\mu + \alpha) \text{rot rot } \mathbf{u} - 2\alpha \text{rot } \Phi = 0, \quad (1)$$

$$J_i \frac{\partial^2 \Phi}{\partial t^2} - (\beta + 2\gamma) \text{grad div } \Phi + (\gamma + \varepsilon) \text{rot rot } \Phi - 2\alpha \text{rot } \mathbf{u} + 4\alpha \Phi = 0, \quad (2)$$

where \mathbf{u} is the displacement vector, Φ is the vector of rotation angles of an element of a medium in the reference frame related to its mass center, J_i are the corresponding inertia moments of elements of a medium, ρ is the density of a medium, λ and μ are Lamé constants corresponding to the classical theory of elasticity, α , β , γ , and ε are the parameters of the Cosserat medium.

In this system, equation (1) characterizes propagation of translational waves (dilatation and shear waves) in a medium, but, in distinct from the classical case, shear waves are linearly interrelated with rotational waves, the propagation of which is described by equation (2). The propagation of rotational wave perturbations in the Cosserat medium occurs by means of a moment (noncentral) interaction of its elements with each other.

Thus, the Cosserat theory is a mathematical description of such a phenomenon as rotational waves. Naturally, the question arises: “Are there any experimental confirmations of existence of rotational waves in solids?”

3. Experimental data on rotation of elements of a medium

In the middle of 1930s and early 1940s, experimentators paid attention to the importance of taking into account the rotational degrees of freedom of the elements (particles) of the medium. Thus, the experiments of B. Bauer and M. Mag are very interesting [7]. They compared the scattering spectra for heavy and light water. From the comparison of spectra of these two substances, which molecules have approximately the same mass, but different moments of inertia, the authors made a conclusion about existence of both translational and rotational oscillations of molecules. J. Bernal and J. Tamm [8] explained the differences between some physical properties of light and heavy water under the assumption about the existence of rotational oscillations.

In 1940 E.F. Gross [7] observed the effect of variation of the wavelength of scattered light in a liquid associated with orientation fluctuations of anisotropic molecules. He remarked that the axes of molecules can rotate by a significant angle, if the oscillation period is much larger than the relaxation time. Later, E.F. Gross and A.V. Korshunov established experimentally [9] that in crystals of some organic substances (for instance, benzene and naphthalene) the scattering spectrum of small frequencies is associated with rotational vibrations of molecules. The scattering spectrum is the most intensive in substances, which molecules have a large optical anisotropy (carbon disulfide, naphthalene, benzene). The crystal lattices of such substances consist of large molecules. In them, the intermolecular forces are usually much larger than the van der Waals forces acting between the molecules; therefore, the molecules can be regarded as solid bodies oscillating with respect to each other. There are translational oscillations of molecules, rotational oscillations, as well as mixed translational-rotational oscillations. Experimental studies of oscillations in such crystals, carried out by Raman scattering, have shown that in the vicinity of the Rayleigh lines there are characteristic scattering lines due to the rotational nature of molecular oscillations [10-11]. In experiments on spectrograms of light scattering in organic substances, estimates have been obtained for the threshold frequency of benzene and naphthalene [10].

It is interesting to note that around the same time, discrete models of media consisting of non-point particles possessing rotational degrees of freedom began to be developed. So, in the late 1930s Ya.I. Frenkel considered a model of a chain of oriented dipoles with fixed centers of gravity and showed that “waves of rotational oscillations” [12] (i.e., orientational waves) can propagate in such a chain. The first model of the interaction of translational and rotational oscillations in a molecular lattice was proposed in 1949 by A.I. Anselm and N.N. Porfiryeva [13]. Only the linear interaction of orientational waves with one type of translational

oscillations – longitudinal waves – was taken into account in this model. Nevertheless, the authors showed that, basically, mixed orientational-translational oscillations, which frequencies depend both on the mass and the moment of inertia of molecules, propagate in molecular crystal lattices. There exist four branches of the rotational-translational oscillation spectrum in a one-dimensional molecular lattice model with two molecules in a unit cell. In the long-wavelength range, one branch (acoustic) corresponds to purely translational oscillations, the second branch – to purely rotational oscillations depending only on the moment of inertia, and the other two ones are responsible for mixed rotational-translational oscillations depending on both the mass and the moment of inertia.

The mentioned above data show that elaboration of a model for a medium necessitates taking into account the rotational motions of elements of the medium, their masses and moments of inertia, because the physical properties of the medium quantitatively depend on such characteristics. Hence, further improvement of models for microstructured media taking into account rotational degrees of freedom of particles is necessary.

4. Acoustic wave propagation in generalized continua

From the beginning of the 1960s generalized models of the Cosserat continuum are intensively developed [14]: the theory of oriented media, asymmetric, multipolar, micromorphic, gradient theories of elasticity. So, on the basis of assumption of the rotational interaction of particles of elongated shape in an anisotropic elastic medium, E.L. Aero and E.V. Kuvshinsky [15, 16] generalized the phenomenological theory of elasticity in order to explain some anomalies in the dynamic behaviour of plastics, to which the classical theory of elasticity did not provide a satisfactory treatment. Later, the idea of an “oriented” continuum, each point of which is assigned a direction (the field of a director), was developed in the theory of liquid crystals [17], where the director waves in liquid crystals are, in fact, analogues of rotational waves in solids, like spin waves in ferromagnets [18]. Chapter 4 of the monograph [18] of A.I. Akhiezer, V.G. Barjahtar and S.V. Peletminsky deals with the analysis of coupled spin and acoustic waves in ferromagnets. Elastic waves are considered in the framework of the classical theory without taking into account microrotations, but it is shown that, due to the relationship between the elastic deformations and the magnetic field of spins, the stress tensor is no longer symmetric, i.e. couple stresses arise in an elastic ferromagnet, when the spin waves are excited. In the monograph by V.E. Lyamov [19] it was shown that the account of microrotations in crystals leads to the appearance of the spatial dispersion and new wave modes.

In the last thirty years, the processes of propagation and interaction of elastic (acoustic) waves and rotational waves in microstructured solids have been extensively studied theoretically (see, e.g., [20–31]). The first experiments on acoustics of microstructured solids were performed yet in 1970 by G.N. Savin et al. [32–33]. The authors established the correlation between the grain size in different metals and aluminum alloys and the dispersion of the acoustic wave. Dispersion of the ultrasound waves was observed by V.I. Erofeev and V.M. Rodyushkin in an artificial composite - ferrite pellets in epoxy resin [34]. The appearance of a wave dispersion “forbidden” by the classical theory of elasticity can be explained, in particular, by the influence of rotational modes. Moreover, A.I. Potapov and V.M. Rodyushkin [35] experimentally observed the transfer of momentum in a microstructured material with the velocity that is distinct from the longitudinal wave velocity. A clear separation of the impact pulse into two components was observed in this case. This fact indicates that pulse is carried by two types of oscillations differing from each other in velocity. Nevertheless, still nobody could observe experimentally the propagation of rotational waves in “laboratory conditions”.

It is probably, such a failure is explained by the fact that in “ordinary” solids the rotational interactions of grains with each other are rather small in comparison with other interactions. This hypothesis is indirectly confirmed, for example, in [26–27], where numerical estimates of

the threshold frequency of the rotational mode of oscillations in some crystals are performed. In particular, in these papers it is shown that for grain sizes of the order of 100 nm for crystals with cubic and hexagonal symmetry, the threshold frequency of rotational waves lies in the range $10^{10} - 10^{11} \text{ sec}^{-1}$. At the same time, with a significant growing of the particle size of the medium, the threshold frequency of rotational waves substantially decreases, i.e. the range of propagation of rotational waves is substantially expanded.

The mentioned above data testify that for a more complete and comprehensive understanding of the rotational mechanics of the particles of a medium, first of all, it is necessary to use such models, which elementary volumes are sufficiently large and, hence, their particles have enough large moments of inertia. Such media include our planet Earth, which elements are geophysical blocks, tectonic plates and geological structures.

5. The role of rotational factor in geodynamics

Many geologists (A.P. Karpinsky, D.I. Mushketov, I.S. Shatsky, B.L. Lichkov, P.S. Voronov, A.L. Yanshin, V.E. Khain et al.) wrote about the importance of rotational movements and their relationship with the stressed state of the Earth [36, p. 11]. Geodynamic models, based on ideas about the importance of rotational movements, began to be developed in the late 1950s (see the works of N.V. Stovas [37], K.F. Tyapkin and M.M. Dovbnich [38], etc.). In these works, the Earth was represented as a single monolithic body, the calculation of stresses in which was carried out taking into account the rotation of the Earth.

Recent data of geological and geophysical research argue that the Earth's crust consists of non-point particles-blocks that are able to rotate. Thus, according to [39-40], the state of the Earth's crust is determined by the "inner motion potential" [41] and "self-energy" [42]. Within the scope of the mechanical concept, motion with such properties can occur only under the influence of own angular momentums of the blocks [41], in fact, their spins. The Earth's rotation around its axis with angular velocity Ω and the rotational movements of crust blocks provided by "own moments" $J\Omega$, where J is the moment of inertia of the spherical block, play an important role in geodynamics.

So, geophysical observations made during a long time interval enabled one, for example, to formulate a conclusion that Easter Island ($300 \times 400 \text{ km}^2$) in the Pacific Ocean for 5 million years (it is time of its existence) has turned almost by 90° [43] that corresponds to the angular velocity $0.5\pi \text{ rad} / 5 \cdot 10^6 \text{ years} \approx 3 \cdot 10^{-7} \text{ rad/year}$. Moreover, Siberian platform performs a rather complex motion as a rigid plate. In the period 2.5-1 billion years ago it was located, mainly, in the equatorial and low northern latitudes, performing quasi-oscillation rotations relative to the meridian with amplitude up to 45° , whereas in the period 1.6-1 billion years ago it turned counterclockwise at the angle of about 90° [44].

Recently, rotational waves are increasingly being studied in problems of geodynamics – one of the branches of the Earth sciences, which "elementary" structures reach the sizes of a planetary scale. Thus, V.N. Nikolayevskiy studied nonlinear interactions of longitudinal waves and rotational waves in the framework of the Cosserat model applying to seismoacoustic and geodynamic problems [45]. In the framework of a gradient-consistent model of a medium with complex structure, he attempted to explain an ultrasound generation during the propagation of seismic waves. Considering the lithosphere of the Earth in the framework of the Cosserat continuum, V.N. Nikolaevsky and his co-authors simulated a lot of geodynamic wave motions observed on the surface of the Earth, including global tectonic waves having, apparently, a rotational nature [46].

As an alternative to the Cosserat theory, A.V. Vikulin with his co-authors developed a "rotational" approach to solving geodynamic problems [47-48]. This approach is based on the following assumptions: an elementary part of the rotating solid body – the Earth's crust block – is, first of all, a rigid non-deformable volume; secondly, its motion occurs under the action of

its own moment; thirdly, such a motion leads to change of the stress state of the crust surrounding the block. In this model, in contrast to the Cosserat theory, the stress tensor is symmetric and the rotational motions of a block generating its own elastic field and interacting with the intrinsic elastic fields of other equal-sized blocks of a chain are described by the sine-Gordon equation in the dimensionless form [48]:

$$\frac{\partial^2 \varphi}{\partial t^2} - \frac{\partial^2 \varphi}{\partial x^2} - \sin \varphi = 0. \quad (3)$$

In the framework of this rotational model it is possible to describe the whole range of geodynamic velocities of rotational waves that are typical both for geophysical and geological – processes from slow rotational waves ($\leq 10^{-2}$ sm/sec) characterizing redistribution of tectonic stresses up to fast seismic waves (1-10 km/sec) [47-48].

6. Conclusions

In fact, a medium consists, as a rule, of relatively large particles that are able to rotate with respect to each other. Account of rotational motions of particles of a medium is necessary when either high-frequency wave processes are researched or a medium consisting of large rigid bodies is considered (especially it concerns geomedia). At present, rotational movements of blocks of the Earth's crust are no longer a hypothesis, but an experimentally established fact that is confirmed by lots of data obtained by various methods and by different groups of researchers in many regions of the Earth. And in geodynamics there is majority of experimental confirmations of the existence of rotational waves that were mathematically described back in 1909 by the Cosserat brothers.

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